Chapter 21

1. Eq. 21-1 gives Coulomb's Law, $F = k\frac{|q_1||q_2|}{r^2}$, which we solve for the distance:

$$r = \sqrt{\frac{k\,|q_1||q_2|}{F}} = \sqrt{\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(26.0\times10^{-6}\,\text{C}\right)\left(47.0\times10^{-6}\,\text{C}\right)}{5.70\,\text{N}}} = 1.39\,\text{m}.$$

2. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2 a_2 = m_1 a_1 \Rightarrow m_2 = \frac{(6.3\times 10^{-7}\,\text{kg})(7.0\,\text{m/s}^2)}{9.0\,\text{m/s}^2} = 4.9\times 10^{-7}\,\text{kg}.$$

(b) The magnitude of the (only) force on particle 1 is

$$F = m_1 a_1 = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{|q|^2}{(0.0032\text{ m})^2}.$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1 \times 10^{-11}$ C.

3. The magnitude of the mutual force of attraction at r = 0.120 m is

$$F = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(3.00\times10^{-6}\,\mathrm{C}\right)\left(1.50\times10^{-6}\,\mathrm{C}\right)}{(0.120\text{ m})^2} = 2.81\,\mathrm{N}.$$

4. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere (q) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2 each having charge q and experiencing a mutual repulsive force $F = kq^2 / r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2, a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}k\frac{q^2}{r^2} = \frac{3}{8}F \quad \Rightarrow \quad \frac{F'}{F} = \frac{3}{8} = 0.375.$$

5. The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

where r is the distance between the charges. We want the value of q that maximizes the function $f(q) = q(Q - q)$. Setting the derivative dF/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

6. The unit Ampere is discussed in §21-4. Using i for current, the charge transferred is

$$q = it = \left(2.5\times10^{4}\,\mathrm{A}\right)\left(20\times10^{-6}\,\mathrm{s}\right) = 0.50\ \mathrm{C}.$$

7. We assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then, the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = -k\frac{q_1 q_2}{r^2}$$

where r = 0.500 m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\frac{(q_1+q_2)^2}{4r^2}.$$

We solve the two force equations simultaneously for q_1 and q_2. The first gives the product

$$q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500\,\text{m})^2\,(0.108\,\text{N})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00\times10^{-12}\,\text{C}^2\,,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\text{m})\sqrt{\frac{0.0360\,\text{N}}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 2.00\times10^{-6}\,\text{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-\left(3.00\times10^{-12}\,\text{C}^2\right)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\text{C}^2}{q_1} = 2.00\times10^{-6}\,\text{C}.$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - \left(2.00\times10^{-6}\,\text{C}\right)q_1 - 3.00\times10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00\times10^{-6}\ \text{C} \pm \sqrt{\left(-2.00\times10^{-6}\ \text{C}\right)^2 - 4\left(-3.00\times10^{-12}\ \text{C}^2\right)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}$ C, and if the negative sign is used, $q_1 = -1.00\times10^{-6}\ \text{C}$.

(a) Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}$ C, we get $q_2 = -1.00\times10^{-6}\ \text{C}$.

(b) If we instead work with the $q_1 = -1.00 \times 10^{-6}$ C root, then we find $q_2 = 3.00\times10^{-6}\ \text{C}$.

Note that since the spheres are identical, the solutions are essentially the same: one sphere originally had charge -1.00×10^{-6} C and the other had charge $+3.00 \times 10^{-6}$ C.

What if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}$ C on one sphere and a charge of -3.00×10^{-6} C on the other also satisfies the conditions of the problem.

8. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice).

(a) The *x*-component of the force experienced by $q_1 = Q$ is

$$F_{1x} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{(Q)(Q)}{\left(\sqrt{2}a\right)^2}\cos 45° + \frac{(|q|)\,(Q)}{a^2}\right) = \frac{Q|q|}{4\pi\varepsilon_0 a^2}\left(-\frac{Q/|q|}{2\sqrt{2}}+1\right)$$

which (upon requiring $F_{1x} = 0$) leads to $Q/|q| = 2\sqrt{2}$, or $Q/q = -2\sqrt{2} = -2.83$.

(b) The *y*-component of the net force on $q_2 = q$ is

$$F_{2y} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q|^2}{\left(\sqrt{2}a\right)^2}\sin 45° - \frac{(|q|)\,(Q)}{a^2}\right) = \frac{|q|^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2\sqrt{2}} - \frac{Q}{|q|}\right)$$

which (if we demand $F_{2y} = 0$) leads to $Q/q = -1/2\sqrt{2}$. The result is inconsistent with that obtained in part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces present are given by Eq. 21-1.

9. The force experienced by q_3 is

$$\vec{F}_3=\vec{F}_{31}+\vec{F}_{32}+\vec{F}_{34}=\frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{\mathrm{j}}+\frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45^\circ\hat{\mathrm{i}}+\sin 45^\circ\hat{\mathrm{j}})+\frac{|q_3||q_4|}{a^2}\hat{\mathrm{i}}\right)$$

(a) Therefore, the x-component of the resultant force on q_3 is

$$F_{3x}=\frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}}+|q_4|\right)=\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(\frac{1}{2\sqrt{2}}+2\right)=0.17\,\mathrm{N}.$$

(b) Similarly, the y-component of the net force on q_3 is

$$F_{3y}=\frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1|+\frac{|q_2|}{2\sqrt{2}}\right)=\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(-1+\frac{1}{2\sqrt{2}}\right)=-0.046\,\mathrm{N}.$$

10. (a) The individual force magnitudes (acting on Q) are, by Eq. 21-1,

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-a/2)^2}$$

which leads to $|q_1| = 9.0\,|q_2|$. Since Q is located between q_1 and q_2, we conclude q_1 and q_2 are like-sign. Consequently, $q_1/q_2 = 9.0$.

(b) Now we have

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-3a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-3a/2)^2}$$

which yields $|q_1| = 25\,|q_2|$. Now, Q is not located between q_1 and q_2, one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1/q_2 = -25$.

11. With rightwards positive, the net force on q_3 is

$$F_3 = F_{13} + F_{23} = k\frac{q_1 q_3}{\left(L_{12}+L_{23}\right)^2} + k\frac{q_2 q_3}{L_{23}^2}\,.$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero $L_{23}= L_{12}$ and canceling k, q_3 and L_{12} leads to

$$\frac{q_1}{4.00} + q_2 = 0 \quad \Rightarrow \quad \frac{q_1}{q_2} = -\,4.00.$$

12. As a result of the first action, both sphere W and sphere A possess charge $\frac{1}{2}q_A$, where q_A is the initial charge of sphere A. As a result of the second action, sphere W has charge

$$\frac{1}{2}\left(\frac{q_A}{2} - 32e\right).$$

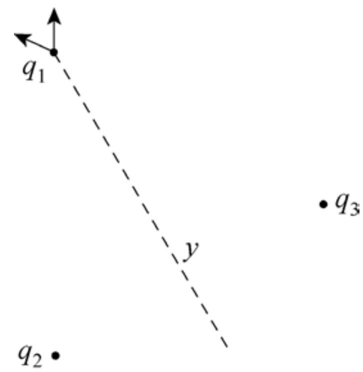
As a result of the final action, sphere W now has charge equal to

$$\frac{1}{2}\left[\frac{1}{2}\left(\frac{q_A}{2} - 32e\right) + 48e\right].$$

Setting this final expression equal to $+18e$ as required by the problem leads (after a couple of algebra steps) to the answer: $q_A = +16e$.

13. (a) Eq. 21-1 gives

$$F_{12} = k\frac{q_1 q_2}{d^2} = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(20.0\times10^{-6}\,\mathrm{C}\right)^2}{(1.50\,\mathrm{m})^2} = 1.60\,\mathrm{N}.$$



(b) On the right, a force diagram is shown as well as our choice of *y* axis (the dashed line).

The *y* axis is meant to bisect the line between q_2 and q_3 in order to make use of the symmetry in the problem (equilateral triangle of side length *d*, equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$\left|F_y\right| = 2\left(k\frac{q^2}{d^2}\right)\cos 30° = 2\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(20.0\times10^{-6}\,\mathrm{C}\right)^2}{(1.50\,\mathrm{m})^2}\cos 30° = 2.77\,\mathrm{N}\,.$$

14. (a) According to the graph, when q_3 is very close to q_1 (at which point we can consider the force exerted by particle 1 on 3 to dominate) there is a (large) force in the positive x direction. This is a repulsive force, then, so we conclude q_1 has the same sign as q_3. Thus, q_3 is a positive-valued charge.

(b) Since the graph crosses zero and particle 3 is *between* the others, q_1 must have the same sign as q_2, which means it is also positive-valued. We note that it crosses zero at r = 0.020 m (which is a distance d = 0.060 m from q_2). Using Coulomb's law at that point, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_3q_2}{d^2} \quad \Rightarrow \quad q_2 = \left(\frac{d}{r}\right)^2 q_1 = \left(\frac{0.060\text{ m}}{0.020\text{ m}}\right)^2 q_1 = 9.0\, q_1\,,$$

or q_2/q_1 = 9.0.

15. (a) There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis which is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3. Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2, where the net force on q_3 has magnitude

$$\left| k\frac{|q_1 q_3|}{L_0^2} - k\frac{|q_2 q_3|}{(L+L_0)^2} \right|$$

with L = 10 cm and L_0 is assumed to be *positive*. We set this equal to zero, as required by the problem, and cancel k and q_3. Thus, we obtain

$$\frac{|q_1|}{L_0^2} - \frac{|q_2|}{(L+L_0)^2} = 0 \;\Rightarrow\; \left(\frac{L+L_0}{L_0}\right)^2 = \left|\frac{q_2}{q_1}\right| = \left|\frac{-3.0\ \mu\text{C}}{+1.0\ \mu\text{C}}\right| = 3.0$$

which yields (after taking the square root)

$$\frac{L+L_0}{L_0} = \sqrt{3} \;\Rightarrow\; L_0 = \frac{L}{\sqrt{3}-1} = \frac{10\ \text{cm}}{\sqrt{3}-1} \approx 14\,\text{cm}$$

for the distance between q_3 and q_1. That is, q_3 should be placed at $x = -14$ cm along the x-axis.

(b) As stated above, $y = 0$.

16. Since the forces involved are proportional to q, we see that the essential difference between the two situations is $F_a \propto q_B + q_C$ (when those two charges are on the same side) versus $F_b \propto -q_B + q_C$ (when they are on opposite sides). Setting up ratios, we have

$$\frac{F_a}{F_b} = \frac{q_B + q_C}{-q_B + q_C} \quad \Rightarrow \quad \frac{2.014\times10^{-23}\ \text{N}}{-2.877\times10^{-24}\ \text{N}} = \frac{1+q_C/q_B}{-1+q_C/q_B} \ .$$

After noting that the ratio on the left hand side is very close to – 7, then, after a couple of algebra steps, we are led to

$$\frac{q_C}{q_B} = \frac{7+1}{7-1} = \frac{8}{6} = 1.333.$$

17. (a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.020 \text{ m} - 0.035 \text{ m})^2 + (0.015 \text{ m} - 0.005 \text{ m})^2} = 0.056 \text{ m}.$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k\frac{|q_1 q_2|}{r_{12}^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-6}\text{C})(4.0\times10^{-6}\text{C})}{(0.056 \text{ m})^2} = 35 \text{ N}.$$

(b) The vector $\vec{F}_{21}$ is directed towards q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \tan^{-1}\left(\frac{1.5 \text{ cm} - 0.5 \text{ cm}}{-2.0 \text{ cm} - 3.5 \text{ cm}}\right) = -10.3° \approx -10°.$$

(c) Let the third charge be located at (x_3, y_3), a distance r from q_2. We note that q_1, q_2 and q_3 must be collinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1, since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k\frac{|q_2 q_3|}{r^2} = k\frac{|q_1 q_2|}{r_{12}^2} \quad \Rightarrow \quad r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645 \text{ m} = 6.45 \text{ cm}.$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0$ cm – (6.45 cm) cos(–10°) = –8.4 cm,

(d) and $y_3 = y_2 - r\sin\theta = 1.5$ cm – (6.45 cm) sin(–10°) = 2.7 cm.

18. (a) For the net force to be in the $+x$ direction, the y components of the individual forces must cancel. The angle of the force exerted by the q_1 = 40 μC charge on $q_3 = 20\mu C$ is 45°, and the angle of force exerted on q_3 by Q is at $-\theta$ where

$$\theta = \tan^{-1}\left(\frac{2.0\text{ cm}}{3.0\text{ cm}}\right) = 33.7°.$$

Therefore, cancellation of y components requires

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2}\text{ m}\right)^2}\sin 45° = k\frac{|Q|\,q_3}{\left(\sqrt{(0.030\text{ m})^2 + (0.020\text{ m})^2}\right)^2}\sin\theta$$

from which we obtain $|Q|$ = 83 μC. Charge Q is "pulling" on q_3, so (since $q_3 > 0$) we conclude Q = –83 μC.

(b) Now, we require that the x components cancel, and we note that in this case, the angle of force on q_3 exerted by Q is $+\theta$ (it is repulsive, and Q is positive-valued). Therefore,

$$k\frac{q_1\, q_3}{\left(0.02\sqrt{2}\text{ m}\right)^2}\cos 45° = k\frac{Qq_3}{\left(\sqrt{(0.030\text{ m})^2 + (0.020\text{ m})^2}\right)^2}\cos\theta$$

from which we obtain Q = 55.2 μC ≈ 55 μC .

19. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. The third charge q_3 must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_3 could not be in equilibrium. Suppose q_3 is at a distance x from q, and $L - x$ from 4.00q. The force acting on it is then given by

$$F_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} - \frac{4qq_3}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_3 = 0$ and solve for x. Canceling common factors yields $1/x^2 = 4/(L-x)^2$ and taking the square root yields $1/x = 2/(L-x)$. The solution is $x = L/3$. With $L = 9.00$ cm, we have $x = 3.00$ cm.

(b) Similarly, the y coordinate of q_3 is $y = 0$.

(c) The force on q is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4.00q^2}{L^2}\right).$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_3:

$$q_3 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q \quad \Rightarrow \quad \frac{q_3}{q} = -\frac{4}{9} = -0.444$$

where $x = L/3$ is used. Note that we may easily verify that the force on 4.00q also vanishes:

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) = 0\,.$$

20. (a) We note that cos(30º) $= \frac{1}{2}\sqrt{3}$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$. We net force on q_1 due to the two charges q_3 and q_4 (with $|q_3| = |q_4| = 1.60 \times 10^{-19}$ C) on the *y* axis has magnitude

$$2\frac{|q_1q_3|}{4\pi\varepsilon_0 r^2}\cos(30°) = \frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2}.$$

This must be set equal to the magnitude of the force exerted on q_1 by $q_2 = 8.00 \times 10^{-19}$ C $= 5.00\,|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1q_2|}{4\pi\varepsilon_0 (D+d)^2} \quad\Rightarrow\quad D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245\,d\,.$$

Given $d = 2.00$ cm, this then leads to $D = 1.92$ cm.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the *y* axis. To offset this, the force exerted by q_2 must be made stronger, so that it must be brought closer to q_1 (keep in mind that Coulomb's law is *inversely* proportional to distance-squared). Thus, *D* must be decreased.

21. If θ is the angle between the force and the x-axis, then

$$\cos\theta = \frac{x}{\sqrt{x^2 + d^2}} .$$

We note that, due to the symmetry in the problem, there is no y component to the net force on the third particle. Thus, F represents the magnitude of force exerted by q_1 or q_2 on q_3. Let $e = +1.60 \times 10^{-19}$ C, then $q_1 = q_2 = +2e$ and $q_3 = 4.0e$ and we have

$$F_{\text{net}} = 2F\cos\theta = \frac{2(2e)(4e)}{4\pi\varepsilon_o (x^2 + d^2)} \frac{x}{\sqrt{x^2 + d^2}} = \frac{4e^2 x}{\pi\varepsilon_o (x^2 + d^2)^{3/2}} .$$

(a) To find where the force is at an extremum, we can set the derivative of this expression equal to zero and solve for x, but it is good in any case to graph the function for a fuller understanding of its behavior – and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum (and will be presented in part (b)) and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x = 0$, which is the smallest value of the net force in the interval 5.0 m $\geq x \geq 0$.

(b) The maximum is found to be at $x = d/\sqrt{2}$ or roughly 12 cm.

(c) The value of the net force at $x = 0$ is $F_{\text{net}} = 0$.

(d) The value of the net force at $x = d/\sqrt{2}$ is $F_{\text{net}} = 4.9 \times 10^{-26}$ N.

22. We note that the problem is examining the force <u>on</u> charge *A*, so that the respective distances (involved in the Coulomb force expressions) between *B* and *A*, and between *C* and *A*, do not change as particle *B* is moved along its circular path. We focus on the endpoints ($\theta = 0°$ and $180°$) of each graph, since they represent cases where the forces (on *A*) due to *B* and *C* are either parallel or antiparallel (yielding maximum or minimum force magnitudes, respectively). We note, too, that since Coulomb's law is inversely proportional to r^2 then the (if, say, the charges were all the same) force due to *C* would be one-fourth as big as that due to *B* (since *C* is twice as far away from *A*). The charges, it turns out, are not the same, so there is also a factor of the charge ratio ξ (the charge of *C* divided by the charge of *B*), as well as the aforementioned ¼ factor. That is, the force exerted by *C* is, by Coulomb's law equal to $\pm\frac{1}{4}\xi$ multiplied by the force exerted by *B*.

(a) The maximum force is $2F_0$ and occurs when $\theta = 180°$ (*B* is to the left of *A*, while *C* is the right of *A*). We choose the minus sign and write

$$2\,F_0 = (1 - \tfrac{1}{4}\xi)\,F_0 \quad \Rightarrow \quad \xi = -4\,.$$

One way to think of the minus sign choice is $\cos(180°) = -1$. This is certainly consistent with the minimum force ratio (zero) at $\theta = 0°$ since that would also imply

$$0 = 1 + \tfrac{1}{4}\xi \quad \Rightarrow \quad \xi = -4\,.$$

(b) The ratio of maximum to minimum forces is $1.25/0.75 = 5/3$ in this case, which implies

$$\frac{5}{3} = \frac{1 + \frac{1}{4}\xi}{1 - \frac{1}{4}\xi} \quad \Rightarrow \quad \xi = 16\,.$$

Of course, this could also be figured as illustrated in part (a), looking at the maximum force ratio by itself and solving, or looking at the minimum force ratio (¾) at $\theta = 180°$ and solving for ξ.

23. The charge dq within a thin shell of thickness dr is $dq = \rho dV = \rho A dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b \int_{r_1}^{r_2} r\, dr = 2\pi b\left(r_2^2 - r_1^2\right).$$

With $b = 3.0\ \mu\text{C/m}^2$, $r_2 = 0.06$ m and $r_1 = 0.04$ m, we obtain $q = 0.038\ \mu\text{C} = 3.8 \times 10^{-8}$ C.

24. The magnitude of the force is

$$F = k\frac{e^2}{r^2} = \left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{\left(2.82\times10^{-10}\ \mathrm{m}\right)^2} = 2.89\times10^{-9}\ \mathrm{N}.$$

25. (a) The magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2}$$

where q is the charge on either of them and r is the distance between them. We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = \left(5.0\times10^{-10}\ \text{m}\right)\sqrt{\frac{3.7\times10^{-9}\ \text{N}}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 3.2\times10^{-19}\ \text{C}.$$

(b) Let n be the number of electrons missing from each ion. Then, $ne = q$, or

$$n = \frac{q}{e} = \frac{3.2\times10^{-9}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 2.$$

26. Keeping in mind that an Ampere is a Coulomb per second (1 A = 1 C/s), and that a minute is 60 seconds, the charge (in absolute value) that passes through the chest is

$$|q| = (0.300 \text{ C/s})(120 \text{ s}) = 36.0 \text{ C}.$$

This charge consists of n electrons (each of which has an absolute value of charge equal to e). Thus,

$$n = \frac{|q|}{e} = \frac{36.0 \text{ C}}{1.60 \text{ x } 10^{-19} \text{ C}} = 2.25 \times 10^{20}.$$

27. Eq. 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{1.0 \times 10^{-7}\,\text{C}}{1.6 \times 10^{-19}\,\text{C}} = 6.3 \times 10^{11}.$$

28. (a) Eq. 21-1 gives

$$F = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.00\times10^{-16}\,\mathrm{C}\right)^2}{\left(1.00\times10^{-2}\ \mathrm{m}\right)^2} = 8.99\times10^{-19}\,\mathrm{N}.$$

(b) If n is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00\times10^{-16}\ \mathrm{C}}{1.60\times10^{-19}\ \mathrm{C}} = 625.$$

29. The unit Ampere is discussed in §21-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi\,(6.37 \times 10^6\ \text{m})^2 = 5.1 \times 10^{14}\ \text{m}^2$ would be

$$i = \left(5.1\times 10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6\times 10^{-19}\ \text{C/proton}\right) = 0.122\ \text{A}.$$

30. Since the graph crosses zero, q_1 must be positive-valued: $q_1 = +8.00e$. We note that it crosses zero at $r = 0.40$ m. Now the asymptotic value of the force yields the magnitude and sign of q_2:

$$\frac{q_1\, q_2}{4\pi\varepsilon_o\, r^2} = F \quad \Rightarrow \quad q_2 = \left(\frac{1.5 \text{ x } 10^{-25}}{k\, q_1}\right) r^2 \;=\; 2.086 \times 10^{-18}\,\text{C} \;=\; 13e\,.$$

31. The volume of 250 cm^3 corresponds to a mass of 250 g since the density of water is 1.0 g/cm^3. This mass corresponds to 250/18 = 14 moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O, so

$$Q = 14 N_A q = 14\left(6.02\times10^{23}\right)(10)\left(1.60\times10^{-19}\,\text{C}\right) = 1.3\times10^{7}\,\text{C}.$$

32. (a) Let x be the distance between particle 1 and particle 3. Thus, the distance between particle 3 and particle 2 is $L - x$. Both particles exert leftward forces on q_3 (so long as it is on the line <u>between</u> them), so the magnitude of the net force on q_3 is

$$F_{\text{net}} = |\vec{F}_{13}| + |\vec{F}_{23}| = \frac{|q_1\, q_3|}{4\pi\varepsilon_o x^2} + \frac{|q_2\, q_3|}{4\pi\varepsilon_o (L-x)^2} = \frac{e^2}{\pi\varepsilon_o}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)$$

with the values of the charges (stated in the problem) plugged in. Finding the value of x which minimizes this expression leads to $x = \frac{1}{4} L$. Thus, $x = 2.00$ cm.

(b) Substituting $x = \frac{1}{4} L$ back into the expression for the net force magnitude and using the standard value for e leads to $F_{\text{net}} = 9.21 \times 10^{-24}$ N.

33. (a) We note that $\tan(30°) = 1/\sqrt{3}$. In the initial (highly symmetrical) configuration, the net force on the central bead is in the $-y$ direction and has magnitude $3F$ where F is the Coulomb's law force of one bead on another at distance d = 10 cm. This is due to the fact that the forces exerted on the central bead (in the initial situation) by the beads on the x axis cancel each other; also, the force exerted "downward" by bead 4 on the central bead is four times larger than the "upward" force exerted by bead 2. This net force along the y axis does not change as bead 1 is now moved, though there is now a nonzero x-component F_x. The components are now related by

$$\tan(30°) = \frac{F_x}{F_y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{F_x}{3F}$$

which implies $F_x = \sqrt{3}\,F$. Now, bead 3 exerts a "leftward" force of magnitude F on the central bead, while bead 1 exerts a "rightward" force of magnitude F'. Therefore,

$$F' - F = \sqrt{3}\,F. \quad \Rightarrow \quad F' = (\sqrt{3} + 1)\,F\,.$$

The fact that Coulomb's law depends inversely on distance-squared then implies

$$r^2 = \frac{d^2}{\sqrt{3}+1} \Rightarrow r = \frac{d}{\sqrt{\sqrt{3}+1}} = \frac{10\text{ cm}}{\sqrt{\sqrt{3}+1}} = \frac{10\text{ cm}}{1.65} = 6.05\text{ cm}$$

where r is the distance between bead 1 and the central bead. This corresponds to $x = -6.05\text{ cm}$.

(b) To regain the condition of high symmetry (in particular, the cancellation of x-components) bead 3 must be moved closer to the central bead so that it, too, is the distance r (as calculated in part(a)) away from it.

34. Let d be the vertical distance from the coordinate origin to $q_3 = -q$ and $q_4 = -q$ on the $+y$ axis, where the symbol q is assumed to be a positive value. Similarly, d is the (positive) distance from the origin $q_4 = -$ on the $-y$ axis. If we take each angle θ in the figure to be positive, then we have $\tan\theta = d/R$ and $\cos\theta = R/r$ (where r is the dashed line distance shown in the figure). The problem asks us to consider θ to be a variable in the sense that, once the charges on the x axis are fixed in place (which determines R), d can then be arranged to some multiple of R, since $d = R\tan\theta$. The aim of this exploration is to show that if q is bounded then θ (and thus d) is also bounded.

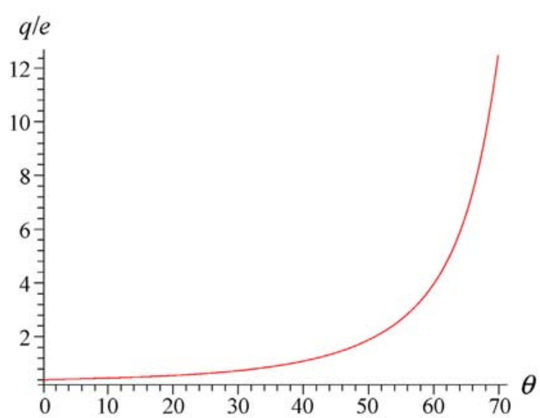
From symmetry, we see that there is no net force in the vertical direction on $q_2 = -e$ sitting at a distance R to the left of the coordinate origin. We note that the net x force caused by q_3 and q_4 on the y axis will have a magnitude equal to

$$2\frac{qe}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{2qe\cos\theta}{4\pi\varepsilon_0 (R/\cos\theta)^2} = \frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} \ .$$

Consequently, to achieve a zero net force along the x axis, the above expression must equal the magnitude of the repulsive force exerted on q_2 by $q_1 = -e$. Thus,

$$\frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} = \frac{e^2}{4\pi\varepsilon_0 R^2} \quad\Rightarrow\quad q = \frac{e}{2\cos^3\theta} \, .$$

Below we plot q/e as a function of the angle (in degrees):



The graph suggests that $q/e < 5$ for $\theta < 60°$, roughly. We can be more precise by solving the above equation. The requirement that $q \le 5e$ leads to

$$\frac{e}{2\cos^3\theta} \le 5e \quad\Rightarrow\quad \frac{1}{(10)^{1/3}} \le \cos\theta$$

which yields $\theta \le 62.34°$. The problem asks for "physically possible values," and it is reasonable to suppose that only positive-integer-multiple values of e are allowed for q. If we let $q = ne$, for $n = 1 \ldots 5$, then θ_N will be found by taking the inverse cosine of the cube root of $(1/2n)$.

(a) The smallest value of angle is $\theta_1 = 37.5°$ (or 0.654 rad).

(b) The second smallest value of angle is $\theta_2 = 50.95°$ (or 0.889 rad).

(c) The third smallest value of angle is $\theta_3 = 56.6°$ (or 0.988 rad).

35. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, we superpose charge *–e* at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

$$F = k\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(3/4)\left(0.40\times10^{-9}\ \text{m}\right)^2} = 1.9\times10^{-9}\ \text{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

36. (a) Since the proton is positively charged, the emitted particle must be a positron (as opposed to the negatively charged electron) in accordance with the law of charge conservation.

(b) In this case, the initial state had zero charge (the neutron is neutral), so the sum of charges in the final state must be zero. Since there is a proton in the final state, there should also be an electron (as opposed to a positron) so that $\Sigma q = 0$.

37. None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

(a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and 9 – 4 = 5 neutrons, so X has 1 + 4 = 5 protons, 1 + 4 = 5 electrons, and 0 + 5 – 1 = 4 neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of 5 + 4 = 9 g/mol: ^{9}B.

(b) ^{12}C has 6 protons, 6 electrons, and 12 – 6 = 6 neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has 6 + 1 = 7 protons, 6 + 1 = 7 electrons, and 6 + 0 = 6 neutrons. It must be nitrogen with a molar mass of 7 + 6 = 13 g/mol: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and 15 – 7 = 8 neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and 4 – 2 = 2 neutrons; so X has 7 + 1 – 2 = 6 protons, 6 electrons, and 8 + 0 – 2 = 6 neutrons. It must be carbon with a molar mass of 6 + 6 = 12: ^{12}C.

38. Let $\vec{F}_{12}$ denotes the force <u>on</u> q_1 exerted <u>by</u> q_2 and F_{12} be its magnitude.

(a) We consider the net force on q_1. $\vec{F}_{12}$ points in the $+x$ direction since q_1 is attracted to q_2. $\vec{F}_{13}$ and $\vec{F}_{14}$ both point in the $-x$ direction since q_1 is repelled by q_3 and q_4. Thus, using d = 0.0200 m, the net force is

$$F_1 = F_{12} - F_{13} - F_{14} = \frac{2e|-e|}{4\pi\varepsilon_0 d^2} - \frac{(2e)(e)}{4\pi\varepsilon_0 (2d)^2} - \frac{(2e)(4e)}{4\pi\varepsilon_0 (3d)^2} = \frac{11}{18}\frac{e^2}{4\pi\varepsilon_0 d^2}$$
$$= \frac{11}{18}\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(2.00\times10^{-2}\,\text{m}\right)^2} = 3.52\times10^{-25}\ \text{N},$$

or $\vec{F}_1 = (3.52\times10^{-25}\ \text{N})\hat{\text{i}}$.

(b) We now consider the net force on q_2. We note that $\vec{F}_{21} = -\vec{F}_{12}$ points in the $-x$ direction, and $\vec{F}_{23}$ and $\vec{F}_{24}$ both point in the $+x$ direction. The net force is

$$F_{23} + F_{24} - F_{21} = \frac{4e|-e|}{4\pi\varepsilon_0 (2d)^2} + \frac{e|-e|}{4\pi\varepsilon_0 d^2} - \frac{2e|-e|}{4\pi\varepsilon_0 d^2} = 0$$

39. If θ is the angle between the force and the x axis, then

$$\cos\theta = \frac{d_2}{\sqrt{d_1^2 + d_2^2}} .$$

Thus, using Coulomb's law for F, we have

$$F_x = F\cos\theta = \frac{q_1 q_2}{4\pi\varepsilon_o (d_1^2 + d_2^2)} \frac{d_2}{\sqrt{d_1^2 + d_2^2}} = 1.31 \times 10^{-22}\ \text{N} .$$

40. For the Coulomb force to be sufficient for circular motion at that distance (where r = 0.200 m and the acceleration needed for circular motion is $a = v^2/r$) the following equality is required:

$$\frac{Qq}{4\pi\varepsilon_0 r^2} = -\frac{mv^2}{r} .$$

With $q = 4.00 \times 10^{-6}$ C, m = 0.000800 kg, v = 50.0 m/s, this leads to

$$Q = -\frac{4\pi\varepsilon_0 rmv^2}{q} = -\frac{(0.200 \text{ m})(8.00\times10^{-4} \text{ kg})(50.0 \text{ m/s})^2}{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.00\times10^{-6}\text{C})} = -1.11\times10^{-5}\text{C} .$$

41. The charge dq within a thin section of the rod (of thickness dx) is $\rho\, A\, dx$ where $A = 4.00\times10^{-4}\,\text{m}^2$ and ρ is the charge per unit volume. The number of (excess) electrons in the rod (of length L = 2.00 m) is $n = q/(-e)$ where e is given in Eq. 21-12.

(a) In the case where $\rho = -\,4.00 \times 10^{-6}$ C/m^3, we have

$$n = \frac{q}{-e} = \frac{\rho A}{-e}\int_0^L dx = \frac{|\rho|\, AL}{e} = \ 2.00\times10^{10}\ .$$

(b) With $\rho = bx^2$ ($b = -2.00 \times 10^{-6}$ C/m^5) we obtain

$$n = \frac{b\, A}{-e}\int_0^L x^2\, dx = \frac{|\, b\,|\, A\, L^3}{3e} = 1.33\times10^{10}.$$

42. Let q_1 be the charge of one part and q_2 that of the other part; thus, $q_1 + q_2 = Q = 6.0$ μC. The repulsive force between them is given by Coulomb's law:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(Q - q_1)}{4\pi\varepsilon_0 r^2} \ .$$

If we maximize this expression by taking the derivative with respect to q_1 and setting equal to zero, we find $q_1 = Q/2$, which might have been anticipated (based on symmetry arguments). This implies $q_2 = Q/2$ also. With $r = 0.0030$ m and $Q = 6.0 \times 10^{-6}$ C, we find

$$F = \frac{(Q/2)(Q/2)}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{Q^2}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\,\text{C}\right)^2}{\left(3.00\times10^{-3}\,\text{m}\right)^2} \approx 9.0\times10^3\ \text{N}\,.$$

43. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = (6.02\times10^{23})(2)(1.60\times10^{-19}\ \text{C}) = 1.9\times10^{5}\ \text{C} = 0.19\ \text{MC}.$$

44. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x. The charge Q on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W, at a distance $x - L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2} - W\left(x - \frac{L}{2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2} = 0.$$

The solution for x is

$$x = \frac{L}{2}\left(1 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2 W}\right).$$

(b) If F_N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2} - \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2} - F_N = 0.$$

We solve for h so that $F_N = 0$. The result is

$$h = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}}.$$

45. Coulomb's law gives

$$F = \frac{|q|^2}{4\pi\varepsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(1.60\times10^{-19}\ \mathrm{C})^2}{9(2.6\times10^{-15}\,\mathrm{m})^2} = 3.8\,\mathrm{N}.$$

46. (a) Since q_A = –2.00 nC and q_C = +8.00 nC Eq. 21-4 leads to

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-2.00\times10^{-9}\ \text{C})(8.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

(b) After making contact with each other, both *A* and *B* have a charge of

$$\frac{q_A+q_B}{2} = \left(\frac{-2.00+(-4.00)}{2}\right)\ \text{nC} = -3.00\ \text{nC}.$$

When *B* is grounded its charge is zero. After making contact with *C*, which has a charge of +8.00 nC, *B* acquires a charge of [0 + (–8.00 nC)]/2 = –4.00 nC, which charge *C* has as well. Finally, we have Q_A = –3.00 nC and Q_B = Q_C = –4.00 nC. Therefore,

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 2.70\times10^{-6}\ \text{N}.$$

(c) We also obtain

$$|\vec{F}_{BC}| = \frac{|q_B q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

47. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.00\,\mathrm{C}\right)^2}{\left(1.00\,\mathrm{m}\right)^2} = 8.99\times10^9\ \mathrm{N}.$$

(b) If r = 1000 m, then

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.00\,\mathrm{C}\right)^2}{\left(1.00\times10^3\,\mathrm{m}\right)^2} = 8.99\times10^3\ \mathrm{N}.$$

48. In experiment 1, sphere C first touches sphere A, and they divided up their total charge (Q/2 plus Q) equally between them. Thus, sphere A and sphere C each acquired charge 3Q/4. Then, sphere C touches B and those spheres split up their total charge (3Q/4 plus –Q/4) so that B ends up with charge equal to Q/4. The force of repulsion between A and B is therefore

$$F_1 = k\frac{(3Q/4)(Q/4)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere C first touches B which leaves each of them with charge Q/8. When C next touches A, sphere A is left with charge 9Q/16. Consequently, the force of repulsion between A and B is

$$F_2 = k\frac{(9Q/16)(Q/8)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{(9/16)(1/8)}{(3/4)(1/4)} = 0.375.$$

49. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 1.6 \times 10^{-25}\,\mathrm{C}$. Amplified by a factor of 29 × 3 × 10^{22} as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = \left(29 \times 3 \times 10^{22}\right)\left(1.6 \times 10^{-25}\,\mathrm{C}\right) = 0.14\,\mathrm{C}$$

in a copper penny. Two such pennies, at r = 1.0 m, would therefore experience a very large force. Eq. 21-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7 \times 10^8\,\mathrm{N}.$$

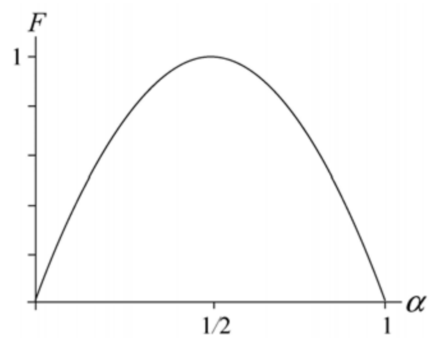
50. Letting $kq^2/r^2 = mg$, we get

$$r = q\sqrt{\frac{k}{mg}} = \left(1.60\times10^{-19}\ \mathrm{C}\right)\sqrt{\frac{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}}{\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)}} = 0.119\,\mathrm{m}.$$

51. The two charges are $q = \alpha Q$ (where α is a pure number presumably less than 1 and greater than zero) and $Q - q = (1 - \alpha)Q$. Thus, Eq. 21-4 gives

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(\alpha Q)((1-\alpha)Q)}{d^2} = \frac{Q^2\alpha(1-\alpha)}{4\pi\varepsilon_0\, d^2}.$$

The graph below, of F versus α, has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{max} = Q^2/16\pi\varepsilon_0\, d^2$.



(a) It is clear that $\alpha = \frac{1}{2}$ = 0.5 gives the maximum value of F.

(b) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\alpha_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.15 \quad \text{and} \quad \alpha_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 0.85.$$

Thus, the smaller value of α is $\alpha_1 = 0.15$,

(c) and the larger value of α is $\alpha_2 = 0.85$.

52. (a) Eq. 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00 \times 10^{-6}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 1.25 \times 10^{13}\ \text{electrons}.$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

53. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = G\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q = \sqrt{4\pi\varepsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(7.36\times10^{22}\ \mathrm{kg}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}}} = 5.7\times10^{13}\ \mathrm{C}.$$

(b) The distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

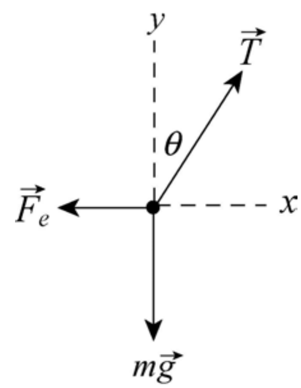
(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$n = \frac{q}{e} = \frac{5.7\times10^{13}\ \mathrm{C}}{1.6\times10^{-19}\ \mathrm{C}} = 3.6\times10^{32}\ \text{ions}.$$

Each ion has a mass of $m_i = 1.67 \times 10^{-27}$ kg, so the total mass needed is

$$m = nm_i = \left(3.6\times10^{32}\right)\left(1.67\times10^{-27}\ \mathrm{kg}\right) = 6.0\times10^{5}\ \mathrm{kg}.$$

54. (a) A force diagram for one of the balls is shown on the right. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T\cos\theta - mg = 0$ and the x component yields $T\sin\theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.



Examination of the geometry of Figure 21-42 leads to

$$\tan\theta = \frac{x/2}{\sqrt{L^2 - (x/2)^2}}.$$

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \quad\Rightarrow\quad x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(b) We solve $x^3 = 2kq^2L/mg$ for the charge (using Eq. 21-5):

$$q = \sqrt{\frac{mgx^3}{2kL}} = \sqrt{\frac{(0.010\,\text{kg})(9.8\,\text{m/s}^2)(0.050\,\text{m})^3}{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.20\,\text{m})}} = \pm 2.4\times10^{-8}\ \text{C}.$$

Thus, the magnitude is $|q| = 2.4\times10^{-8}$ C.

55. (a) If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other.

(b) A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\varepsilon_0 mg}\right]^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = \left(\frac{1}{4}\right)^{1/3} (5.0\text{ cm}) = 3.1\text{ cm}.$$

56. Regarding the forces on q_3 exerted by q_1 and q_2, one must "push" and the other must "pull" in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\frac{|q_1||q_3|}{(L_{12}+L_{23})^2}=k\frac{|q_2||q_3|}{(L_{23})^2}.$$

With $L_{23}=2.00L_{12}$, the above expression simplifies to $\frac{|q_1|}{9}=\frac{|q_2|}{4}$. Therefore, $q_1=-9q_2/4$, or $q_1/q_2=-2.25$.

57. The mass of an electron is m = 9.11 × 10^{-31} kg, so the number of electrons in a collection with total mass M = 75.0 kg is

$$n = \frac{M}{m} = \frac{75.0\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = 8.23\times10^{31}\,\text{electrons}.$$

The total charge of the collection is

$$q = -\,ne = -\left(8.23\times10^{31}\right)\left(1.60\times10^{-19}\,\text{C}\right) = -1.32\times10^{13}\,\text{C}.$$

58. We note that, as result of the fact that the Coulomb force is inversely proportional to r^2, a particle of charge Q which is distance d from the origin will exert a force on some charge q_o at the origin of equal strength as a particle of charge $4Q$ at distance $2d$ would exert on q_o. Therefore, $q_6 = +8e$ on the $-y$ axis could be replaced with a $+2e$ closer to the origin (at half the distance); this would add to the $q_5 = +2e$ already there and produce $+4e$ below the origin which exactly cancels the force due to $q_2 = +4e$ above the origin.

Similarly, $q_4 = +4e$ to the far right could be replaced by a $+e$ at half the distance, which would add to $q_3 = +e$ already there to produce a $+2e$ at distance d to the right of the central charge q_7. The horizontal force due to this $+2e$ is cancelled exactly by that of $q_1 = +2e$ on the $-x$ axis, so that the net force on q_7 is zero.

59. (a) Charge Q_1 = +80 × 10^{-9} C is on the y axis at y = 0.003 m, and charge $Q_2 = +80\times10^{-9}$ C is on the y axis at y = –0.003 m. The force on particle 3 (which has a charge of q = +18 × 10^{-9} C) is due to the vector sum of the repulsive forces from Q_1 and Q_2. In symbols, $\vec{F}_{3\,1} + \vec{F}_{3\,2} = \vec{F}_3$, where

$$|\vec{F}_{31}| = k\frac{q_3|q_1|}{r_{3\,1}^2}\,, \quad |\vec{F}_{3\,2}| = k\frac{q_3 q_2}{r_{3\,2}^2}.$$

Using the Pythagorean theorem, we have r_{31} = r_{32} = 0.005 m. In magnitude-angle notation (particularly convenient if one uses a vector-capable calculator in polar mode), the indicated vector addition becomes

$$\vec{F}_3 = (0.518\angle -37°) + (0.518\angle 37°) = (0.829\angle 0°)\,.$$

Therefore, the net force is $\vec{F}_3 = (0.829 \text{ N})\hat{\text{i}}$.

(b) Switching the sign of Q_2 amounts to reversing the direction of its force on q. Consequently, we have

$$\vec{F}_3 = (0.518\angle -37°) + (0.518\angle -143°) = (0.621\angle -90°)\,.$$

Therefore, the net force is $\vec{F}_3 = -(0.621 \text{ N})\hat{\text{j}}$.

60. The individual force magnitudes are found using Eq. 21-1, with SI units (so $a = 0.02\ \text{m}$) and k as in Eq. 21-5. We use magnitude-angle notation (convenient if one uses a vector-capable calculator in polar mode), listing the forces due to +4.00q, +2.00q, and –2.00q charges:

$$\left(4.60\times10^{-24}\ \angle 180°\right)+\left(2.30\times10^{-24}\ \angle -90°\right)+\left(1.02\times10^{-24}\ \angle -145°\right)=\left(6.16\times10^{-24}\ \angle -152°\right)$$

(a) Therefore, the net force has magnitude 6.16×10^{-24} N.

(b) The direction of the net force is at an angle of –152° (or 208° measured counterclockwise from the +x axis).

61. The magnitude of the net force on the $q = 42 \times 10^{-6}$ C charge is

$$k\frac{q_1\, q}{0.28^2} + k\frac{|q_2|q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9}$ C and $|q_2| = 40 \times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\mathrm{N}}{100 \times 10^3\ \mathrm{m/s^2}} = 2.2 \times 10^{-6}\,\mathrm{kg}.$$

62. For the net force on $q_1 = +Q$ to vanish, the x force component due to $q_2 = q$ must exactly cancel the force of attraction caused by $q_4 = -2Q$. Consequently,

$$\frac{Qq}{4\pi\varepsilon_0 a^2} = \frac{Q|2Q|}{4\pi\varepsilon_0(\sqrt{2}a)^2}\cos 45° = \frac{Q^2}{4\pi\varepsilon_0\sqrt{2}a^2}$$

or $q = Q/\sqrt{2}$. This implies that $q/Q = 1/\sqrt{2} = 0.707$.

63. We are looking for a charge q which, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 21-1, and without loss of generality we assume $q > 0$. The charges q_1 (+6 μC), q_2 (–4 μC), and q_3 (unknown), are located on the $+x$ axis, so that we know $\vec{F}_1$ points towards $-x$, $\vec{F}_2$ points towards $+x$, and $\vec{F}_3$ points towards $-x$ if $q_3 > 0$ and points towards $+x$ if $q_3 < 0$. Therefore, with r_1 = 8 m, r_2 = 16 m and r_3 = 24 m, we have

$$0 = -k\frac{q_1 q}{r_1^2} + k\frac{|q_2| q}{r_2^2} - k\frac{q_3 q}{r_3^2} .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC. Thus, we obtain q_3 = –45 μC.

64. Charge $q_1 = -80 \times 10^{-6}$ C is at the origin, and charge $q_2 = +40 \times 10^{-6}$ C is at $x = 0.20$ m. The force on $q_3 = +20 \times 10^{-6}$ C is due to the attractive and repulsive forces from q_1 and q_2, respectively. In symbols, $\vec{F}_{3\,\text{net}} = \vec{F}_{3\,1} + \vec{F}_{3\,2}$, where

$$\left|\vec{F}_{31}\right| = k\frac{q_3\,|\,q_1\,|}{r_{3\,1}^2}, \quad |\vec{F}_{32}\,| = k\frac{q_3 q_2}{r_{3\,2}^2}.$$

(a) In this case $r_{31} = 0.40$ m and $r_{32} = 0.20$ m, with $\vec{F}_{31}$ directed towards $-x$ and $\vec{F}_{32}$ directed in the $+x$ direction. Using the value of k in Eq. 21-5, we obtain

$$\begin{aligned}\vec{F}_{3\,\text{net}} &= -\left|\vec{F}_{31}\right|\hat{\mathrm{i}} + |\vec{F}_{32}\,|\,\hat{\mathrm{i}} = \left(-k\frac{q_3\,|\,q_1\,|}{r_{3\,1}^2} + k\frac{q_3 q_2}{r_{3\,2}^2}\right)\hat{\mathrm{i}} = kq_3\left(-\frac{|\,q_1\,|}{r_{3\,1}^2} + \frac{q_2}{r_{3\,2}^2}\right)\hat{\mathrm{i}} \\ &= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(20\times10^{-6}\,\mathrm{C})\left(\frac{-80\times10^{-6}\,\mathrm{C}}{(0.40\mathrm{m})^2} + \frac{+40\times10^{-6}\,\mathrm{C}}{(0.20\mathrm{m})^2}\right)\hat{\mathrm{i}} \\ &= (89.9\ \mathrm{N})\hat{\mathrm{i}}\ .\end{aligned}$$

(b) In this case $r_{31} = 0.80$ m and $r_{32} = 0.60$ m, with $\vec{F}_{31}$ directed towards $-x$ and $\vec{F}_{3\,2}$ towards $+x$. Now we obtain

$$\begin{aligned}\vec{F}_{3\,\text{net}} &= -\left|\vec{F}_{31}\right|\hat{\mathrm{i}} + |\vec{F}_{32}\,|\,\hat{\mathrm{i}} = \left(-k\frac{q_3\,|\,q_1\,|}{r_{3\,1}^2} + k\frac{q_3 q_2}{r_{3\,2}^2}\right)\hat{\mathrm{i}} = kq_3\left(-\frac{|\,q_1\,|}{r_{3\,1}^2} + \frac{q_2}{r_{3\,2}^2}\right)\hat{\mathrm{i}} \\ &= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(20\times10^{-6}\,\mathrm{C})\left(\frac{-80\times10^{-6}\,\mathrm{C}}{(0.80\mathrm{m})^2} + \frac{+40\times10^{-6}\,\mathrm{C}}{(0.60\mathrm{m})^2}\right)\hat{\mathrm{i}} \\ &= -(2.50\ \mathrm{N})\hat{\mathrm{i}}\ .\end{aligned}$$

(c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3\,\text{net}} = 0$. Writing $r_{31} = x$ and $r_{32} = x - 0.20$ m, we equate $\left|\vec{F}_{3\,1}\right|$ and $\left|\vec{F}_{3\,2}\right|$, and after canceling common factors, arrive at

$$\frac{|\,q_1\,|}{x^2} = \frac{q_2}{\left(x - 0.20\ \mathrm{m}\right)^2}.$$

This can be further simplified to

$$\frac{(x - 0.20\ \mathrm{m})^2}{x^2} = \frac{q_2}{|\,q_1\,|} = \frac{1}{2}.$$

Taking the (positive) square root and solving, we obtain $x = 0.683$ m. If one takes the negative root and 'solves', one finds the location where the net force *would* be zero *if* q_1 and q_2 were of like sign (which is not the case here).

(d) From the above, we see that $y = 0$.

65. We are concerned with the charges in the nucleus (not the "orbiting" electrons, if there are any). The nucleus of Helium has 2 protons and that of Thorium has 90.

(a) Eq. 21-1 gives

$$F = k\frac{q^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(2(1.60\times10^{-19}\ \mathrm{C}))(90(1.60\times10^{-19}\ \mathrm{C}))}{(9.0\times10^{-15}\ \mathrm{m})^2} = 5.1\times10^2\ \mathrm{N}.$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1\times10^2\ \mathrm{N}}{4\left(1.67\times10^{-27}\ \mathrm{kg}\right)} = 7.7\times10^{28}\ \mathrm{m/s^2}\ .$$

66. Let the two charges be q_1 and q_2. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 21-1:

$$1.0\,\text{N} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)q_1q_2}{\left(2.0\,\text{m}\right)^2}.$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2, since it does not matter which is which. We get 1.2×10^{-5} C and 3.8×10^{-5} C. Thus, the charge on the sphere with the smaller charge is 1.2×10^{-5} C.

67. When sphere C touches sphere A, they divide up their total charge (Q/2 plus Q) equally between them. Thus, sphere A now has charge 3Q/4, and the magnitude of the force of attraction between A and B becomes

$$F = k\frac{(3Q/4)(Q/4)}{d^2} = 4.68\times10^{-19}\,\text{N}.$$

68. With $F = m_e g$, Eq. 21-1 leads to

$$y^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.8\text{m/s}^2\right)}$$

which leads to $y = \pm 5.1$ m. We choose $y = -5.1$ m since the second electron must be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

69. (a) If a (negative) charged particle is placed a distance x to the right of the $+2q$ particle, then its attraction to the $+2q$ particle will be exactly balanced by its repulsion from the $-5q$ particle is we require

$$\frac{5}{(L+x)^2} = \frac{2}{x^2}$$

which is obtained by equating the Coulomb force magnitudes and then canceling common factors. Cross-multiplying and taking the square root, we obtain

$$\frac{x}{L+x} = \sqrt{\frac{2}{5}}$$

which can be rearranged to produce

$$x = \frac{L}{\sqrt{2/5}-1} \approx 1.72\ L$$

(b) The y coordinate of particle 3 is $y = 0$.

70. The net charge carried by John whose mass is *m* is roughly

$$
\begin{aligned}
q &= (0.0001)\frac{mN_A Ze}{M} \\
&= (0.0001)\frac{(90\,\text{kg})(6.02\times10^{23}\text{ molecules/mol})(18\text{ electron proton pairs/molecule})(1.6\times10^{-19}\,\text{C})}{0.018\,\text{kg/mol}} \\
&= 8.7\times10^{5}\,\text{C},
\end{aligned}
$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k\frac{q(q/2)}{d^2} = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(8.7\times10^{5}\,\text{C})^2}{2(30\,\text{m})^2} \approx 4\times10^{18}\,\text{N}.$$

Thus, the order of magnitude of the electrostatic force is 10^{18} N .

Chapter 22

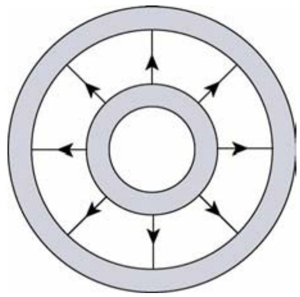
1. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so $\hat{\text{i}}$ points right in the figure), we find

$$\vec{F} = \left(+1.6\times10^{-19}\,\text{C}\right)\left(-40\frac{\text{N}}{\text{C}}\hat{\text{i}}\right) = (-6.4\times10^{-18}\,\text{N})\,\hat{\text{i}}$$

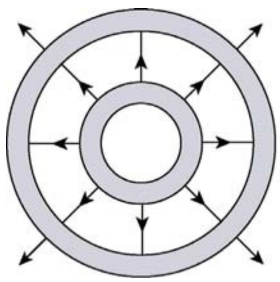
which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction $(-\hat{\text{i}})$ is leftward.

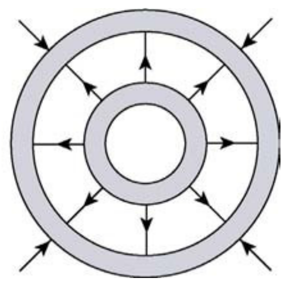
(b) As the discussion in §22-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

2. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge which resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).





3. Since the magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2 (2.0\,\text{N/C})}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times 10^{-11}\,\text{C}.$$

4. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\mathrm{N/C})(1.00\,\mathrm{m})^2}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = 1.11\times10^{-10}\,\mathrm{C}.$$

5. Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E=\frac{q}{4\pi\varepsilon_0 R^2}$$

where q is the magnitude of the total charge and R is the sphere radius.

(a) The magnitude of the total charge is Ze, so

$$E=\frac{Ze}{4\pi\varepsilon_0 R^2}=\frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)(94)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(6.64\times10^{-15}\ \mathrm{m}\right)^2}=3.07\times10^{21}\ \mathrm{N/C}.$$

(b) The field is normal to the surface and since the charge is positive, it points outward from the surface.

6. With x_1 = 6.00 cm and x_2 = 21.00 cm, the point midway between the two charges is located at x = 13.5 cm. The values of the charge are $q_1 = -q_2 = -2.00 \times 10^{-7}$ C, and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0 (x-x_1)^2}\,\hat{\mathrm{i}} = -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})|-2.00\times10^{-7}\,\mathrm{C}|}{\left(0.135\ \mathrm{m}-0.060\ \mathrm{m}\right)^2}\,\hat{\mathrm{i}} = -(3.196\times10^5\ \mathrm{N/C})\hat{\mathrm{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0 (x-x_2)^2}\,\hat{\mathrm{i}} = -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-7}\,\mathrm{C})}{\left(0.135\ \mathrm{m}-0.210\ \mathrm{m}\right)^2}\,\hat{\mathrm{i}} = -(3.196\times10^5\ \mathrm{N/C})\hat{\mathrm{i}}$$

Thus, the net electric field is

$$\vec{E}_{\mathrm{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \mathrm{N/C})\hat{\mathrm{i}}$$

7. At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\ q_1$ located at x_2 = 70 cm has a greater magnitude than $q_1 = 2.1 \times 10^{-8}$ C located at x_1 = 20 cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{\left(x-x_1\right)^2}\right).$$

If the field is to vanish, then

$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{\left(x-x_1\right)^2} \quad \Rightarrow \quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{\left(x-x_1\right)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

$$\frac{x - 70\text{ cm}}{x - 20\text{ cm}} = \pm 2.0\,.$$

Choosing –2.0 for consistency, the value of x is found to be $x = -30$ cm.

8. (a) The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}} .$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > L$ region:

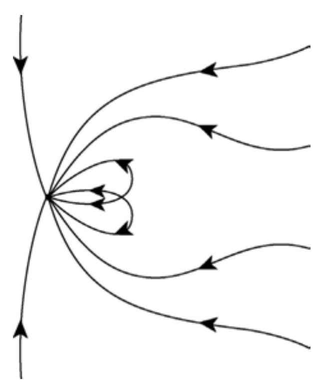
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



9. The x component of the electric field at the center of the square is given by

$$
\begin{aligned}
E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= 0.
\end{aligned}
$$

Similarly, the y component of the electric field is

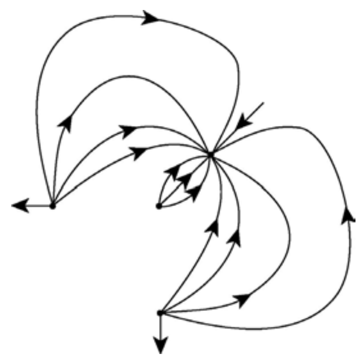
$$
\begin{aligned}
E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\ \text{C})}{(0.050\ \text{m})^2/2}\frac{1}{\sqrt{2}} = 1.02\times10^5\ \text{N/C}.
\end{aligned}
$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\ \text{N/C})\hat{\text{j}}$.

10. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\text{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\text{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown below:



11. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using $d = 3.00$ m and $y = 4.00$ m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.00\ \text{m})}{[(3.00\ \text{m})^2+(4.00\ \text{m})^2]^{3/2}} .$$

$$= 1.38\times10^{-10}\ \text{N/C} .$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

12. For it to be possible for the net field to vanish at some $x > 0$, the two individual fields (caused by q_1 and q_2) must point in opposite directions for $x > 0$. Given their locations in the figure, we conclude they are therefore oppositely charged. Further, since the net field points more strongly leftward for the small positive x (where it is very close to q_2) then we conclude that q_2 is the negative-valued charge. Thus, q_1 is a positive-valued charge. We write each charge as a multiple of some positive number ξ (not determined at this point). Since the problem states the absolute value of their ratio, and we have already inferred their signs, we have $q_1 = 4\,\xi$ and $q_2 = -\xi$. Using Eq. 22-3 for the individual fields, we find

$$E_{\text{net}} = E_1 + E_2 = \frac{4\,\xi}{4\pi\varepsilon_0 (L+x)^2} - \frac{\xi}{4\pi\varepsilon_0 x^2}$$

for points along the positive x axis. Setting $E_{\text{net}} = 0$ at $x = 20$ cm (see graph) immediately leads to $L = 20$ cm.

(a) If we differentiate E_{net} with respect to x and set equal to zero (in order to find where it is maximum), we obtain (after some simplification) that location:

$$x = \left(\frac{2}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{4} + \frac{1}{3}\right)L = 1.70(20\text{ cm}) = 34\text{ cm}.$$

We note that the result for part (a) does not depend on the particular value of ξ.

(b) Now we are asked to set $\xi = 3e$, where $e = 1.60 \times 10^{-19}$ C, and evaluate E_{net} at the value of x (converted to meters) found in part (a). The result is 2.2×10^{-8} N/C .

13. By symmetry we see the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\,\text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the x axis.

14. The field of each charge has magnitude

$$E = \frac{kq}{r^2} = k\frac{e}{(0.020\,\text{m})^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1.60\times10^{-19}\,\text{C}}{(0.020\ \text{m})^2} = 3.6\times10^{-6}\ \text{N/C}.$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector-capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\text{net}}$ as follows:

$$(E\angle -20^\circ) + (E\angle 130^\circ) + (E\angle -100^\circ) + (E\angle -150^\circ) + (E\angle 0^\circ).$$

This yields $(3.93\times10^{-6}\angle -76.4^\circ)$, with the N/C unit understood.

(a) The result above shows that the magnitude of the net electric field is $|\vec{E}_{\text{net}}| = 3.93\times10^{-6}$ N/C.

(b) Similarly, the direction of $\vec{E}_{\text{net}}$ is –76.4° from the x axis.

15. (a) The electron e_c is a distance $r = z = 0.020$ m away. Thus,

$$E_C = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{(0.020 \text{ m})^2} = 3.60\times10^{-6} \text{ N/C}.$$

(b) The horizontal components of the individual fields (due to the two e_s charges) cancel, and the vertical components add to give

$$\begin{aligned} E_{\text{s,net}} &= \frac{2ez}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19} \text{ C})(0.020 \text{ m})}{[(0.020 \text{ m})^2+(0.020 \text{ m})^2]^{3/2}} \\ &= 2.55\times10^{-6} \text{ N/C}. \end{aligned}$$

(c) Calculation similar to that shown in part (a) now leads to a stronger field $E_c = 3.60\times10^{-4}$ N/C from the central charge.

(d) The field due to the side charges may be obtained from calculation similar to that shown in part (b). The result is $E_{\text{s, net}} = 7.09 \times 10^{-7}$ N/C.

(e) Since E_c is inversely proportional to z^2, this is a simple result of the fact that z is now much smaller than in part (a). For the net effect due to the side charges, it is the "trigonometric factor" for the y component (here expressed as $z/\sqrt{r}$) which shrinks almost linearly (as z decreases) for very small z, plus the fact that the x components cancel, which leads to the decreasing value of $E_{\text{s, net}}$.

16. The net field components along the x and y axes are

$$E_{\text{net},x} = \frac{q_1}{4\pi\varepsilon_0 R^2} - \frac{q_2 \cos\theta}{4\pi\varepsilon_0 R^2}, \quad E_{\text{net},y} = -\frac{q_2 \sin\theta}{4\pi\varepsilon_0 R^2}.$$

The magnitude is the square root of the sum of the components-squared. Setting the magnitude equal to $E = 2.00 \times 10^5$ N/C, squaring and simplifying, we obtain

$$E^2 = \frac{q_1^2 + q_1^2 - 2q_1 q_2 \cos\theta}{(4\pi\varepsilon_0 R^2)^2}.$$

With R = 0.500 m, q_1 = 2.00 × 10^{-6} C and q_2 = 6.00 × 10^{-6} C, we can solve this expression for cos θ and then take the inverse cosine to find the angle:

$$\theta = \cos^{-1}\left(\frac{q_1^2 + q_1^2 - (4\pi\varepsilon_0 R^2)^2 E^2}{2q_1 q_2}\right).$$

There are two answers.

(a) The positive value of angle is θ = 67.8°.

(b) The positive value of angle is θ = – 67.8°.

17. We make the assumption that bead 2 is in the lower half of the circle, partly because it would be awkward for bead 1 to "slide through" bead 2 if it were in the path of bead 1 (which is the upper half of the circle) and partly to eliminate a second solution to the problem (which would have opposite angle and charge for bead 2). We note that the net y component of the electric field evaluated at the origin is negative (points *down*) for all positions of bead 1, which implies (with our assumption in the previous sentence) that bead 2 is a negative charge.

(a) When bead 1 is on the $+y$ axis, there is no x component of the net electric field, which implies bead 2 is on the $-y$ axis, so its angle is $-90°$.

(b) Since the downward component of the net field, when bead 1 is on the $+y$ axis, is of largest magnitude, then bead 1 must be a positive charge (so that its field is in the same direction as that of bead 2, in that situation). Comparing the values of E_y at 0° and at 90° we see that the absolute values of the charges on beads 1 and 2 must be in the ratio of 5 to 4. This checks with the 180° value from the E_x graph, which further confirms our belief that bead 1 is positively charged. In fact, the 180° value from the E_x graph allows us to solve for its charge (using Eq. 22-3):

$$q_1 = 4\pi\varepsilon_o r \square E = 4\pi(8.854 \times 10^{-12}\,\tfrac{\mathrm{C}^2}{\mathrm{N\,m}^2})(0.60\ \mathrm{m})^2\,(5.0 \times 10^4\,\tfrac{\mathrm{N}}{\mathrm{C}}) = 2.0 \times 10^{-6}\,\mathrm{C}\,.$$

(c) Similarly, the 0° value from the E_y graph allows us to solve for the charge of bead 2:

$$q_2 = 4\pi\varepsilon_o r \square E = 4\pi(8.854 \times 10^{-12}\,\tfrac{\mathrm{C}^2}{\mathrm{N\,m}^2})(0.60\ \mathrm{m})^2\,(-4.0 \times 10^4\,\tfrac{\mathrm{N}}{\mathrm{C}}) = -1.6 \times 10^{-6}\,\mathrm{C}\,.$$

18. According to the problem statement, E_{act} is Eq. 22-5 (with $z = 5d$)

$$E_{\text{act}} = \frac{q}{4\pi\varepsilon_0(4.5d)^2} - \frac{q}{4\pi\varepsilon_0(5.5d)^2} = \frac{160}{9801}\cdot\frac{q}{4\pi\varepsilon_0 d^2}$$

and E_{approx} is

$$E_{\text{approx}} = \frac{2qd}{4\pi\varepsilon_0(5d)^3} = \frac{2}{125}\cdot\frac{q}{4\pi\varepsilon_0 d^2}.$$

The ratio is

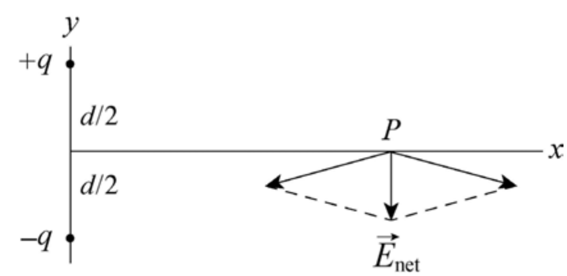
$$\frac{E_{\text{approx}}}{E_{\text{act}}} = 0.9801 \approx 0.98.$$

19. (a) Consider the figure below. The magnitude of the net electric field at point P is

$$\left|\vec{E}_{\text{net}}\right| = 2E_1 \sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(d/2\right)^2 + r^2}\right]\frac{d/2}{\sqrt{\left(d/2\right)^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[\left(d/2\right)^2 + r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$



(b) From the figure, it is clear that the net electric field at point P points in the $-\hat{\jmath}$ direction, or $-90°$ from the $+x$ axis.

20. Referring to Eq. 22-6, we use the binomial expansion (see Appendix E) but keeping higher order terms than are shown in Eq. 22-7:

$$E = \frac{q}{4\pi\varepsilon_o z^2}\left(\left(1+\frac{d}{z}+\frac{3}{4}\frac{d^2}{z^2}+\frac{1}{2}\frac{d^3}{z^3}+\ldots\right)-\left(1-\frac{d}{z}+\frac{3}{4}\frac{d^2}{z^2}-\frac{1}{2}\frac{d^3}{z^3}+\ldots\right)\right)$$

$$= \frac{q\,d}{2\pi\varepsilon_o z^3}+\frac{q\,d^3}{4\pi\varepsilon_o z^5}+\ldots$$

Therefore, in the terminology of the problem, $E_{\text{next}} = q\,d^3/\,4\pi\varepsilon_0 z^5$.

21. Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then, the field produced by the right dipole of the pair is $qd/2\pi\varepsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$. Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

to obtain

$$E = \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Let $Q = 2qd^2$. We have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

22. (a) We use the usual notation for the linear charge density: $\lambda = q/L$. The arc length is $L = r\theta$ with θ is expressed in radians. Thus,

$$L = (0.0400\text{ m})(0.698\text{ rad}) = 0.0279\text{ m}.$$

With $q = -300(1.602 \times 10^{-19}\text{ C})$, we obtain $\lambda = -1.72 \times 10^{-15}$ C/m.

(b) We consider the same charge distributed over an area $A = \pi r^2 = \pi(0.0200\text{ m})^2$ and obtain $\sigma = q/A = -3.82 \times 10^{-14}\text{ C/m}^2$.

(c) Now the area is four times larger than in the previous part ($A_{\text{sphere}} = 4\pi r^2$) and thus obtain an answer that is one-fourth as big:

$$\sigma = q/A_{\text{sphere}} = -9.56 \times 10^{-15}\text{ C/m}^2.$$

(d) Finally, we consider that same charge spread throughout a volume of $V = 4\pi r^3/3$ and obtain the charge density $\rho = q/V = -1.43 \times 10^{-12}\text{ C/m}^3$.

23. We use Eq. 22-3, assuming both charges are positive. At P, we have

$$E_{\text{left ring}} = E_{\text{right ring}} \quad \Rightarrow \quad \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2 + R^2\right)^{3/2}} = \frac{q_2 (2R)}{4\pi\varepsilon_0 [(2R)^2 + R^2]^{3/2}}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

24. Studying Sample Problem 22-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\theta}^{\theta}$$

along the symmetry axis, with λ = $q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$|\vec{E}| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\pi/4}^{\pi/4} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}.$$

That produced by the positive quarter-circle points at – 45°, and that of the negative quarter-circle points at +45°.

(a) The magnitude of the net field is

$$E_{\text{net},x} = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}\right)\cos 45° = \frac{1}{4\pi\varepsilon_0}\frac{4|q|}{\pi r^2}$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)4(4.50\times10^{-12}\,\text{C})}{\pi(5.00\times10^{-2}\ \text{m})^2} = 20.6\ \text{N/C}.$$

(b) By symmetry, the net field points vertically downward in the $-\hat{\text{j}}$ direction, or −90° counterclockwise from the $+x$ axis.

25. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda \cdot \pi R$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\text{net}} = 2\left(-\hat{\text{j}}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_{-90^\circ}^{90^\circ} = -\left(\frac{q}{\varepsilon_0 \pi^2 R^2}\right)\hat{\text{j}}.$$

(a) With $R = 8.50 \times 10^{-2}$ m and $q = 1.50 \times 10^{-8}$ C, $|\vec{E}_{\text{net}}| = 23.8$ N/C.

(b) The net electric field $\vec{E}_{\text{net}}$ points in the $-\hat{\text{j}}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

26. We find the maximum by differentiating Eq. 22-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right)=\frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}}=0$$

which leads to $z = R/\sqrt{2}$. With R = 2.40 cm, we have z = 1.70 cm.

27. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod,

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\ \text{C}}{0.0815\ \text{m}} = -5.19\times10^{-14}\ \text{C/m}.$$

(b) We position the x axis along the rod with the origin at the left end of the rod, as shown in the diagram.

Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\, dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}.$$

The total electric field produced at P by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a(L+a)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)},$$

upon substituting $-q = \lambda L$. With $q = 4.23\times10^{-15}$ C, $L = 0.0815$ m and $a = 0.120$ m, we obtain $E_x = -1.57\times10^{-3}\ \text{N/C}$, or $|E_x| = 1.57\times10^{-3}\ \text{N/C}$.

(c) The negative sign in E_x indicates that the field points in the $-x$ direction, or $-180°$ counterclockwise form the $+x$ axis.

(d) If a is much larger than L, the quantity $L + a$ in the denominator can be approximated by a and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

Since $a = 50\ \text{m} \gg L = 0.0815\ \text{m}$, the above approximation applies and we have $E_x = -1.52\times10^{-8}\ \text{N/C}$, or $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

(e) For a particle of charge $-q = -4.23\times10^{-15}\ \text{C}$, the electric field at a distance a = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

28. We use Eq. 22-16, with "q" denoting the charge on the larger ring:

$$\frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}+\frac{qz}{4\pi\varepsilon_0[z^2+(3R)^2]^{3/2}}=0 \;\Rightarrow\; q=-Q\left(\frac{13}{5}\right)^{3/2}=-4.19Q\,.$$

Note: we set $z = 2R$ in the above calculation.

29. The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi(2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi(3R)/2$. The charge per unit length for each arc is $\lambda = q/L$ where each charge q is specified in the figure. Following the steps that lead to Eq. 22-21 in Sample Problem 22-3, we find

$$E_{\text{net}} = \frac{\lambda_1(2\sin 45°)}{4\pi\varepsilon_0 r_1} + \frac{\lambda_2(2\sin 45°)}{4\pi\varepsilon_0 r_2} + \frac{\lambda_3(2\sin 45°)}{4\pi\varepsilon_0 r_3} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

which yields $E_{\text{net}} = 1.62 \times 10^6$ N/C .

(b) The direction is – 45º, measured counterclockwise from the $+x$ axis.

30. (a) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center.

(b) The result ($E = 0$) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z$□as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r$□at distant points).

(c) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right)=\frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}}=0 \quad\Rightarrow\quad z=+\frac{R}{\sqrt{2}}=0.707R\,.$$

(d) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{\max} = 3.46 \times 10^7$ N/C.

31. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ if θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

The problem asks for the ratio $E_{\text{particle}} / E_{\text{arc}}$ where E_{particle} is given by Eq. 22-3:

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{Q/4\pi\varepsilon_0 R^2}{2Q\sin(\theta/2)/4\pi\varepsilon_0 R^2\theta} = \frac{\theta}{2\sin(\theta/2)}.$$

With $\theta = \pi$, we have

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{\pi}{2} \approx 1.57.$$

32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

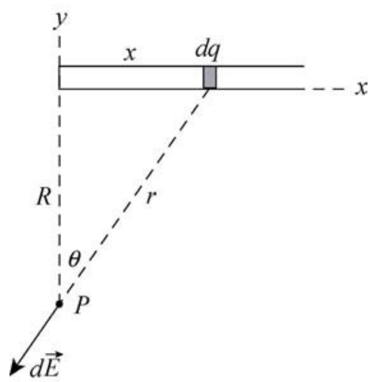
$$
\begin{aligned}
|\vec{E}| &= 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\,dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right) \\
&= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2} \\
&= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}
\end{aligned}
$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C, $L = 0.145$ m and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C.

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.

33. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the following diagram. It contains charge $dq = \lambda\, dx$ and is a distance r from P. The magnitude of the field it produces at P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}.$$



The x and the y components are

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\cos\theta,$$

respectively. We use θ as the variable of integration and substitute $r = R/\cos\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\, d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}.$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

34. From Eq. 22-26, we obtain

$$E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)=\frac{5.3\times10^{-6}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}\left[1-\frac{12\,\mathrm{cm}}{\sqrt{(12\,\mathrm{cm})^2+(2.5\,\mathrm{cm})^2}}\right]=6.3\times10^{3}\ \mathrm{N/C}.$$

35. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \Rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}.$$

Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$, or $z = R/\sqrt{3}$. With $R = 0.600$ m, we have $z = 0.346$ m.

36. We write Eq. 22-26 as

$$\frac{E}{E_{\max}} = 1 - \frac{z}{(z^2 + R^2)^{1/2}}$$

and note that this ratio is $\frac{1}{2}$ (according to the graph shown in the figure) when z = 4.0 cm. Solving this for R we obtain $R = z\sqrt{3}$ = 6.9 cm.

37. We use Eq. 22-26, noting that the disk in figure (b) is effectively equivalent to the disk in figure (a) plus a concentric smaller disk (of radius $R/2$) with the opposite value of σ. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}}\right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + R^2}}\right) .$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1-2/\sqrt{4+1/4}}{1-2/\sqrt{4+1}} = \frac{1-2/\sqrt{17/4}}{1-2/\sqrt{5}} = \frac{0.0299}{0.1056} = 0.283$$

or approximately 28%.

38. From $dA = 2\pi r\, dr$ (which can be thought of as the differential of $A = \pi r^2$) and $dq = \sigma\, dA$ (from the definition of the surface charge density σ), we have

$$dq = \left(\frac{Q}{\pi R^2}\right) 2\pi r\, dr$$

where we have used the fact that the disk is uniformly charged to set the surface charge density equal to the total charge (Q) divided by the total area (πR^2). We next set r = 0.0050 m and make the approximation $dr \approx 30 \times 10^{-6}$ m. Thus we get $dq \approx 2.4 \times 10^{-16}$ C.

39. The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{4}\ \text{N/C}\right)}{9.11 \times 10^{-31}\ \text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2 .$$

40. Eq. 22-28 gives

$$\vec{E} = \frac{\vec{F}}{q} = \frac{m\vec{a}}{(-e)} = -\left(\frac{m}{e}\right)\vec{a}$$

using Newton's second law.

(a) With *east* being the $\hat{\text{i}}$ direction, we have

$$\vec{E} = -\left(\frac{9.11\times10^{-31}\text{ kg}}{1.60\times10^{-19}\text{ C}}\right)\left(1.80\times10^{9}\text{ m/s}^2\,\hat{\text{i}}\right) = (-0.0102\text{ N/C})\,\hat{\text{i}}$$

which means the field has a magnitude of 0.0102 N/C

(b) The result shows that the field $\vec{E}$ is directed in the –x direction, or westward.

41. We combine Eq. 22-9 and Eq. 22-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2kep}{z^3}$$

where we have used Eq. 21-5 for the constant k in the last step. Thus, we obtain

$$F = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(3.6\times10^{-29}\text{C}\cdot\text{m}\right)}{\left(25\times10^{-9}\text{m}\right)^3} = 6.6\times10^{-15}\ \text{N}\,.$$

If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

42. (a) Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \;\Rightarrow\; \left|\vec{E}\right| = \frac{mg}{2e}.$$

Substituting the values given in the problem, we obtain

$$\left|\vec{E}\right| = \frac{mg}{2e} = \frac{(6.64\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)}{2(1.6\times10^{-19}\text{C})} = 2.03\times10^{-7}\text{ N/C}.$$

(b) Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

43. (a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus,

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6}\ \text{N}}{2.0 \times 10^{-9}\ \text{C}} = 1.5 \times 10^{3}\ \text{N/C}.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_{el} = eE = \left(1.60\times10^{-19}\ \text{C}\right)\left(1.5\times10^{3}\ \text{N/C}\right) = 2.4\times10^{-16}\text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = \left(1.67\times10^{-27}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right) = 1.6\times10^{-26}\text{N}.$$

The force is downward.

(e) The ratio of the forces is

$$\frac{F_{el}}{F_g} = \frac{2.4\times10^{-16}\ \text{N}}{1.64\times10^{-26}\ \text{N}} = 1.5\times10^{10}.$$

44. (a) $F_e = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

45. (a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(2.00\times10^{4}\ \mathrm{N/C}\right)}{1.67\times10^{-27}\ \mathrm{kg}} = 1.92\times10^{12}\ \mathrm{m/s^2}\ .$$

(b) We assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2\left(1.92\times10^{12}\ \mathrm{m/s^2}\right)\left(0.0100\ \mathrm{m}\right)} = 1.96\times10^{5}\ \mathrm{m/s}.$$

46. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^6\text{ m/s}\right)^2}{-2\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^3\text{ N/C}\right)} = 7.12\times10^{-2}\text{ m.}$$

(b) Eq. 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2\left(7.12\times10^{-2}\text{ m}\right)}{5.00\times10^6\text{ m/s}} = 2.85\times10^{-8}\text{ s.}$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\begin{aligned}\frac{\Delta K}{K_i} &= \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}\\ &= \frac{-2\left(1.60\times10^{-19}\text{C}\right)\left(1.00\times10^3\text{ N/C}\right)\left(8.00\times10^{-3}\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(5.00\times10^6\text{ m/s}\right)^2} = -0.112.\end{aligned}$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

47. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$q = -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi\left(1.64\times10^{-6}\,\text{m}\right)^3\left(851\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)}{3\left(1.92\times10^5\,\text{N/C}\right)} = -8.0\times10^{-19}\,\text{C}$$

and $q/e = (-8.0 \times 10^{-19}\text{ C})/(1.60 \times 10^{-19}\text{ C}) = -5$, or $q = -5e$.

48. We assume there are no forces or force-components along the x direction. We combine Eq. 22-28 with Newton's second law, then use Eq. 4-21 to determine time t followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the a_y of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as v_{0x} and v_{0y} respectively.

(a) We have $\vec{a} = q\vec{E}/m = -(e/m)\vec{E}$ which leads to

$$\vec{a} = -\left(\frac{1.60\times10^{-19}\,\mathrm{C}}{9.11\times10^{-31}\mathrm{kg}}\right)\left(120\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{\mathrm{j}} = -(2.1\times10^{13}\ \mathrm{m/s^2})\,\hat{\mathrm{j}}.$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$t = \frac{\Delta x}{v_{0x}} = \frac{0.020\,\mathrm{m}}{1.5\times10^5\ \mathrm{m/s}} = 1.3\times10^{-7}\,\mathrm{s}$$
$$v_y = v_{0y} + a_y t = 3.0\times10^3\ \mathrm{m/s} + \left(-2.1\times10^{13}\ \mathrm{m/s^2}\right)\left(1.3\times10^{-7}\,\mathrm{s}\right)$$

which leads to $v_y = -2.8\times10^6$ m/s. Therefore, the final velocity is

$$\vec{v} = (1.5\times10^5\ \mathrm{m/s})\,\hat{\mathrm{i}} - (2.8\times10^6\ \mathrm{m/s})\,\hat{\mathrm{j}}.$$

49. (a) We use $\Delta x = v_{avg}t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2\left(2.0\times 10^{-2}\,\text{m}\right)}{1.5\times 10^{-8}\,\text{s}} = 2.7\times 10^{6}\ \text{m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta xm}{et^2} = \frac{2\left(2.0\times 10^{-2}\,\text{m}\right)\left(9.11\times 10^{-31}\,\text{kg}\right)}{\left(1.60\times 10^{-19}\,\text{C}\right)\left(1.5\times 10^{-8}\,\text{s}\right)^2} = 1.0\times 10^{3}\ \text{N/C}.$$

50. Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with g replaced with $a = eE/m = 8.78 \times 10^{11}$ m/s^2). Thus, Eq. 4-21 gives

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{3.00 \text{ m}}{(2.00\times10^6 \text{ m/s})\cos 40.0°} = 1.96\times10^{-6} \text{ s}.$$

This leads (using Eq. 4-23) to

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - at = (2.00\times10^6 \text{ m/s})\sin 40.0° - (8.78\times10^{11} \text{ m/s}^2)(1.96\times10^{-6} \text{ s}) \\ &= -4.34\times10^5 \text{ m/s}. \end{aligned}$$

Since the x component of velocity does not change, then the final velocity is

$$\vec{v} = (1.53 \times 10^6 \text{ m/s})\,\hat{\text{i}} - (4.34 \times 10^5 \text{ m/s})\,\hat{\text{j}}\ .$$

51. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$
\begin{aligned}
x &= \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)}L = \left(\frac{m_e}{m_e + m_p}\right)L \\
&= \left(\frac{9.11\times10^{-31}\,\text{kg}}{9.11\times10^{-31}\,\text{kg} + 1.67\times10^{-27}\,\text{kg}}\right)(0.050\,\text{m}) \\
&= 2.7\times10^{-5}\,\text{m}.
\end{aligned}
$$

52. We are given $\sigma = 4.00 \times 10^{-6}$ C/m^2 and various values of z (in the notation of Eq. 22-26 which specifies the field E of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance R is

$$a = \frac{e\,\sigma\,(2-\sqrt{2}\,)}{4\,m\,\varepsilon_o} = 1.16 \times 10^{16} \text{ m/s}^2 \ .$$

(b) At a distance R/100, $a = \dfrac{e\,\sigma\,(10001-\sqrt{10001}\,)}{20002\,m\,\varepsilon_o} = 3.94 \times 10^{16} \text{ m/s}^2$.

(c) At a distance R/1000, $a = \dfrac{e\,\sigma\,(1000001-\sqrt{1000001}\,)}{2000002\,m\,\varepsilon_o} = 3.97 \times 10^{16} \text{ m/s}^2$.

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

53. (a) Using Eq. 22-28, we find

$$\begin{aligned}\vec{F} &= \left(8.00\times10^{-5}\,\text{C}\right)\left(3.00\times10^{3}\,\text{N/C}\right)\hat{\text{i}} + \left(8.00\times10^{-5}\,\text{C}\right)\left(-600\,\text{N/C}\right)\hat{\text{j}} \\ &= \left(0.240\,\text{N}\right)\hat{\text{i}} - \left(0.0480\,\text{N}\right)\hat{\text{j}}.\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(0.240\,\text{N}\right)^2 + \left(-0.0480\,\text{N}\right)^2} = 0.245\,\text{N}.$$

(b) The angle the force $\vec{F}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\,\text{N}}{0.240\,\text{N}}\right) = -11.3°$$

measured counterclockwise from the $+x$ axis.

(c) With m = 0.0100 kg, the (x, y) coordinates at t = 3.00 s can be found by combining Newton's second law with the kinematics equations of Chapters 2–4. The x coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{\left(0.240\text{ N}\right)\left(3.00\text{ s}\right)^2}{2\left(0.0100\text{ kg}\right)} = 108\,\text{m}.$$

(d) Similarly, the y coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{\left(-0.0480\text{ N}\right)\left(3.00\text{ s}\right)^2}{2\left(0.0100\text{ kg}\right)} = -21.6\,\text{m}.$$

54. (a) Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the same direction as the velocity leads to deceleration. Thus, with $t = 1.5 \times 10^{-9}$ s, we find

$$v = v_0 - |a|t = v_0 - \frac{eE}{m}t = 4.0\times10^4 \text{ m/s} - \frac{(1.6\times10^{-19}\text{ C})(50\text{ N/C})}{9.11\times10^{-31}\text{ kg}}(1.5\times10^{-9}\text{ s})$$

$$= 2.7\times10^4 \text{ m/s}.$$

(b) The displacement is equal to the distance since the electron does not change its direction of motion. The field is uniform, which implies the acceleration is constant. Thus,

$$d = \frac{v+v_0}{2}t = 5.0\times10^{-5}\text{ m}.$$

55. We take the charge $Q = 45.0\,\text{pC}$ of the bee to be concentrated as a particle at the center of the sphere. The magnitude of the induced charges on the sides of the grain is $|q| = 1.000$ pC.

(a) The electrostatic force on the grain by the bee is

$$F = \frac{kQq}{(d+D/2)^2} + \frac{kQ(-q)}{(D/2)^2} = -kQ|q|\left[\frac{1}{(D/2)^2} - \frac{1}{(d+D/2)^2}\right]$$

where $D = 1.000$ cm is the diameter of the sphere representing the honeybee, and $d = 40.0\mu\text{m}$ is the diameter of the grain. Substituting the values, we obtain

$$F = -\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(5.00\times10^{-3}\ \text{m})^2} - \frac{1}{(5.04\times10^{-3}\ \text{m})^2}\right]$$
$$= -2.56\times10^{-10}\ \text{N}.$$

The negative sign implies that the force between the bee and the grain is attractive. The magnitude of the force is $|F| = 2.56\times10^{-10}$ N.

(b) Let $|Q'| = 45.0$ pC be the magnitude of the charge on the tip of the stigma. The force on the grain due to the stigma is

$$F' = \frac{k|Q'|q}{(d+D')^2} + \frac{k|Q'|(-q)}{(D')^2} = -k|Q'||q|\left[\frac{1}{(D')^2} - \frac{1}{(d+D')^2}\right]$$

where $D' = 1.000$ mm is the distance between the grain and the tip of the stigma. Substituting the values given, we have

$$F' = -\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(1.000\times10^{-3}\ \text{m})^2} - \frac{1}{(1.040\times10^{-3}\ \text{m})^2}\right]$$
$$= -3.06\times10^{-8}\ \text{N}.$$

The negative sign implies that the force between the grain and the stigma is attractive. The magnitude of the force is $|F'| = 3.06\times10^{-8}$ N.

(c) Since $|F'| > |F|$, the grain will move to the stigma.

56. (a) Eq. 22-33 leads to $\tau = pE\sin 0° = 0.$

(b) With $\theta = 90°$, the equation gives

$$\tau = pE = \left(2\left(1.6\times10^{-19}\ \text{C}\right)\left(0.78\times10^{-9}\ \text{m}\right)\right)\left(3.4\times10^{6}\ \text{N/C}\right) = 8.5\times10^{-22}\ \text{N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE\sin 180° = 0.$

57. (a) The magnitude of the dipole moment is

$$p = qd = \left(1.50 \times 10^{-9}\ \text{C}\right)\left(6.20 \times 10^{-6}\ \text{m}\right) = 9.30 \times 10^{-15}\ \text{C} \cdot \text{m}.$$

(b) Following the solution to part (c) of Sample Problem 22-5, we find

$$U\left(180°\right) - U\left(0\right) = 2pE = 2\left(9.30 \times 10^{-15}\,\text{C} \cdot \text{m}\right)\left(1100\ \text{N/C}\right) = 2.05 \times 10^{-11}\,\text{J}.$$

58. Using Eq. 22-35, considering θ as a variable, we note that it reaches its maximum value when $\theta = -90°$: $\tau_{max} = pE$. Thus, with E = 40 N/C and $\tau_{max} = 100 \times 10^{-28}$ N·m (determined from the graph), we obtain the dipole moment: $p = 2.5 \times 10^{-28}$ C·m.

59. Eq. 22-35 $(\tau = -pE\sin\theta)$ captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace sin θ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}.$$

60. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p} \cdot \vec{E} = -1.00 \times 10^{-28}\ \text{J}.$$

If $E = 20$ N/C, we find $p = 5.0 \times 10^{-28}$ C·m.

61. Following the solution to part (c) of Sample Problem 22-5, we find

$$\begin{aligned} W &= U(\theta_0+\pi)-U(\theta_0) = -pE\left(\cos(\theta_0+\pi)-\cos(\theta_0)\right) = 2pE\cos\theta_0 \\ &= 2(3.02\times10^{-25}\,\mathrm{C\cdot m})(46.0\ \mathrm{N/C})\cos 64.0° \\ &= 1.22\times10^{-23}\ \mathrm{J}. \end{aligned}$$

62. Our approach (based on Eq. 22-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values:

$$18.08 \times 10^{-19}\text{ C} - 16.48 \times 10^{-19}\text{ C} = 1.60 \times 10^{-19}\text{ C}$$

which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

datum1	$\frac{6.563\times10^{-19}\text{C}}{e_{\text{approx}}}=4.10 \Rightarrow n_1=4$	datum6	$\frac{18.08\times10^{-19}\text{C}}{e_{\text{appeox}}}=11.30 \Rightarrow n_6=11$
datum2	$\frac{8.204\times10^{-19}\text{C}}{e_{\text{approx}}}=5.13 \Rightarrow n_2=5$	datum7	$\frac{19.71\times10^{-19}\text{C}}{e_{\text{approx}}}=12.32 \Rightarrow n_7=12$
datum3	$\frac{11.50\times10^{-19}\text{C}}{e_{\text{approx}}}=7.19 \Rightarrow n_3=7$	datum8	$\frac{22.89\times10^{-19}\text{C}}{e_{\text{approx}}}=14.31 \Rightarrow n_8=14$
datum4	$\frac{13.13\times10^{-19}\text{C}}{e_{\text{approx}}}=8.21 \Rightarrow n_4=8$	datum9	$\frac{26.13\times10^{-19}\text{C}}{e_{\text{approx}}}=16.33 \Rightarrow n_9=16$
datum5	$\frac{16.48\times10^{-19}\text{C}}{e_{\text{approx}}}=10.30 \Rightarrow n_5=10$		

Next, we construct a new data set (e_1, e_2, e_3 …) by dividing the given data by the respective exact integers n_i (for i = 1, 2, 3 …):

$$(e_1, e_2, e_3 \ldots) = \left(\frac{6.563\times10^{-19}\,\text{C}}{n_1}, \frac{8.204\times10^{-19}\,\text{C}}{n_2}, \frac{11.50\times10^{-19}\,\text{C}}{n_3} \ldots \right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\,\text{C},\ 1.6408\times10^{-19}\,\text{C},\ 1.64286\times10^{-19}\,\text{C}\ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}} = e_{\text{avg}} \pm \Delta e = (1.641 \pm 0.004)\times10^{-19}\,\text{C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}} - \Delta e = 1.637 \times 10^{-19}$ C which is still about 2% too high.

63. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} .$$

Thus, the problem requires $E_{\text{arc}} = \frac{1}{2}\,E_{\text{particle}}$ where E_{particle} is given by Eq. 22-3. Hence,

$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \Rightarrow \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta = 3.791$ rad $\approx 217°$.

64. Most of the individual fields, caused by diametrically opposite charges, will cancel, except for the pair that lie on the x axis passing through the center. This pair of charges produces a field pointing to the right

$$\vec{E} = \frac{3q}{4\pi\varepsilon_0 d^2}\hat{\text{i}} = \frac{3e}{4\pi\varepsilon_0 d^2}\hat{\text{i}} = \frac{3\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(0.020\text{m}\right)^2}\hat{\text{i}} = 1.08\times10^{-5}\ \text{N/C}\,.$$

65. (a) From symmetry, we see the net field component along the x axis is zero; the net field component along the y axis points upward. With $\theta = 60°$,

$$E_{\text{net},y} = 2\frac{Q\sin\theta}{4\pi\varepsilon_0 a^2} \ .$$

Since $\sin(60°) = \sqrt{3}/2$, we can write this as $E_{\text{net}} = kQ\sqrt{3}/a^2$ (using the notation of the constant k defined in Eq. 21-5). Numerically, this gives roughly 47 N/C.

(b) From symmetry, we see in this case that the net field component along the y axis is zero; the net field component along the x axis points rightward. With $\theta = 60°$,

$$E_{\text{net},x} = 2\frac{Q\cos\theta}{4\pi\varepsilon_0 a^2} .$$

Since $\cos(60°) = 1/2$, we can write this as $E_{\text{net}} = kQ/a^2$ (using the notation of Eq. 21-5). Thus, $E_{\text{net}} \approx 27$ N/C.

66. The two closest charges produce fields at the midpoint which cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2} = \frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2} .$$

67. (a) Since the two charges in question are of the same sign, the point x = 2.0 mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x' - x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \;\Rightarrow\; E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0 (x'-x)^2}.$$

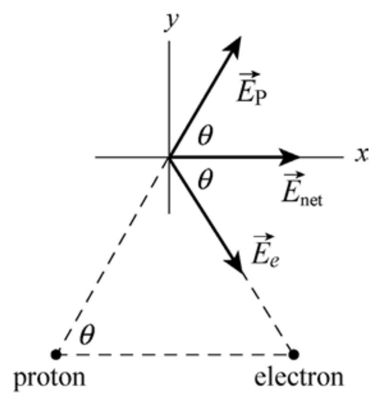
Thus, we obtain $x' = 3x = 3(2.0 \text{ mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at x = 2.0 mm. Therefore, the net field points in the negative x direction, or 180°, measured counterclockwise from the $+x$ axis.

68. We denote the electron with subscript e and the proton with p. From the figure below we see that

$$|\vec{E}_e| = |\vec{E}_p| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the y axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$|\vec{E}_{net}| = E_x = 2E_e \cos\theta = 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2\left(8.99\times10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(1.6\times10^{-19}\ \text{C})}{(2.0\times10^{-6}\ \text{m})^2}\cos 60°$$

$$= 3.6\times10^2\ \text{N/C}.$$

69. On the one hand, the conclusion (that $Q = +1.00\ \mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 22-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the y axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q \sin 30° = q$ and thus to the conclusion mentioned above.

70. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6$ N/C. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2\sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2 E}{2k} = \frac{\left(2.5\times10^{-2}\,\text{m}\right)^2\left(3.0\times10^6\,\text{N/C}\right)}{2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = 1.0\times10^{-7}\,\text{C}\,,$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$.

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

$$n = \frac{\pi\left(2.5\times10^{-2}\,\text{m}\right)^2}{0.015\times10^{-18}\,\text{m}^2} = 1.3\times10^{17}.$$

(c) The fraction is

$$\frac{q}{Ne} = \frac{1.0\times10^{-7}\,\text{C}}{\left(1.3\times10^{17}\right)\left(1.6\times10^{-19}\,\text{C}\right)} \approx 5.0\times10^{-6}.$$

71. (a) Using the density of water (ρ = 1000 kg/m^3), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\ \text{m}\right)^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87\times10^{-15}\ \text{N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)} = 120.$$

72. Eq. 22-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that θ_i = 110° and θ_f = 70.0°. Therefore,

$$\Delta U = -pE\left(\cos 70.0° - \cos 110°\right) = -3.28\times10^{-21}\ \text{J}.$$

73. Studying Sample Problem 22-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\theta}^{\theta}$$

along the symmetry axis, where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0\,\text{m}$. Therefore, $\theta = \ell/r = 4.0/2.0 = 2.0\,\text{rad}$. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{(q/\ell)}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-1.0\text{ rad}}^{1.0\text{ rad}} = 38\text{ N/C} .$$

74. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\,\text{kg}}\right)\left(1.40\times10^{6}\,\frac{\text{N}}{\text{C}}\right) = 2.46\times10^{17}\ \text{m/s}^2 .$$

(b) With $v = \frac{c}{10} = 3.00\times10^{7}\ \text{m/s}$, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00\times10^{7}\ \text{m/s}}{2.46\times10^{17}\ \text{m/s}^2} = 1.22\times10^{-10}\ \text{s}.$$

(c) Eq. 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00\times10^{7}\ \text{m/s}\right)^2}{2\left(2.46\times10^{17}\ \text{m/s}^2\right)} = 1.83\times10^{-3}\ \text{m}.$$

75. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o’clock ($-q$) and seven o’clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o’clock position. Similarly, the net field due to just the charges in the six o’clock ($-6q$) and twelve o’clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o’clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o’clock, eight o’clock … twelve o’clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o’clock. Therefore, $\vec{E}_{\text{resultant}}$ points towards the nine-thirty position.

76. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 22-16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}}$$

where m is the mass of the electron.

77. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 22-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\,\text{N}}{150\ \text{N/C}} = 0.029\,\text{C}\,,$$

or $q = -0.029$ C.

(b) The feasibility of this experiment may be studied by using Eq. 22-3 (using k for $1/4\pi\varepsilon_0$). We have $E = k|q|/r^2$ with

$$\rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is 4.4/9.8 ≈ 0.45 kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \;\Rightarrow\; E = k\frac{|q|}{r^2} \approx 2\times10^{11}\ \text{N/C}$$

which is much too large a field to maintain in air.

78. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem,

$$p = \left(1.60\times10^{-19}\text{ C}\right)\left(4.30\times10^{-9}\text{ m}\right) = 6.88\times10^{-28}\text{ C}\cdot\text{m}.$$

The dipole moment is a vector that points from the negative toward the positive charge.

79. From the second measurement (at (2.0, 0)) we see that the charge must be somewhere on the x axis. A line passing through (3.0, 3.0) with slope $\tan^{-1}(3/4)$ will intersect the x axis at $x = -1.0$. Thus, the location of the particle is specified by the coordinates (in cm): (–1.0, 0).

(a) The x coordinate is $x = -1.0$ cm.

(b) Similarly, the y coordinate is $y = 0$.

(c) Using $k = 1/4\pi\varepsilon_0$, the field magnitude measured at (2.0, 0) (which is r = 0.030 m from the charge) is

$$\left|\vec{E}\right| = k\frac{q}{r^2} = 100 \ \text{N/C}.$$

Therefore,

$$q = \frac{\left|\vec{E}\right| r^2}{k} = \frac{(100 \ \text{N/C})(0.030 \ \text{m})^2}{8.99\times10^9 \ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.0\times10^{-11}\,\text{C}.$$

80. We interpret the linear charge density, $\lambda = |Q|/L$, to indicate a positive quantity (so we can relate it to the magnitude of the field). Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(|Q|/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(|Q|/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2|Q|\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} .$$

With $|Q| = 6.25\times10^{-12}$ C, $\theta = 2.40$ rad $= 137.5°$ and $R = 9.00\times10^{-2}$ m, the magnitude of the electric field is $E = 5.39$ N/C.

81. (a) From Eq. 22-38 (and the facts that $\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=1$ and $\hat{\mathrm{j}}\cdot\hat{\mathrm{i}}=0$), the potential energy is

$$
\begin{aligned}
U &= -\vec{p}\cdot\vec{E} = -\left[\left(3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\cdot\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right] \\
&= -1.49\times10^{-26}\ \mathrm{J}.
\end{aligned}
$$

(b) From Eq. 22-34 (and the facts that $\hat{\mathrm{i}}\times\hat{\mathrm{i}}=0$ and $\hat{\mathrm{j}}\times\hat{\mathrm{i}}=-\hat{\mathrm{k}}$), the torque is

$$
\begin{aligned}
\vec{\tau} &= \vec{p}\times\vec{E} = \left[\left(3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\times\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right] \\
&= \left(-1.98\times10^{-26}\,\mathrm{N\cdot m}\right)\hat{\mathrm{k}}.
\end{aligned}
$$

(c) The work done is

$$
\begin{aligned}
W &= \Delta U = \Delta\left(-\vec{p}\cdot\vec{E}\right) = \left(\vec{p}_i-\vec{p}_f\right)\cdot\vec{E} \\
&= \left[\left(3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}\right)-\left(-4.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}\right)\right]\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\cdot\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right] \\
&= 3.47\times10^{-26}\ \mathrm{J}.
\end{aligned}
$$

82. (a) From symmetry, we see the net force component along the y axis is zero.

(b) The net force component along the x axis points rightward. With $\theta = 60°$,

$$F_3 = 2\frac{q_3 q_1 \cos\theta}{4\pi\varepsilon_0 a^2}.$$

Since $\cos(60°) = 1/2$, we can write this as

$$F_3 = \frac{kq_3q_1}{a^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-12}\ \mathrm{C})(2.00\times10^{-12}\ \mathrm{C})}{(0.0950\ \mathrm{m})^2} = 9.96\times10^{-12}\ \mathrm{N}.$$

83. A small section of the distribution that has charge dq is $\lambda\ dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 (x - x_P)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\text{m}} \frac{\lambda\ dx}{4\pi\varepsilon_0 \left(x - x_P\right)^2}\,\hat{\text{i}}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0} \int_{-4.0\,\text{m}}^{-1.0\,\text{m}} \frac{du}{u^2}\,\hat{\text{i}} = \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{-1}{-1.0\,\text{m}} - \frac{-1}{-4.0\,\text{m}} \right) \hat{\text{i}}$$

which yields 61 N/C in the $+x$ direction.

84. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$ are figured from Eq. 22-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

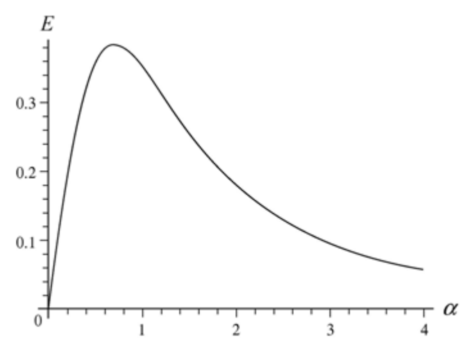
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2+d^2}\right)\left(\frac{x}{\sqrt{x^2+d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0 d^2}\left(\frac{\alpha}{(\alpha^2+1)^{3/2}}\right).$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set $d = 1$ m and $q = 5.56 \times 10^{-11}$ C.



(c) From the graph, we estimate E_{max} occurs at about $\alpha = 0.71$. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 2.0$. Further numerical exploration leads to the values: $\alpha = 0.2047$ and $\alpha = 1.9864$.

85. (a) For point A, we have (in SI units)

$$\begin{aligned}\vec{E}_A &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2}+\frac{q_2}{4\pi\varepsilon_0 r_2^2}\right]\left(-\hat{\mathrm{i}}\right)\\ &= \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\mathrm{C}\right)}{\left(5.00\times10^{-2}\right)^2}\left(-\hat{\mathrm{i}}\right)+\frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(2\times5.00\times10^{-2}\right)^2}\left(\hat{\mathrm{i}}\right)\ .\\ &= (-1.80\,\mathrm{N/C})\hat{\mathrm{i}}.\end{aligned}$$

(b) Similar considerations leads to

$$\begin{aligned}\vec{E}_B &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2}+\frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\mathrm{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}+\frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}\\ &= (43.2\,\mathrm{N/C})\hat{\mathrm{i}}.\end{aligned}$$

(c) For point C, we have

$$\begin{aligned}\vec{E}_C &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2}-\frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\mathrm{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}-\frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}\\ &= -(6.29\,\mathrm{N/C})\hat{\mathrm{i}}.\end{aligned}$$

(d) Although a sketch is not shown here, it would be somewhat similar to Fig. 22-5 in the textbook except that there would be twice as many field lines "coming into" the negative charge (which would destroy the simple up/down symmetry seen in Fig. 22-5).

86. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{(1.60\times10^{-19}\,\mathrm{C})(2.00\times10^{3}\,\mathrm{N/C})}{9.11\times10^{-31}\,\mathrm{kg}} = 3.51\times10^{14}\,\mathrm{m/s^2}\,.$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta,\quad y = v_0 t\sin\theta - \frac{1}{2}at^2,\qquad \text{and}\qquad v_y = v_0\sin\theta - at.$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{v_0^2\sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} = \frac{(6.00\times10^{6}\,\mathrm{m/s})^2\sin^2 45°}{2(3.51\times10^{14}\,\mathrm{m/s^2})} = 2.56\times10^{-2}\,\mathrm{m}.$$

Since this is greater than $d = 2.00$ cm, the electron might hit the upper plate.

(b) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0\sin\theta = (6.00\times10^{6}\,\mathrm{m/s})\sin 45° = 4.24\times10^{6}\,\mathrm{m/s}$$

and

$$2ad = 2(3.51\times10^{14}\,\mathrm{m/s^2})(0.0200\,\mathrm{m}) = 1.40\times10^{13}\,\mathrm{m^2/s^2}$$

the solution to $d = v_0 t\sin\theta - \frac{1}{2}at^2$ is

$$t = \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} = \frac{(4.24\times10^{6}\,\mathrm{m/s}) - \sqrt{(4.24\times10^{6}\,\mathrm{m/s})^2 - 1.40\times10^{13}\,\mathrm{m^2/s^2}}}{3.51\times10^{14}\,\mathrm{m/s^2}}$$
$$= 6.43\times10^{-9}\,\mathrm{s}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$x = v_0 t\cos\theta = (6.00\times10^{6}\,\mathrm{m/s})(6.43\times10^{-9}\,\mathrm{s})\cos 45° = 2.72\times10^{-2}\,\mathrm{m}.$$

This is less than L so the electron hits the upper plate at $x = 2.72$ cm.

87. (a) If we subtract each value from the next larger value in the table, we find a set of numbers which are suggestive of a basic unit of charge: 1.64×10^{-19}, 3.3×10^{-19}, 1.63×10^{-19}, 3.35×10^{-19}, 1.6×10^{-19}, 1.63×10^{-19}, 3.18×10^{-19}, 3.24×10^{-19}, where the SI unit Coulomb is understood. These values are either close to a common $e \approx 1.6 \times 10^{-19}\,\text{C}$ value or are double that. Taking this, then, as a crude approximation to our experimental e we divide it into all the values in the original data set and round to the nearest integer, obtaining n = 4,5,7,8,10,11,12,14, and 16.

(b) When we perform a least squares fit of the original data set versus these values for n we obtain the linear equation:

$$q = 7.18 \times 10^{-21} + 1.633 \times 10^{-19} n \, .$$

If we dismiss the constant term as unphysical (representing, say, systematic errors in our measurements) then we obtain $e = 1.63 \times 10^{-19}$ when we set $n = 1$ in this equation.

88. Since both charges are positive (and aligned along the z axis) we have

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-d/2)^2} + \frac{q}{(z+d/2)^2}\right].$$

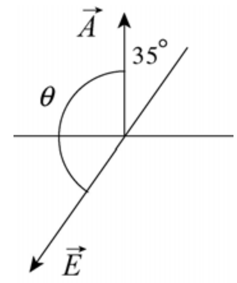
For $z \gg d$ we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$\left|\vec{E}_{\text{net}}\right| \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z^2} + \frac{q}{z^2}\right) = \frac{2q}{4\pi\varepsilon_0 z^2}.$$

Chapter 23

1. The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below. The angle θ between them is 180° – 35° = 145°, so the electric flux through the area is

$$\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta = \left(1800\ \text{N/C}\right)\left(3.2\times10^{-3}\ \text{m}\right)^2 \cos 145° = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}.$$



2. We use $\Phi = \int \vec{E} \cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube y = 2.0 m and $d\vec{A} = (dA)\hat{\mathrm{j}}$. Therefore, we have $\vec{E} = 4\hat{\mathrm{i}} - 3\left((2.0)^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}\right) \cdot (dA)\hat{\mathrm{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2 \ \mathrm{N \cdot m^2/C} = -72 \ \mathrm{N \cdot m^2/C}.$$

(b) On the bottom face of the cube y = 0 and $d\vec{A} = (dA)\left(-\hat{\mathrm{j}}\right)$. Therefore, we have $E = 4\hat{\mathrm{i}} - 3\left(0^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2 \ \mathrm{N \cdot m^2/C} = +24 \ \mathrm{N \cdot m^2/C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{i}}\right)$. So

$$\Phi = \int_{\text{left}} \hat{E} \cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\mathrm{i}} + E_y\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2 \ \mathrm{N \cdot m^2/C} = -16 \ \mathrm{N \cdot m^2/C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{k}}\right)$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 N·m^2/C. Thus the net flux through the cube is

$$\Phi = (-72 + 24 - 16 + 0 + 0 + 16) \ \mathrm{N \cdot m^2/C} = -48 \ \mathrm{N \cdot m^2/C}.$$

3. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\mathrm{j}} = (1.40\mathrm{m})^2\,\hat{\mathrm{j}}$.

(a) $\Phi = (6.00\ \mathrm{N/C})\hat{\mathrm{i}} \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = 0.$

(b) $\Phi = (-2.00\ \mathrm{N/C})\hat{\mathrm{j}} \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = -3.92\ \mathrm{N \cdot m^2/C}.$

(c) $\Phi = \left[(-3.00\ \mathrm{N/C})\hat{\mathrm{i}} + (400\,\mathrm{N/C})\hat{\mathrm{k}}\right] \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

4. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude $(20)(3.0)^2$). With "inward" flux being negative, the result is $\Phi = -486\ \text{N}\cdot\text{m}^2/\text{C}$. Gauss' law then leads to

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-486\ \ \text{N}\cdot\text{m}^2/\text{C}) = -4.3\times10^{-9}\,\text{C}.$$

5. We use Gauss' law: $\varepsilon_0\Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C}.$$

6. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus, the flux through the netting is

$$\Phi' = -\Phi = -\pi a^2 E = -\pi (0.11\ \text{m})^2 (3.0\times10^{-3}\ \text{N/C}) = -1.1\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C}\,.$$

7. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with a proton of charge $q = +1.6\times10^{-19}$ C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \text{N}\cdot\text{m}^2/\text{C}.$$

8. We note that only the smaller shell contributes a (non-zero) field at the designated point, since the point is inside the radius of the large sphere (and E = 0 inside of a spherical charge), and the field points towards the $-x$ direction. Thus, with R = 0.020 m (the radius of the smaller shell), L = 0.10 m and x = 0.020 m, we obtain

$$\vec{E} = E(-\hat{\text{j}}) = -\frac{q}{4\pi\varepsilon_0 r^2}\hat{\text{j}} = -\frac{4\pi R^2\sigma_2}{4\pi\varepsilon_0 (L-x)^2}\hat{\text{j}} = -\frac{R^2\sigma_2}{\varepsilon_0 (L-x)^2}\hat{\text{j}}$$

$$= -\frac{(0.020\text{ m})^2(4.0\times10^{-6}\,\text{C/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.10\text{ m}-0.020\text{ m})^2}\hat{\text{j}} = \left(-2.8\times10^4\text{ N/C}\right)\hat{\text{j}}.$$

9. Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_l be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$q = \varepsilon_0 \Phi = \varepsilon_0 A(E_\ell - E_u) = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(100\ \text{m})^2(100\ \text{N/C} - 60.0\ \text{N/C})$$
$$= 3.54\times10^{-6}\ \text{C} = 3.54\,\mu\text{C}.$$

10. (a) The total surface area bounding the bathroom is

$$A = 2(2.5\times 3.0) + 2(3.0\times 2.0) + 2(2.0\times 2.5) = 37\ \text{m}^2.$$

The absolute value of the total electric flux, with the assumptions stated in the problem, is

$$|\Phi| = |\sum \vec{E}\cdot\vec{A}| = |\vec{E}|\,A = (600\ \text{N/C})(37\ \text{m}^2) = 22\times 10^3\ \text{N}\cdot\text{m}^2/\text{C}.$$

By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\text{enc}}| = \varepsilon_0 |\Phi| = 2.0\times 10^{-7}\ \text{C}$. Therefore, with volume $V = 15\ \text{m}^3$, and recognizing that we are dealing with negative charges, the charge density is

$$\rho = \frac{|q_{\text{enc}}|}{V} = \frac{2.0\times 10^{-7}\ \text{C}}{15\ \text{m}^3} = 1.3\times 10^{-8}\ \text{C/m}^3.$$

(b) We find $(|q_{\text{enc}}|/e)/V = (2.0 \times 10^{-7}\ \text{C}/1.6 \times 10^{-19}\ \text{C})/15\ \text{m}^3 = 8.2 \times 10^{10}$ excess electrons per cubic meter.

11. (a) Let $A = (1.40 \text{ m})^2$. Then

$$\Phi = \left(3.00y\,\hat{\text{j}}\right)\cdot\left(-A\,\hat{\text{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\text{j}}\right)\cdot\left(\text{A}\,\hat{\text{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\text{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\text{i}} + 6.00\hat{\text{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still 8.23 $\text{N}\cdot\text{m}^2/\text{C}$.

(d) The charge is again given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

12. Eq. 23-6 (Gauss' law) gives $\varepsilon_o\Phi = q_{enc}$.

(a) Thus, the value $\Phi = 2.0\times10^5\ \mathrm{N\cdot m^2/C}$ for small r leads to

$$q_{\text{central}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(2.0\times10^5\ \mathrm{N\cdot m^2/C}) = 1.77\times10^{-6}\,\mathrm{C} \approx 1.8\times10^{-6}\,\mathrm{C}.$$

(b) The next value that Φ takes is $\Phi = -4.0\times10^5\ \mathrm{N\cdot m^2/C}$, which implies $q_{\text{enc}} = -3.54\times10^{-6}\,\mathrm{C}$. But we have already accounted for some of that charge in part (a), so the result for part (b) is

$$q_A = q_{\text{enc}} - q_{\text{central}} = -\,5.3\times10^{-6}\,\mathrm{C}.$$

(c) Finally, the large r value for Φ is $\Phi = 6.0\times10^5\ \mathrm{N\cdot m^2/C}$, which implies $q_{\text{total enc}} = 5.31\times10^{-6}\,\mathrm{C}$. Considering what we have already found, then the result is $q_{\text{total enc}} - q_A - q_{\text{central}} = +8.9\mu C$.

13. The total flux through any surface that completely surrounds the point charge is q/ε_0.

(a) If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero.

(b) The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$. Thus, the multiple is $1/24 = 0.0417$.

14. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only. In Si units, we have

$$E_{\text{non-constant}} = 3x\,\hat{\mathrm{i}}\ .$$

The face of the cube located at $x = 0$ (in the yz plane) has area $A = 4\ \mathrm{m}^2$ (and it "faces" the $+\hat{\mathrm{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{non-constant}}A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area A (and this one "faces" the $-\hat{\mathrm{i}}$ direction) and a contribution to the flux:

$$-E_{\text{non-constant}}A = -(3)(-2)(4) = 24\ \mathrm{N{\cdot}m/C^2}.$$

Thus, the net flux is $\Phi = 0 + 24 = 24\ \mathrm{N{\cdot}m/C^2}$. According to Gauss' law, we therefore have $q_{\text{enc}} = \varepsilon_o\,\Phi = 2.13 \times 10^{-10}$ C.

15. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only:

$$E_{\text{non-constant}} = (-4.00y^2)\,\hat{\text{i}} \text{ (in SI units)} .$$

The face of the cube located at y = 4.00 has area A = 4.00 m^2 (and it "faces" the $+\hat{\text{j}}$ direction) and has a "contribution" to the flux equal to

$$E_{\text{non-constant}}A = (-4)(4^2)(4) = -256 \text{ N}\cdot\text{m/C}^2.$$

The face of the cube located at y = 2.00 m has the same area A (however, this one "faces" the $-\hat{\text{j}}$ direction) and a contribution to the flux:

$$-E_{\text{non-constant}}A = -\,(-4)(2^2)(4) = 64 \text{ N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = (-256 + 64)$ N·m/C^2 = -192 N·m/C^2. According to Gauss's law, we therefore have

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-192\ \ \text{N}\cdot\text{m}^2/\text{C}) = -1.70\times10^{-9}\,\text{C}.$$

16. The total electric flux through the cube is $\Phi = \oint \vec{E} \cdot d\vec{A}$. The net flux through the two faces parallel to the *yz* plane is

$$\Phi_{yz} = \iint \left[E_x(x = x_2) - E_x(x = x_1) \right] dydz = \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz \left[10 + 2(4) - 10 - 2(1) \right]$$

$$= 6 \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz = 6(1)(2) = 12.$$

Similarly, the net flux through the two faces parallel to the *xz* plane is

$$\Phi_{xz} = \iint \left[E_y(y = y_2) - E_y(y = y_1) \right] dxdz = \int_{x_1=1}^{x_2=4} dy \int_{z_1=1}^{z_2=3} dz [-3 - (-3)] = 0,$$

and the net flux through the two faces parallel to the *xy* plane is

$$\Phi_{xy} = \iint \left[E_z(z = z_2) - E_z(z = z_1) \right] dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy \left(3b - b \right) = 2b(3)(1) = 6b.$$

Applying Gauss' law, we obtain

$$q_{\text{enc}} = \varepsilon_0 \Phi = \varepsilon_0 (\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0 (6.00b + 0 + 12.0) = 24.0 \varepsilon_0$$

which implies that $b = 2.00 \ \text{N/C} \cdot \text{m}$.

17. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times10^{-6}\ \text{C}}{\pi(1.3\ \text{m})^2} = 4.5\times10^{-7}\ \text{C/m}^2.$$

(b) Eq. 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5\times10^{-7}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.1\times10^{4}\ \text{N/C}.$$

18. Eq. 23-6 (Gauss' law) gives $\varepsilon_o\Phi = q_{enc}$.

(a) The value $\Phi = -9.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$ for small r leads to $q_{central} = -\ 7.97 \times 10^{-6}\ \text{C}$ or roughly – 8.0 μC.

(b) The next (non-zero) value that Φ takes is $\Phi = +4.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$, which implies $q_{enc} = 3.54\times10^{-6}\,\text{C}$. But we have already accounted for some of that charge in part (a), so the result is

$$q_A = q_{enc} - q_{central} = 11.5 \times 10^{-6}\ \text{C} \approx 12\ \mu\text{C}\,.$$

(c) Finally, the large r value for Φ is $\Phi = -2.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$, which implies $q_{\text{total enc}} = -1.77\times10^{-6}\,\text{C}$. Considering what we have already found, then the result is

$$q_{\text{total enc}} - q_A - q_{central} = -5.3\ \mu\text{C}.$$

19. (a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere (which is $4\pi r^2$, where r is the radius). Thus,

$$q = 4\pi r^2\sigma = 4\pi\left(\frac{1.2\text{ m}}{2}\right)^2\left(8.1\times10^{-6}\text{ C/m}^2\right) = 3.7\times10^{-5}\text{ C}.$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss's law:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times10^{-5}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 4.1\times10^6\text{ N}\cdot\text{m}^2/\text{C}.$$

20. Using Eq. 23-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3\times10^5 \ \text{N/C}\right)\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right) = 2.0\times10^{-6} \ \text{C/m}^2.$$

21. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}$C.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_\omega = \left(10\times10^{-6}\ \text{C}\right) - \left(-3.0\times10^{-6}\ \text{C}\right) = +1.3\times10^{-5}\ \text{C}.$$

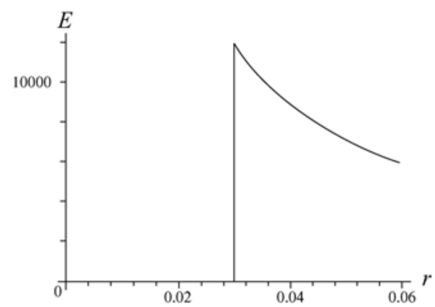
22. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E} \cdot d\vec{A} = 2\pi r E = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r \varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R = 0.0600$ m, we obtain

$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^{3}\ \text{N/C}.$$

(c) The plot of E vs. r is shown below.



Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r \varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^{4}\ \text{N/C}.$$

23. The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 23-12. Thus,

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.5\times10^{4}\ \mathrm{N/C}\right)\left(2.0\ \mathrm{m}\right) = 5.0\times10^{-6}\ \mathrm{C/m}.$$

24. We combine Newton's second law ($F = ma$) with the definition of electric field ($F = qE$) and with Eq. 23-12 (for the field due to a line of charge). In terms of magnitudes, we have (if $r = 0.080$ m and $\lambda = 6.0 \times 10^{-6}\,\text{C/m}$)

$$ma = eE = \frac{e\lambda}{2\pi\varepsilon_o r} \quad \Rightarrow \quad a = \frac{e\lambda}{2\pi\varepsilon_o r\, m} = 2.1 \times 10^{17} \text{ m/s}^2 \; .$$

25. (a) The side surface area A for the drum of diameter D and length h is given by $A = \pi Dh$. Thus,

$$q = \sigma A = \sigma \pi Dh = \pi \varepsilon_0 EDh = \pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.3\times10^5\ \text{N/C}\right)\left(0.12\ \text{m}\right)\left(0.42\ \text{m}\right)$$
$$= 3.2\times10^{-7}\ \text{C}.$$

(b) The new charge is

$$q' = q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D'h'}{\pi Dh}\right) = \left(3.2\times10^{-7}\,\text{C}\right)\left[\frac{(8.0\ \text{cm})(28\ \text{cm})}{(12\ \text{cm})(42\ \text{cm})}\right] = 1.4\times10^{-7}\ \text{C}.$$

26. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0(x + L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0(x - L/2)} .$$

Setting this equal to zero and solving for x we find

$$x = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)\frac{L}{2} = \left(\frac{6.0\mu\text{C/m} - (-2.0\mu\text{C/m})}{6.0\mu\text{C/m} + (-2.0\mu\text{C/m})}\right)\frac{8.0\text{ cm}}{2} = 8.0\text{ cm} .$$

27. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q_{enc} = Q1 + Q_2 = -Q_1 = -3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(20.0\times1.30\times10^{-3}\text{m})} = -0.214\ \text{N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in E indicates that the field points inward.

(c) Next, for $r = 5.00\ R_1$, the charge enclosed by the Gaussian surface is $q_{enc} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\ \text{N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge Q_1, the inner surface of the shell must have charge $Q_{in} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{out} = Q_2 - Q_{in} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.

28. As we approach $r = 3.5$ cm from the inside, we have

$$E_{\text{internal}} = \frac{2\lambda}{4\pi\varepsilon_0 r} = 1000 \text{ N/C} .$$

And as we approach $r = 3.5$ cm from the outside, we have

$$E_{\text{external}} = \frac{2\lambda}{4\pi\varepsilon_0 r} + \frac{2\lambda'}{4\pi\varepsilon_0 r} = -3000 \text{ N/C} \ .$$

Considering the difference ($E_{\text{external}} - E_{\text{internal}}$) allows us to find λ' (the charge per unit length on the larger cylinder). Using $r = 0.035$ m, we obtain $\lambda' = -5.8 \times 10^{-9}$ C/m.

29. We denote the inner and outer cylinders with subscripts i and o, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\ \text{m})} = 2.3\times10^{6}\ \text{N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r = 8.0\ \text{cm}) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m} - 7.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})} = -4.5\times10^{5}\ \text{N/C},$$

or $|E(r = 8.0\ \text{cm})| = 4.5\times10^{5}\ \text{N/C}$.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

30. (a) In Eq. 23-12, $\lambda = q/L$ where q is the net charge enclosed by a cylindrical Gaussian surface of radius r. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$\left|\vec{E}\right| = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2(2.0\times10^{-9}\text{C/m})}{4\pi\varepsilon_0(0.15\text{ m})} = 2.4\times10^{2}\text{ N/C.}$$

(b) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where q is the charge on the rod at the center). Therefore, with $r_i = 0.05$ m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -\frac{2.0\times10^{-9}\text{C/m}}{2\pi(0.050\text{ m})} = -6.4\times10^{-9}\text{ C/m}^2$$

for the inner surface.

(c) With $r_o = 0.10$ m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2\times10^{-9}\text{ C/m}^2.$$

31. We denote the radius of the thin cylinder as R = 0.015 m. Using Eq. 23-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6$ nC/m is the linear charge density of the wire and λ' is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \Rightarrow \lambda' = \sigma(2\pi R).$$

Now, E_{net} outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times10^{-6}\ \text{C/m}}{(2\pi)(0.015\ \text{m})} = 3.8\times10^{-8}\ \text{C/m}^2.$$

32. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi\, r\, L$ where L is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi\, r^2\, L$, or expressed more appropriate to our needs: $dV = 2\pi\, r\, L\, dr$. The charge enclosed is, with $A = 2.5\times10^{-6}\,\text{C/m}^5$,

$$q_{\text{enc}} = \int_0^r A r^2\, 2\pi r\, L\, dr = \frac{\pi}{2} A L r^4 .$$

By Gauss' law, we find $\Phi = |\vec{E}|\,(2\pi r L) = q_{\text{enc}}/\varepsilon_0;$ we thus obtain $\left|\vec{E}\right| = \dfrac{A r^3}{4\varepsilon_0}.$

(a) With $r = 0.030$ m, we find $|\vec{E}| = 1.9$ N/C.

(b) Once outside the cylinder, Eq. 23-12 is obeyed. To find $\lambda = q/L$ we must find the total charge q. Therefore,

$$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A r^2\, 2\pi\, r\, L\, dr = 1.0\times10^{-11}\ \text{C/m}.$$

And the result, for $r = 0.050$ m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0\, r = 3.6$ N/C.

33. In the region between sheets 1 and 2, the net field is $E_1 - E_2 + E_3 = 2.0 \times 10^5$ N/C .

In the region between sheets 2 and 3, the net field is at its greatest value:

$$E_1 + E_2 + E_3 = 6.0 \times 10^5 \text{ N/C} .$$

The net field vanishes in the region to the right of sheet 3, where $E_1 + E_2 = E_3$. We note the implication that σ_3 is negative (and is the largest surface-density, in magnitude). These three conditions are sufficient for finding the fields:

$$E_1 = 1.0 \times 10^5 \text{ N/C} , \; E_2 = 2.0 \times 10^5 \text{ N/C} , \; E_3 = 3.0 \times 10^5 \text{ N/C} .$$

From Eq. 23-13, we infer (from these values of E)

$$\frac{|\sigma_3|}{|\sigma_2|} = \frac{3.0 \text{ x } 10^5 \text{ N/C}}{2.0 \text{ x } 10^5 \text{ N/C}} = 1.5 .$$

Recalling our observation, above, about σ_3, we conclude $\dfrac{\sigma_3}{\sigma_2} = -1.5$.

34. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22}\ \text{C/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 2.00\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\ \text{N/C})\hat{\text{j}}$.

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\ \text{N/C})\hat{\text{j}}$.

35. (a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus,

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\ \text{C}}{2(0.080\ \text{m})^2} = 4.69\times10^{-4}\ \text{C/m}^2.$$

The magnitude of the field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.69\times10^{-4}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^{7}\ \text{N/C}.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where r is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\ \text{C}\right)}{\left(30\ \text{m}\right)^2} = 60\ \text{N/C}.$$

36. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density $\sigma = 4.50\ \times 10^{-12}\ \text{C/m}^2$ plus a small circular pad of radius R = 1.80 cm located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for $\vec{E}_2$, the net electric field $\vec{E}$ at a distance z = 2.56 cm along the central axis is then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\text{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)\hat{\text{k}} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2 + R^2}}\hat{\text{k}}$$

$$= \frac{(4.50\times 10^{-12}\ \text{C/m}^2)(2.56\times 10^{-2}\ \text{m})}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\sqrt{(2.56\times 10^{-2}\ \text{m})^2 + (1.80\times 10^{-2}\ \text{m})^2}}\hat{\text{k}} = (0.208\ \text{N/C})\ \hat{\text{k}}$$

37. We use Eq. 23-13.

(a) To the left of the plates:

$$\vec{E} = (\sigma/2\varepsilon_0)(-\hat{\mathrm{i}}) \text{ (from the right plate) } + (\sigma/2\varepsilon_0)\hat{\mathrm{i}} \text{ (from the left one)} = 0.$$

(b) To the right of the plates:

$$\vec{E} = (\sigma/2\varepsilon_0)\hat{\mathrm{i}} \text{ (from the right plate)} + (\sigma/2\varepsilon_0)(-\hat{\mathrm{i}}) \text{ (from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\mathrm{i}}) = -\left(\frac{7.00\times10^{-22}\ \mathrm{C/m^2}}{8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}}\right)\hat{\mathrm{i}} = \left(-7.91\times10^{-11}\ \mathrm{N/C}\right)\hat{\mathrm{i}}.$$

38. We use the result of part (c) of Problem 23-35 to obtain the surface charge density.

$$E = \sigma / \varepsilon_0 \Rightarrow \sigma = \varepsilon_0 E = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(55\ \text{N/C}) = 4.9\times10^{-10}\ \text{C/m}^2.$$

Since the area of the plates is $A = 1.0\ \text{m}^2$, the magnitude of the charge on the plate is $Q = \sigma A = 4.9\times10^{-10}\ \text{C}$.

39. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

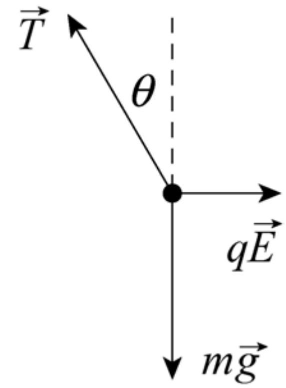
$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(1.60\times10^{-17}\ \mathrm{J}\right)}{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(2.0\times10^{-6}\ \mathrm{C/m^2}\right)} = 4.4\times10^{-4}\ \mathrm{m}.$$

40. The field due to the sheet is $E = \frac{\sigma}{2\varepsilon_0}$. The force (in magnitude) on the electron (due to that field) is $F = eE$, and assuming it's the *only* force then the acceleration is

$$a = \frac{e\sigma}{2\varepsilon_0 m} = \text{slope of the graph } (= 2.0 \times 10^5 \text{ m/s divided by } 7.0 \times 10^{-12} \text{ s}) .$$

Thus we obtain $\sigma = 2.9 \times 10^{-6}$ C/m^2.

41. The forces acting on the ball are shown in the diagram on the right. The gravitational force has magnitude *mg*, where *m* is the mass of the ball; the electrical force has magnitude *qE*, where *q* is the charge on the ball and *E* is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by *T*. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T\sin\theta = 0$$

and the sum of the vertical components yields

$$T\cos\theta - mg = 0\,.$$

The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N.m^2}\right)\left(1.0\times10^{-6}\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)\tan 30°}{2.0\times10^{-8}\ \mathrm{C}}$$
$$= 5.0\times10^{-9}\ \mathrm{C/m^2}.$$

42. The point where the individual fields cancel cannot be in the region between the sheet and the particle ($-d < x < 0$) since the sheet and the particle have opposite-signed charges. The point(s) could be in the region to the right of the particle ($x > 0$) and in the region to the left of the sheet ($x < d$); this is where the condition

$$\frac{|\sigma|}{2\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

must hold. Solving this with the given values, we find $r = x = \pm\sqrt{3/2\pi} \approx \pm\, 0.691$ m.

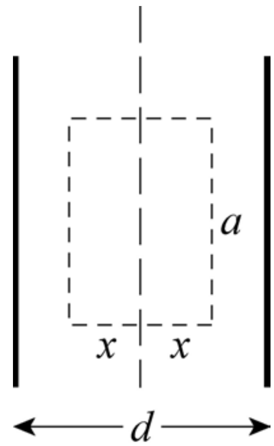
If $d = 0.20$ m (which is less than the magnitude of r found above), then neither of the points ($x \approx \pm\, 0.691$ m) is in the "forbidden region" between the particle and the sheet. Thus, both values are allowed. Thus, we have

(a) $x = 0.691$ m on the positive axis, and

(b) $x = -\,0.691$ m on the negative axis.

(c) If, however, $d = 0.80$ m (greater than the magnitude of r found above), then one of the points ($x \approx -0.691$ m) is in the "forbidden region" between the particle and the sheet and is disallowed. In this part, the fields cancel only at the point $x \approx +0.691$ m.

43. We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram below. It is centered at the central plane of the slab, so the left and right faces are each a distance x from the central plane. We take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares.



The electric field is normal to the left and right faces and is uniform over them. Since ρ = 5.80 fC/m^3 is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$. The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2 x\rho$. Gauss' law yields

$$2\varepsilon_0 Ea^2 = 2a^2 x\rho.$$

We solve for the magnitude of the electric field: $E = \rho x/\varepsilon_0$.

(a) For x =0, E =0.

(b) For x = 2.00 mm = 2.00 × 10^{-3} m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(2.00\times10^{-3}\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.31\times10^{-6}\ \text{N/C}.$$

(c) For $x = d/2$ = 4.70 mm = 4.70 × 10^{-3} m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(4.70\times10^{-3}\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 3.08\times10^{-6}\ \text{N/C}.$$

(d) For x = 26.0 mm = 2.60 × 10^{-2} m, we take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2 d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2 d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(9.40\times10^{-3}\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.08\times10^{-6}\ \text{N/C}.$$

44. (a) The flux is still $-750\ \mathrm{N\cdot m^2/C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge q:

$$q = \varepsilon_0\Phi = \left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(-750\ \mathrm{N\cdot m^2/C}\right) = -6.64\times10^{-9}\ \mathrm{C}.$$

45. Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = |q|/4\pi\varepsilon_0 r^2$, where $|q|$ is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus,

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.15\text{ m})^2(3.0\times10^3\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 7.5\times10^{-9}\text{ C}.$$

The field points inward, toward the sphere center, so the charge is negative, i.e., $q = -7.5\times10^{-9}\,\text{C}$.

46. We determine the (total) charge on the ball by examining the maximum value (E = 5.0 × 10^7 N/C) shown in the graph (which occurs at r = 0.020 m). Thus, from $E = q/4\pi\varepsilon_0 r^2$, we obtain

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.020\text{ m})^2(5.0\times10^7\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.2\times10^{-6}\text{C} \ .$$

47. (a) Since $r_1 = 10.0$ cm $< r = 12.0$ cm $< r_2 = 15.0$ cm,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-8}\text{ C}\right)}{\left(0.120\text{ m}\right)^2} = 2.50\times10^4 \text{ N/C}.$$

(b) Since $r_1 < r_2 < r = 20.0$ cm,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00+2.00\right)\left(1\times10^{-8}\text{ C}\right)}{\left(0.200\text{ m}^2\right)} = 1.35\times10^4 \text{ N/C}.$$

48. The point where the individual fields cancel cannot be in the region between the shells since the shells have opposite-signed charges. It cannot be inside the radius R of one of the shells since there is only one field contribution there (which would not be canceled by another field contribution and thus would not lead to zero net field). We note shell 2 has greater magnitude of charge ($|\sigma_2|A_2$) than shell 1, which implies the point is not to the right of shell 2 (any such point would always be closer to the larger charge and thus no possibility for cancellation of equal-magnitude fields could occur). Consequently, the point should be in the region to the left of shell 1 (at a distance $r > R_1$ from its center); this is where the condition

$$E_1 = E_2 \quad \Rightarrow \quad \frac{|q_1|}{4\pi\varepsilon_0 r^2} = \frac{|q_2|}{4\pi\varepsilon_0 (r+L)^2}$$

or

$$\frac{\sigma_1 A_1}{4\pi\varepsilon_0 r^2} = \frac{|\sigma_2| A_2}{4\pi\varepsilon_0 (r+L)^2} \ .$$

Using the fact that the area of a sphere is $A = 4\pi R^2$, this condition simplifies to

$$r = \frac{L}{(R_2/R_1)\sqrt{|\sigma_2|/\sigma_1} \ - \ 1} \ = \ 3.3 \text{ cm} \ .$$

We note that this value satisfies the requirement $r > R_1$. The answer, then, is that the net field vanishes at $x = -r = -3.3$ cm.

49. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi \, A \int_a^{r_g} r \, dr = 2\pi \, A \left(r_g^2 - a^2 \right).$$

The total charge inside the Gaussian surface is

$$q + q_s = q + 2\pi \, A \left(r_g^2 - a^2 \right).$$

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi \varepsilon_0 E r_g^2 = q + 2\pi \, A \left(r_g^2 - a^2 \right).$$

We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r_g^2} + 2\pi \, A - \frac{2\pi A a^2}{r_g^2} \right].$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$. With $a = 2.00 \times 10^{-2}$ m and $q = 45.0 \times 10^{-15}$ C, we have $A = 1.79 \times 10^{-11} \, \mathrm{C/m^2}$.

50. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For $r = 1.50a$, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{2.375}{2.25}\right) = 7.32\ \text{N/C}.$$

(e) For $r = b = 2.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{4}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{4}\right) = 12.1\ \text{N/C}.$$

(f) For $r \ge b$ we have $E = q_{\text{total}} / 4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for $r = 3.00b = 6.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{36}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{36}\right) = 1.35\ \text{N/C}.$$

51. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{2(2.00\times10^{-2}\,\mathrm{m})^2} = 5.62\times10^{-2}\ \mathrm{N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{(2.00\times10^{-2}\,\mathrm{m})^2} = 0.112\ \mathrm{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{(1.50\times2.00\times10^{-2}\,\mathrm{m})^2} = 0.0499\ \mathrm{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0$. Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E}\cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

52. Let E_A designate the magnitude of the field at $r = 2.4$ cm. Thus $E_A = 2.0 \times 10^7$ N/C, and is totally due to the particle. Since $E_{\text{particle}} = q/4\pi\varepsilon_0 r^2$, then the field due to the particle at any other point will relate to E_A by a ratio of distances squared. Now, we note that at $r = 3.0$ cm the total contribution (from particle and shell) is 8.0×10^7 N/C. Therefore,

$$E_{\text{shell}} + E_{\text{particle}} = E_{\text{shell}} + (2.4/3)^2 E_A = 8.0 \times 10^7 \text{ N/C} .$$

Using the value for E_A noted above, we find $E_{\text{shell}} = 6.6 \times 10^7$ N/C. Thus, with $r = 0.030$ m, we find the charge Q using $E_{\text{shell}} = Q/4\pi\varepsilon_0 r^2$:

$$Q = 4\pi\varepsilon_0 r^2 E_{\text{shell}} = \frac{r^2 E_{\text{shell}}}{k} = \frac{(0.030 \text{ m})^2 (6.6\times10^7 \text{ N/C})}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 6.6\times10^{-6}\,\text{C} .$$

53. We use

$$E(r) = \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r)4\pi r^2 dr$$

to solve for $\rho(r)$ and obtain

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\left[r^2 E(r)\right] = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\left(Kr^6\right) = 6K\varepsilon_0 r^3.$$

54. Applying Eq. 23-20, we have

$$E_1 = \frac{|q_1|}{4\pi\varepsilon_0 R^3} r_1 = \frac{|q_1|}{4\pi\varepsilon_0 R^3}\left(\frac{R}{2}\right) = \frac{1}{2}\frac{|q_1|}{4\pi\varepsilon_0 R^2} .$$

Also, outside sphere 2 we have

$$E_2 = \frac{|q_2|}{4\pi\varepsilon_0 r^2} = \frac{|q_2|}{4\pi\varepsilon_0 (1.50R)^2} .$$

Equating these and solving for the ratio of charges, we arrive at $\frac{q_2}{q_1} = \frac{9}{8} = 1.125$.

55. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi \int_0^R dr\; r^2\; \rho = Q.$$

Substituting the expression $\rho = \rho_s r/R$, with ρ_s= 14.1 pC/m^3, and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q$$

or

$$Q = \pi\rho_s R^3 = \pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})^3 = 7.78\times10^{-15}\ \text{C}.$$

(b) At $r = 0$, the electric field is zero ($E = 0$) since the enclosed charge is zero.

At a certain point within the sphere, at some distance r from the center, the field (see Eq. 23-8 through Eq. 23-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2}$$

where q_{enc} is given by an integral similar to that worked in part (a):

$$q_{\text{enc}} = 4\pi\int_0^r dr\, r^2 \rho = 4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r^4}{4}\right).$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^4}{Rr^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^2}{R}.$$

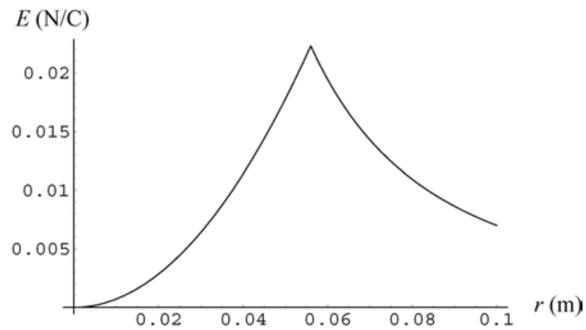
(c) For $r = R/2.00$, where $R = 5.60$ cm, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s (R/2.00)^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R}{4.00} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})}{4.00}$$
$$= 5.58\times10^{-3}\ \text{N/C}.$$

(d) For $r = R$, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R^2}{R} = \frac{\pi\rho_s R}{4\pi\varepsilon_0} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})$$
$$= 2.23\times10^{-2}\ \text{N/C}.$$

(e) The electric field strength as a function of r is depicted below:



56. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0 (2\pi r L)} = \frac{|\rho| r}{2\varepsilon_0}.$$

(b) We note from the above expression that the magnitude of the radial field grows with r.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of r which encloses charged material is $r_{\max} = R$. Therefore, with $|\rho| = 0.0011 \text{ C/m}^3$ and $R = 0.050$ m, we obtain

$$E_{\max} = \frac{|\rho| R}{2\varepsilon_0} = \frac{(0.0011 \text{ C/m}^3)(0.050 \text{ m})}{2(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 3.1\times10^6 \text{ N/C}.$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

57. (a) Since the volume contained within a radius of $\frac{1}{2}R$ is one-eighth the volume contained within a radius of R, so the charge at $0 < r < R/2$ is $Q/8$. The fraction is 1/8 = 0.125.

(b) At $r = R/2$, the magnitude of the field is

$$E = \frac{Q/8}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}$$

and is equivalent to *half* the field at the surface. Thus, the ratio is 0.500.

58. Since the charge distribution is uniform, we can find the total charge q by multiplying ρ by the spherical volume ($\frac{4}{3}\pi r^3$) with $r = R = 0.050$ m. This gives $q = 1.68$ nC.

(a) Applying Eq. 23-20 with $r = 0.035$ m, we have $E_{\text{internal}} = \dfrac{|q|r}{4\pi\varepsilon_0 R^3} = 4.2\times10^3 \text{ N/C}$.

(b) Outside the sphere we have (with $r = 0.080$ m)

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.68\times10^{-9}\text{C})}{(0.080 \text{ m})^2} = 2.4\times10^3 \text{ N/C} .$$

59. The initial field (evaluated "just outside the outer surface" which means it is evaluated at $r = 0.20$ m) is related to the charge q on the hollow conductor by Eq. 23-15. After the point charge Q is placed at the geometric center of the hollow conductor, the final field at that point is a combination of the initial and that due to Q (determined by Eq. 22-3).

(a) $q = 4\pi\varepsilon_o r^2 E_{\text{initial}} = +2.0 \times 10^{-9}$ C.

(b) $Q = 4\pi\varepsilon_o r^2(E_{\text{final}} - E_{\text{initial}}) = -1.2 \times 10^{-9}$ C.

(c) In order to cancel the field (due to Q) within the conducting material, there must be an amount of charge equal to $-Q$ distributed uniformly on the inner surface. Thus, the answer is $+1.2 \times 10^{-9}$ C.

(d) Since the total excess charge on the conductor is q and is located on the surfaces, then the outer surface charge must equal the total minus the inner surface charge. Thus, the answer is $2.0 \times 10^{-9}\text{ C} - 1.2 \times 10^{-9}\text{ C} = +0.80 \times 10^{-9}$ C.

60. The field at the proton's location (but not caused by the proton) has magnitude E. The proton's charge is e. The ball's charge has magnitude q. Thus, as long as the proton is at $r \geq R$ then the force on the proton (caused by the ball) has magnitude

$$F = eE = e\left(\frac{q}{4\pi\varepsilon_0 r^2}\right) = \frac{e\,q}{4\pi\varepsilon_0 r^2}$$

where r is measured from the center of the ball (to the proton).This agrees with Coulomb's law from Chapter 22. We note that if $r = R$ then this expression becomes

$$F_R = \frac{e\,q}{4\pi\varepsilon_0 R^2}\,.$$

(a) If we require $F = \frac{1}{2}F_R$, and solve for r, we obtain $r = \sqrt{2}\,R$. Since the problem asks for the measurement from the surface then the answer is $\sqrt{2}\,R - R = 0.41R$.

(b) Now we require $F_{\text{inside}} = \frac{1}{2}F_R$ where $F_{\text{inside}} = eE_{\text{inside}}$ and E_{inside} is given by Eq. 23-20. Thus,

$$e\left(\frac{q}{4\pi\varepsilon_0 R^2}\right)r = \frac{1}{2}\,\frac{e\,q}{4\pi\varepsilon_0 R^2} \quad \Rightarrow \quad r = \tfrac{1}{2}R = 0.50\,R\,.$$

61. (a) At $x = 0.040$ m, the net field has a rightward ($+x$) contribution (computed using Eq. 23-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward ($-x$) contribution (again computed using Eq. 23-13) from the charge in the region from $x = 0.040 \text{ m}$ to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$\left|\vec{E}\right| = \frac{\rho(0.090\,\text{m})}{2\varepsilon_0} - \frac{\rho(0.010\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\,\text{C/m}^3)(0.090\,\text{m} - 0.010\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.4\,\text{N/C}.$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$\left|\vec{E}\right| = \frac{\rho(0.100\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\,\text{C/m}^3)(0.100\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.8\,\text{N/C}.$$

62. From Gauss's law, we have

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} = \frac{(8.0\times10^{-9}\,\text{C/m}^2)\pi(0.050\text{ m})^2}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 7.1\,\text{N}\cdot\text{m}^2/\text{C} \; .$$

63. (a) For $r < R$, $E = 0$ (see Eq. 23-16).

(b) For r slightly greater than R,

$$E_R = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(2.00\times10^{-7}\,\mathrm{C}\right)}{\left(0.250\mathrm{m}\right)^2} = 2.88\times10^4\ \mathrm{N/C}.$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = \left(2.88\times10^4\ \mathrm{N/C}\right)\left(\frac{0.250\ \mathrm{m}}{3.00\ \mathrm{m}}\right)^2 = 200\,\mathrm{N/C}.$$

64. (a) There is no flux through the sides, so we have two contributions to the flux, one from the $x = 2$ end (with $\Phi_2 = +(2 + 2)(\pi (0.20)^2) = 0.50$ N·m^2/C) and one from the $x = 0$ end (with $\Phi_0 = -(2)(\pi (0.20)^2)$).

(b) By Gauss' law we have $q_{enc} = \varepsilon_0 (\Phi_2 + \Phi_0) = 2.2 \times 10^{-12}$ C.

65. Since the fields involved are uniform, the precise location of *P* is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\mathrm{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}\,\mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} = 5.65\times10^4\ \mathrm{N/C}.$$

In unit-vector notation, we have $\vec{E} = (5.65\times10^4\ \mathrm{N/C})\hat{\mathrm{j}}$.

66. Let $\Phi_0 = 10^3 \text{N}\cdot\text{m}^2/\text{C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6} \Phi_n = \sum_{n=1}^{6} (-1)^n n\,\Phi_0 = \Phi_0(-1+2-3+4-5+6) = 3\Phi_0 \,.$$

Thus, the net charge enclosed is

$$q = \varepsilon_0 \Phi = 3\varepsilon_0 \Phi_0 = 3\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10^3\,\text{N}\cdot\text{m}^2/\text{C}\right) = 2.66\times10^{-8}\,\text{C}.$$

67. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the *xy* plane and the rest of the hemisphere is in the $z > 0$ half space.

(a) $\Phi = \pi R^2 \left(-\hat{\mathrm{k}}\right) \cdot E\hat{\mathrm{k}} = -\pi R^2 E = -\pi(0.0568\ \mathrm{m})^2(2.50\ \mathrm{N/C}) = -0.0253\ \mathrm{N\cdot m^2/C}.$

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E = 0.0253\ \mathrm{N\cdot m^2/C}.$

68. (a) The direction of the electric field at P_1 is away from q_1 and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.0\times10^{-7}\ \mathrm{C})}{(0.015\,\mathrm{m})^2} = 4.0\times10^6\ \mathrm{N/C}.$$

(b) $\vec{E}=0$, since P_2 is inside the metal.

69. We use Eqs. 23-15, 23-16 and the superposition principle.

(a) $E = 0$ in the region inside the shell.

(b) $E = q_a / 4\pi\varepsilon_0 r^2$.

(c) $E = (q_a + q_b) / 4\pi\varepsilon_0 r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

70. The net enclosed charge q is given by

$$q = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(-48\ \mathrm{N\cdot m^2/C}\right) = -4.2\times10^{-10}\,\mathrm{C}.$$

71. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = eq/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{eq}{r^2} = \frac{mv^2}{r}$$

so

$$q = \frac{4\pi\varepsilon_0 mv^2 r}{e} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^{5}\text{ m/s}\right)^2\left(0.0100\text{ m}\right)}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-9}\text{ C}\right)} = 1.04\times10^{-9}\text{ C}.$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04 \times 10^{-9}$ C.

72. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius r of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume q is positive for brevity), then

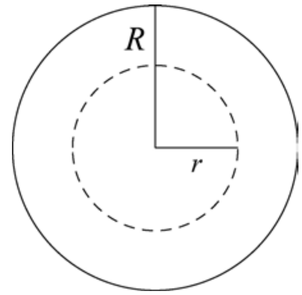
$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 22-3).

73. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius R and length L, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by q. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this Φ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

$$q = 2\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.014\ \mathrm{m})(0.16\ \mathrm{m})\ (2.9\times10^{4}\ \mathrm{N/C}) = 3.6\times10^{-9}\ \mathrm{C}.$$

74. (a) The diagram shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle).



Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dotted circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell \rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder.

If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \;\Rightarrow\; E = \frac{\rho r}{2\varepsilon_0}.$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \;\Rightarrow\; E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r}.$$

75. (a) The mass flux is $wd\rho v$ = (3.22 m) (1.04 m) (1000 kg/m^3) (0.207 m/s) = 693 kg/s.

(b) Since water flows only through area wd, the flux through the larger area is still 693 kg/s.

(c) Now the mass flux is $(wd/2)\rho v$ = (693 kg/s)/2 = 347 kg/s.

(d) Since the water flows through an area (wd/2), the flux is 347 kg/s.

(e) Now the flux is $\left(wd\cos\theta\right)\rho v = \left(693\,\text{kg/s}\right)\left(\cos 34°\right) = 575\ \text{kg/s}$.

76. (a) We use $m_e g = eE = e\sigma/\varepsilon_0$ to obtain the surface charge density.

$$\sigma = \frac{m_e g \varepsilon_0}{e} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(9.8\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}{1.60\times10^{-19}\ \mathrm{C}} = 4.9\times10^{-22}\ \mathrm{C/m^2}\,.$$

(b) Downward (since the electric force exerted on the electron must be upward).

77. (a) From Gauss' law, we get

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{encl}}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{\left(4\pi\rho r^3/3\right)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0}.$$

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)\left(\vec{r}-\vec{a}\right)}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

78. (a) The cube is totally within the spherical volume, so the charge enclosed is

$$q_{\text{enc}} = \rho\, V_{\text{cube}} = (500 \times 10^{-9}\ \text{C/m}^3)(0.0400\ \text{m})^3 = 3.20 \times 10^{-11}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 3.62\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is

$$q_{\text{enc}} = \rho\, V_{\text{sphere}} = 4.5 \times 10^{-10}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 51.1\ \text{N}\cdot\text{m}^2/\text{C}$.

79. (a) In order to have net charge –10 μC when –14 μC is known to be on the outer surface, then there must be +4.0 μC on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) In order to cancel the electric field inside the conducting material, the contribution from the +4 μC on the inner surface must be canceled by that of the charged particle in the hollow. Thus, the particle's charge is –4.0 μC.

80. (a) Outside the sphere, we use Eq. 23-15 and obtain

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}=\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(6.00\times10^{-12}\,\mathrm{C})}{(0.0600\ \mathrm{m})^2}=15.0\,\mathrm{N/C}.$$

(b) With q = +6.00 × 10^{-12} C, Eq. 23-20 leads to $E=25.3\,\mathrm{N/C}$.

81. (a) The field maximum occurs at the outer surface:

$$E_{\text{max}} = \left(\frac{|q|}{4\pi\varepsilon_0 r^2}\right)_{\text{at } r = R} = \frac{|q|}{4\pi\varepsilon_0 R^2}$$

Applying Eq. 23-20, we have

$$E_{\text{internal}} = \frac{|q|}{4\pi\varepsilon_0 R^3} r = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = \frac{R}{4} = 0.25\, R\ .$$

(b) Outside sphere 2 we have

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_0 r^2} = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = 2.0R\ .$$

82. The field due to a sheet of charge is given by Eq. 23-13. Both sheets are horizontal (parallel to the *xy* plane), producing vertical fields (parallel to the *z* axis). At points above the $z = 0$ sheet (sheet *A*), its field points upward (towards $+z$); at points above the $z = 2.0$ sheet (sheet *B*), its field does likewise. However, below the $z = 2.0$ sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 - 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.82\times10^2\ \text{N/C}.$$

(b) The magnitude of the net field at points above both sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 + 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.21\times10^2\ \text{N/C}.$$

Chapter 24

1. If the electric potential is zero at infinity then at the surface of a uniformly charged sphere it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. Thus $q = 4\pi\varepsilon_0 RV$ and the number of electrons is

$$n = \frac{|q|}{e} = \frac{4\pi\varepsilon_0 R|V|}{e} = \frac{\left(1.0\times10^{-6}\ \text{m}\right)\left(400\ \text{V}\right)}{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)} = 2.8\times10^{5}\ .$$

2. The magnitude is $\Delta U = e\Delta V = 1.2 \times 10^9$ eV = 1.2 GeV.

3. (a) An Ampere is a Coulomb per second, so

$$84\ \mathrm{A}\cdot\mathrm{h} = \left(84\,\frac{\mathrm{C}\cdot\mathrm{h}}{\mathrm{s}}\right)\left(3600\,\frac{\mathrm{s}}{\mathrm{h}}\right) = 3.0\times10^{5}\ \mathrm{C}.$$

(b) The change in potential energy is $\Delta U = q\Delta V = (3.0 \times 10^5\ \mathrm{C})(12\ \mathrm{V}) = 3.6 \times 10^6\ \mathrm{J}$.

4. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -\,(3.94 \times 10^{-19}\ \text{J})/(-1.60 \times 10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (Since C and B are on the same equipotential line).

5. The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by

$$\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x.$$

Thus,

$$\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(50\,\text{V})}{0.10\times10^{-6}\,\text{C/m}^2} = 8.8\times10^{-3}\,\text{m}.$$

6. (a) $E = F/e = \left(3.9\times10^{-15}\ \text{N}\right)/\left(1.60\times10^{-19}\ \text{C}\right) = 2.4\times10^{4}\ \text{N/C} = 2.4\times10^{4}\ \text{V/m}.$

(b) $\Delta V = E\Delta s = \left(2.4\times10^{4}\ \text{N/C}\right)\left(0.12\ \text{m}\right) = 2.9\times10^{3}\ \text{V}.$

7. (a) The work done by the electric field is

$$W=\int_i^f q_0\vec{E}\cdot d\vec{s}=\frac{q_0\sigma}{2\varepsilon_0}\int_0^d dz=\frac{q_0\sigma d}{2\varepsilon_0}=\frac{(1.60\times10^{-19}\,\mathrm{C})(5.80\times10^{-12}\ \mathrm{C/m^2})(0.0356\ \mathrm{m})}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}$$

$$=1.87\times10^{-21}\,\mathrm{J}.$$

(b) Since $V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0$, with V_0 set to be zero on the sheet, the electric potential at P is

$$V=-\frac{\sigma z}{2\varepsilon_0}=-\frac{(5.80\times10^{-12}\ \mathrm{C/m^2})(0.0356\ \mathrm{m})}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}=-1.17\times10^{-2}\ \mathrm{V}.$$

8. (a) By Eq. 24-18, the change in potential is the negative of the “area” under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E} \cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30$ V.

(b) For any region within $0 < x < 3\,\text{m}, -\int \vec{E} \cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E} \cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{max} = 40$ V.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3$ m such that the “area” from $x = 3$ m to $x = X$ is 40 V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40.$$

Therefore, $X = 5.5$ m.

9. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E} \cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E} \cdot d\vec{s} = -4.00 \int_0^4 x\,dx = -4.00 \left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0$ V.

10. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\mathrm{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}\ \mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} + \frac{25\times10^{-9}\ \mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}\right)\hat{\mathrm{i}} = -(4.2\times10^{3}\ \mathrm{N/C})\hat{\mathrm{i}}\,.$$

In the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}\,\mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}\hat{\mathrm{i}} + \frac{25\times10^{-9}\ \mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}\hat{\mathrm{i}} = -(1.4\times10^{3}\ \mathrm{N/C})\hat{\mathrm{i}}\,.$$

Therefore, by Eq. 24-18, we have

$$\begin{aligned}\Delta V &= -\int_0^{0.8} \vec{E}\cdot d\vec{s} = -\int_0^{0.5} \left|\vec{E}_{\text{in}}\right| dx - \int_{0.5}^{0.8} \left|\vec{E}_{\text{out}}\right| dx = -\left(4.2\times10^{3}\right)\left(0.5\right) - \left(1.4\times10^{3}\right)\left(0.3\right)\\ &= 2.5\times10^{3}\ \mathrm{V}.\end{aligned}$$

11. (a) The potential as a function of r is

$$V(r)=V(0)-\int_0^r E(r)dr=0-\int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr=-\frac{qr^2}{8\pi\varepsilon_0 R^3}$$

$$=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\ \text{C})(0.0145\ \text{m})^2}{2(0.0231\ \text{m})^3}=-2.68\times10^{-4}\ \text{V}.$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have

$$V(R)=-\frac{q}{8\pi\varepsilon_0 R}=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\ \text{C})}{2(0.0231\ \text{m})}=-6.81\times10^{-4}\ \text{V}.$$

12. (a) The potential difference is

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} = \left(1.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{2.0\ \text{m}} - \frac{1}{1.0\ \text{m}}\right)$$

$$= -4.5\times10^{3}\ \text{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

13. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\ \text{V})(0.15\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 3.3\times10^{-9}\ \text{C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times10^{-9}\ \text{C}}{4\pi(0.15\ \text{m})^2} = 1.2\times10^{-8}\ \text{C}/\text{m}^2.$$

14. The charge is

$$q = 4\pi\varepsilon_0 RV = \frac{(10\,\text{m})(-1.0\,\text{V})}{8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2} = -1.1\times10^{-9}\,\text{C}.$$

15. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d}\right] = \frac{q}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{2(4.00\times10^{-2}\ \text{m})}$$
$$= 5.62\times10^{-4}\,\text{V}.$$

The zero of the electric potential was taken to be at infinity.

16. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

17. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

18. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(6.00\times10^{-12}\ \mathrm{C})}{0.39\ \mathrm{m}} = 2.21\ \mathrm{V}.$$

19. (a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\varepsilon_0 R$. Thus

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(30 \times 10^{-12}\text{ C}\right)}{500 \text{ V}} = 5.4 \times 10^{-4} \text{ m}.$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500 \text{ V}) \approx 790 \text{ V}.$$

20. When the charge q_2 is infinitely far away, the potential at the origin is due only to the charge q_1 :

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 d} = 5.76 \times 10^{-7}\,\text{V}.$$

Thus, $q_1/d = 6.41 \times 10^{-17}$ C/m. Next, we note that when q_2 is located at $x = 0.080$ m, the net potential vanishes ($V_1 + V_2 = 0$). Therefore,

$$0 = \frac{kq_2}{0.08\text{ m}} + \frac{kq_1}{d}$$

Thus, we find $q_2 = -(q_1/d)(0.08\text{ m}) = -5.13 \times 10^{-18}\,\text{C} = -32\,e$.

21. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.47\times3.34\times10^{-30}\ \text{C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\ \text{m}\right)^2} = 1.63\times10^{-5}\ \text{V}.$$

22. From Eq. 24-30 and Eq. 24-14, we have (for $\theta_i = 0°$)

$$W_a = q\Delta V = e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{p\cos\theta_i}{4\pi\varepsilon_0 r^2}\right) = \frac{ep\cos\theta}{4\pi\varepsilon_0 r^2}(\cos\theta - 1)$$

with $r = 20 \times 10^{-9}$ m. For $\theta = 180°$ the graph indicates $W_a = -4.0 \times 10^{-30}$ J, from which we can determine p. The magnitude of the dipole moment is therefore 5.6×10^{-37} C·m.

23. (a) All the charge is the same distance *R* from *C*, so the electric potential at *C* is

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R}-\frac{6Q_1}{R}\right)=-\frac{5Q_1}{4\pi\varepsilon_0 R}=-\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{8.20\times10^{-2}\ \mathrm{m}}=-2.30\ \mathrm{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from *P*. That distance is $\sqrt{R^2+D^2}$, so the electric potential at *P* is

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2+D^2}}-\frac{6Q_1}{\sqrt{R^2+D^2}}\right]=-\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2+D^2}}$$

$$=-\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{\sqrt{(8.20\times10^{-2}\ \mathrm{m})^2+(6.71\times10^{-2}\ \mathrm{m})^2}}=-1.78\ \mathrm{V}.$$

24. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(25.6\times10^{-12}\,\text{C})}{3.71\times10^{-2}\ \text{m}}$$
$$= -6.20\ \text{V}.$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

25. (a) From Eq. 24-35, we find the potential to be

$$V = 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2+\sqrt{(L^2/4)+d^2}}{d}\right]$$

$$= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.68\times10^{-12}\,\mathrm{C/m})\ln\left[\frac{(0.06\ \mathrm{m}/2)+\sqrt{(0.06\ \mathrm{m})^2/4+(0.08\ \mathrm{m})^2}}{0.08\ \mathrm{m}}\right]$$

$$= 2.43\times10^{-2}\ \mathrm{V}.$$

(b) The potential at P is $V = 0$ due to superposition.

26. Using Gauss' law, $q = \varepsilon_0\Phi = +495.8$ nC. Consequently,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.958\times10^{-7}\ \mathrm{C})}{0.120\ \mathrm{m}} = 3.71\times10^4\ \mathrm{V}.$$

27. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 24-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R}$$

$$= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.21\times10^{-12}\ \mathrm{C})}{2.00\ \mathrm{m}} = 3.24\times10^{-2}\ \mathrm{V}.$$

28. The dipole potential is given by Eq. 24-30 (with $\theta = 90°$ in this case)

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos 90°}{4\pi\varepsilon_0 r^2} = 0$$

since $\cos(90°) = 0$. The potential due to the short arc is $q_1 / 4\pi\varepsilon_0 r_1$ and that caused by the long arc is $q_2 / 4\pi\varepsilon_0 r_2$. Since $q_1 = +2$ μC, $r_1 = 4.0$ cm, $q_2 = -3$ μC, and $r_2 = 6.0$ cm, the potentials of the arcs cancel. The result is zero.

29. The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2+D^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+D^2}} = \frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

The total potential at P is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r dr}{\sqrt{r^2+D^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\,\Bigg|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right].$$

The potential V_{sq} at P due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right] = \frac{(7.73\times10^{-15}\,\text{C/m}^2)}{8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\ \text{m})^2+(0.259\ \text{m})^2}-0.259\ \text{m}\right]$$
$$= 4.71\times10^{-5}\ \text{V}.$$

30. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(56.1\times10^{-15}\,\text{C})}{0.12\ \text{m}}\ln\left(1+\frac{0.12\ \text{m}}{0.025\ \text{m}}\right) = 7.39\times10^{-3}\ \text{V}.$$

31. Letting d denote 0.010 m, we have

$$V = \frac{Q_1}{4\pi\varepsilon_0 d} + \frac{3Q_1}{8\pi\varepsilon_0 d} - \frac{3Q_1}{16\pi\varepsilon_0 d} = \frac{Q_1}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(30\times10^{-9}\,\mathrm{C})}{2(0.01\ \mathrm{m})} = 1.3\times10^4\ \mathrm{V}.$$

32. Eq. 24-32 applies with $dq = \lambda\, dx = bx\, dx$ (along $0 \le x \le 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36 \text{ V}.$$

(b) Now $r = \sqrt{x^2 + d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bxdx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Big|_0^{0.20} = 18 \text{ V}.$$

33. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain

$$\begin{aligned} V &= \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{xdx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right] \\ &= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(28.9\times10^{-12}\,\mathrm{C/m^2})\left[0.120\ \mathrm{m} - (0.030\ \mathrm{m})\ln\left(1+\frac{0.120\ \mathrm{m}}{0.030\ \mathrm{m}}\right)\right] \\ &= 1.86\times10^{-2}\ \mathrm{V}. \end{aligned}$$

34. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{\mathrm{i}} = -\frac{d}{dx}(1500x^2)\hat{\mathrm{i}} = (-3000x)\hat{\mathrm{i}} = (-3000\,\mathrm{V/m^2})(0.0130\,\mathrm{m})\hat{\mathrm{i}} = (-39\,\mathrm{V/m})\hat{\mathrm{i}}.$$

(a) Thus, the magnitude of the electric field is $E = 39$ V/m.

(b) The direction of $\vec{E}$ is $-\hat{\mathrm{i}}$, or toward plate 1.

35. We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y.$$

We evaluate at x = 3.0 m and y = 2.0 m to obtain

$$\vec{E} = (-12 \text{ V/m})\hat{\text{i}} + (12 \text{ V/m})\hat{\text{j}}.$$

36. The magnitude of the electric field is given by

$$|E| = \left| -\frac{\Delta V}{\Delta x} \right| = \frac{2(5.0\text{V})}{0.015\text{m}} = 6.7 \times 10^2 \text{ V/m}.$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly towards the negatively charged one.

37. The electric field (along some axis) is the (negative of the) derivative of the potential V with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{\partial V}{\partial x} = -\left(\frac{-500\text{ V}}{0.20\text{ m}}\right) = 2500\text{ V/m} = 2500\text{ N/C}$$

$$E_y = -\frac{\partial V}{\partial y} = -\left(\frac{300\text{ V}}{0.30\text{ m}}\right) = -1000\text{ V/m} = -1000\text{ N/C}.$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of –21.8º (with respect to the positive x axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of q has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180º to that of $\vec{E}$). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\text{C})[(2500\text{ N/C})\hat{\text{i}} - (1000\text{ N/C})\hat{\text{j}}] = (-4.0\times10^{-16}\text{N})\hat{\text{i}} + (1.60\times10^{-16}\text{ N})\hat{\text{j}}.$$

38. (a) From the result of Problem 24-30, the electric potential at a point with coordinate x is given by

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right).$$

At $x = d$ we obtain

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L}{d}\right)=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\,\text{C})}{0.135\ \text{m}}\ln\left(1+\frac{0.135\ \text{m}}{d}\right)$$
$$=(2.90\times10^{-3}\ \text{V})\ln\left(1+\frac{0.135\ \text{m}}{d}\right).$$

(b) We differentiate the potential with respect to x to find the x component of the electric field:

$$E_x=-\frac{\partial V}{\partial x}=-\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right)=-\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right)=-\frac{Q}{4\pi\varepsilon_0 x(x-L)}$$
$$=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\,\text{C})}{x(x+0.135\ \text{m})}=-\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})},$$

or

$$|E_x|=\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})}.$$

(c) Since $E_x<0$, its direction relative to the positive x axis is $180°$.

(d) At $x = d = 6.20$ cm, we obtain

$$|E_x|=\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{(0.0620\ \text{m})(0.0620\ \text{m}+0.135\ \text{m})}=0.0321\ \text{N/C}.$$

(e) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus, the transverse component of the electric field E_y is zero.

39. We apply Eq. 24-41:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

which, at (x, y, z) = (3.00 m, –2.00 m, 4.00 m), gives

$$(E_x, E_y, E_z) = (64.0 \text{ V/m}, -96.0 \text{ V/m}, 96.0 \text{ V/m}).$$

The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150\,\text{V/m} = 150\,\text{N/C}.$$

40. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus,

$$\begin{aligned}
V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}\,dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right)\\
&= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(\sqrt{(0.100\ \text{m})^2+(0.0356\ \text{m})^2}-0.0356\ \text{m}\right)\\
&= 3.16\times10^{-2}\ \text{V}.
\end{aligned}$$

(b) The y component of the field there is

$$\begin{aligned}
E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right).\\
&= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(1-\frac{0.0356\ \text{m}}{\sqrt{(0.100\ \text{m})^2+(0.0356\ \text{m})^2}}\right)\\
&= 0.298\ \text{N/C}.
\end{aligned}$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

41. We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right)=\frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right)=\frac{2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.30\times10^{-12}\ \mathrm{C})^2}{0.640\ \mathrm{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$

$$= -1.92\times10^{-13}\ \mathrm{J}.$$

42. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with $r = 0.020$ m)

$$W = \frac{(3e - 2e + 2e)(6e)}{4\pi\varepsilon_0 r} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(18)(1.60\times10^{-19}\ \mathrm{C})^2}{0.020\ \mathrm{m}} = 2.1\times10^{-25}\ \mathrm{J}.$$

43. We apply conservation of energy for the particle with $q = 7.5 \times 10^{-6}$ C (which has zero initial kinetic energy):

$$U_0 = K_f + U_f,$$

where $U = \dfrac{qQ}{4\pi\varepsilon_0 r}$.

(a) The initial value of r is 0.60 m and the final value is (0.6 + 0.4) m = 1.0 m (since the particles repel each other). Conservation of energy, then, leads to $K_f = 0.90$ J.

(b) Now the particles attract each other so that the final value of r is 0.60 – 0.40 = 0.20 m. Use of energy conservation yields $K_f = 4.5$ J in this case.

44. (a) We use Eq. 24-43 with $q_1 = q_2 = -e$ and $r = 2.00$ nm:

$$U = k\frac{q_1 q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(1.60\times10^{-19}\ \mathrm{C})^2}{2.00\times10^{-9}\ \mathrm{m}} = 1.15\times10^{-19}\ \mathrm{J}.$$

(b) Since $U > 0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

45. (a) Let $\ell = 0.15\,\text{m}$ be the length of the rectangle and $w = 0.050$ m be its width. Charge q_1 is a distance ℓ from point A and charge q_2 is a distance w, so the electric potential at A is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\ell}+\frac{q_2}{w}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}+\frac{2.0\times10^{-6}\,\text{C}}{0.050\text{ m}}\right)$$
$$= 6.0\times10^4\,\text{V}.$$

(b) Charge q_1 is a distance w from point b and charge q_2 is a distance ℓ, so the electric potential at B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{w}+\frac{q_2}{\ell}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.050\text{ m}}+\frac{2.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}\right)$$
$$= -7.8\times10^5\,\text{V}.$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at A and U_B is the potential energy when q_3 is at B, then the work done in moving the charge from B to A is

$$W = U_A - U_B = q_3(V_A - V_B) = (3.0 \times 10^{-6}\text{ C})(6.0 \times 10^4\text{ V} + 7.8 \times 10^5\text{ V}) = 2.5\text{ J}.$$

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

46. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{q_2 Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$

47. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for v is

$$v = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(3.1\times10^{-6}\,\mathrm{C})^2}{20\times10^{-6}\mathrm{kg}}\left(\frac{1}{0.90\times10^{-3}\,\mathrm{m}} - \frac{1}{2.5\times10^{-3}\,\mathrm{m}}\right)}$$

$$= 2.5\times10^3\ \mathrm{m/s}.$$

48. Let r = 1.5 m, x = 3.0 m, q_1 = –9.0 nC, and q_2 = –6.0 pC. The work done by an external agent is given by

$$
\begin{aligned}
W = \Delta U &= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + x^2}}\right) \\
&= \left(-9.0\times 10^{-9}\ \text{C}\right)\left(-6.0\times 10^{-12}\,\text{C}\right)\left(8.99\times 10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot\left[\frac{1}{1.5\ \text{m}} - \frac{1}{\sqrt{(1.5\ \text{m})^2 + (3.0\ \text{m})^2}}\right] \\
&= 1.8\times 10^{-10}\,\text{J}.
\end{aligned}
$$

49. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in chapter 14). Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C, $q = 10000e$, and $r = 0.010$ m. This yields $v = 22490$ m/s $\approx 2.2\times10^4$ m/s .

50. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2} m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19}\,\text{C})(125\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 6.63\times10^{6}\ \text{m/s}.$$

51. (a) The potential energy is

$$U=\frac{q^2}{4\pi\varepsilon_0 d}=\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.0\times10^{-6}\ \mathrm{C}\right)^2}{1.00\ \mathrm{m}}=0.225\ \mathrm{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F=\frac{q^2}{4\pi\varepsilon_0 d^2}=\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.0\times10^{-6}\ \mathrm{C}\right)^2}{\left(1.00\ \mathrm{m}\right)^2}=0.225\ \mathrm{N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A=\frac{F}{m_A}=\frac{0.225\ \mathrm{N}}{5.0\times10^{-3}\ \mathrm{kg}}=45.0\ \mathrm{m/s^2}$$

and the acceleration of sphere B is

$$a_B=\frac{F}{m_B}=\frac{0.225\ \mathrm{N}}{10\times10^{-3}\ \mathrm{kg}}=22.5\ \mathrm{m/s^2}.$$

(c) Energy is conserved. The initial potential energy is $U = 0.225$ J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2$, where v_A and v_B are the final velocities. Thus,

$$U=\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2.$$

Momentum is also conserved, so

$$0=m_Av_A+m_Bv_B.$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B=-(m_A/m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain

$$U=\tfrac{1}{2}(m_A/m_B)(m_A+m_B)v_A^2.$$

Thus,

$$v_A=\sqrt{\frac{2Um_B}{m_A(m_A+m_B)}}=\sqrt{\frac{2(0.225\ \mathrm{J})(10\times10^{-3}\ \mathrm{kg})}{(5.0\times10^{-3}\ \mathrm{kg})(5.0\times10^{-3}\ \mathrm{kg}+10\times10^{-3}\ \mathrm{kg})}}=7.75\ \mathrm{m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B} v_A = -\left(\frac{5.0\times10^{-3}\ \text{kg}}{10\times10^{-3}\ \text{kg}}\right)(7.75\ \text{m/s}) = -3.87\ \text{m/s},$$

or $|v_B| = 3.87$ m/s.

52. When particle 3 is at $x = 0.10$ m, the total potential energy vanishes. Using Eq. 24-43, we have (with meters understood at the length unit)

$$0 = \frac{q_1 q_2}{4\pi\varepsilon_0 d} + \frac{q_1 q_3}{4\pi\varepsilon_0 (d + 0.10 \text{ m})} + \frac{q_3 q_2}{4\pi\varepsilon_0 (0.10 \text{ m})}$$

This leads to

$$q_3 \left(\frac{q_1}{d + 0.10 \text{ m}} + \frac{q_2}{0.10 \text{ m}} \right) = -\frac{q_1 q_2}{d}$$

which yields $q_3 = -5.7$ μC.

53. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, where m is the mass of an electron and v is the initial speed of the moving electron. The final kinetic energy is zero. Thus,

$$K_i = U_f \quad \Rightarrow \quad \frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d.$$

Hence,

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0 dm}} = \sqrt{\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(4)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{(0.010\ \mathrm{m})\left(9.11\times10^{-31}\ \mathrm{kg}\right)}} = 3.2\times10^2\ \mathrm{m/s}.$$

54. (a) When the proton is released, its energy is $K + U = 4.0$ eV + 3.0 eV (the latter value is inferred from the graph). This implies that if we draw a horizontal line at the 7.0 Volt "height" in the graph and find where it intersects the voltage plot, then we can determine the turning point. Interpolating in the region between 1.0 cm and 3.0 cm, we find the turning point is at roughly $x = 1.7$ cm.

(b) There is no turning point towards the right, so the speed there is nonzero, and is given by energy conservation:

$$v = \sqrt{\frac{2(7.0\text{ eV})}{m}} = \sqrt{\frac{2(7.0\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})}{1.67 \times 10^{-27}\text{ kg}}} = 20\text{ km/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the proton follows immediately from $\vec{F} = q\vec{E}$, where $q = +e$ for the proton. In the region just to the left of $x = 3.0$ cm, the field is $\vec{E} = (+300\text{ V/m})\hat{\text{i}}$ and the force is $F = +4.8 \times 10^{-17}$ N.

(d) The force $\vec{F}$ points in the $+x$ direction, as the electric field $\vec{E}$.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = (-200\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 3.2 \times 10^{-17}$ N.

(f) The force $\vec{F}$ points in the $-x$ direction, as the electric field $\vec{E}$.

55. (a) The electric field between the plates is leftward in Fig, 24-52 since it points towards lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \ .$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 90 \times 10^3$ m/s, $V_1 = -70$ V and $V_2 = -50$ V, we obtain the final speed $v = 6.53 \times 10^4$ m/s. We note that the value of d is not used in the solution.

56. From Eq. 24-30 and Eq. 24-7, we have (for $\theta = 180°$)

$$U = qV = -e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{ep}{4\pi\varepsilon_0 r^2}$$

where r = 0.020 m. Using energy conservation, we set this expression equal to 100 eV and solve for *p*. The magnitude of the dipole moment is therefore $p = 4.5 \times 10^{-12}\ \mathrm{C\cdot m}$.

57. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2} m_e v_i^2$, where $v_i = 3.2 \times 10^5$ m/s. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2} m_e v_i^2 ,$$

or

$$r = \frac{2e^2}{3(4\pi\varepsilon_0) m_e v_i^2} = \frac{2\left(1.6\times10^{-19}\ \text{C}\right)^2 \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{3\left(9.11\times10^{-19}\ \text{kg}\right)\left(3.2\times10^5\ \text{m/s}\right)^2} = 1.6\times10^{-9}\,\text{m}.$$

58. (a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative x direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is 1.0×10^7 m/s.

59. We apply conservation of energy for particle 3 (with $q' = -15 \times 10^{-6}$ C):

$$K_0 + U_0 = K_f + U_f$$

where (letting $x = \pm 3$ m and $q_1 = q_2 = 50 \times 10^{-6}$ C $= q$)

$$U = \frac{q_1 q'}{4\pi\varepsilon_0\sqrt{x^2+y^2}} + \frac{q_2 q'}{4\pi\varepsilon_0\sqrt{x^2+y^2}} = \frac{2qq'}{4\pi\varepsilon_0\sqrt{x^2+y^2}} \ .$$

(a) We solve for K_f (with $y_0 = 4$ m):

$$K_f = K_0 + U_0 - U_f = 1.2 \text{ J} + \frac{2qq'}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_0^2}} - \frac{1}{|x|}\right) = 3.0 \text{ J} \ .$$

(b) We set $K_f = 0$ and solve for y (choosing the negative root, as indicated in the problem statement):

$$K_0 + U_0 = U_f \quad \Rightarrow \quad 1.2 \text{ J} + \frac{2qq'}{4\pi\varepsilon_0\sqrt{x^2+y_0^2}} = \frac{2qq'}{4\pi\varepsilon_0\sqrt{x^2+y^2}} \ .$$

This yields $y = -8.5$ m.

60. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +2.16 \times 10^{-13}\ \text{J}.$$

With R = 0.0800 m, we find $Q = -1.20 \times 10^{-5}$ C.

(b) The work is the same, so the increase in the potential energy is $\Delta U = +2.16 \times 10^{-13}$ J.

61. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where N = odd and N = even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$U_{1,N=\text{even}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)}+\frac{1}{2}\right),\quad U_{1,N=\text{odd}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right)$$

where $\theta = \frac{2\pi}{N}$. For configuration 2, we find

$$U_{2,N=\text{even}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)}+2\right),\ U_{2,N=\text{odd}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)}+\frac{5}{2}\right)$$

where $\theta' = \frac{2\pi}{N-1}$. The results are all of the form

$$U_{1\text{or}2}\ \frac{ke^2}{2R}\times\text{a pure number.}$$

In our table, below, we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) $N = 12$ is the smallest value such that $U_2 < U_1$.

(b) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two — one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

62. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also +400 V.

63. If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = q/4\pi\varepsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus,

$$q = 4\pi\varepsilon_0 rV = \frac{(0.15\text{ m})(1500\text{ V})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.5\times10^{-8}\text{C}.$$

64. (a) Since the two conductors are connected V_1 and V_2 must be equal to each other.

Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$, or

(b) $q_1/q = 1/3 = 0.333$.

(c) Similarly, $q_2/q = 2/3 = 0.667$.

(d) The ratio of surface charge densities is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2.00.$$

65. (a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0\, d/2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C} - 3.0\times10^{-8}\,\text{C}\right)}{1.0\ \text{m}} = -1.8\times10^2\,\text{V}.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}}\right] = 2.9\times10^3\,\text{V}.$$

(c) The potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{0.030\,\text{m}}\right] = -8.9\times10^3\,\text{V}.$$

66. Since the charge distribution is spherically symmetric we may write

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r},$$

where q_{enc} is the charge enclosed in a sphere of radius r centered at the origin.

(a) For $r = 4.00$ m, $R_2 = 1.00$ m and $R_1 = 0.500$ m, with $r > R_2 > R_1$ we have

$$E(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(4.00\ \text{m})^2}=1.69\times10^3\ \text{V/m}.$$

(b) For $R_2 > r = 0.700$ m $> R_2$

$$E(r)=\frac{q_1}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{C})}{(0.700\ \text{m})^2}=3.67\times10^4\ \text{V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, $E = 0$.

The electric potential may be obtained using Eq. 24-18:

$$V(r)-V(r')=\int_r^{r'}E(r)\,dr.$$

(d) For $r = 4.00$ m $> R_2 > R_1$, we have

$$V(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(4.00\ \text{m})}=6.74\times10^3\ \text{V}.$$

(e) For $r = 1.00$ m $= R_2 > R_1$, we have

$$V(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(1.00\ \text{m})}=2.70\times10^4\ \text{V}.$$

(f) For $R_2 > r = 0.700$ m $> R_2$,

$$\begin{aligned}V(r)&=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.700\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)\\&=3.47\times10^4\ \text{V}.\end{aligned}$$

(g) For $R_2 > r = 0.500$ m $= R_2$,

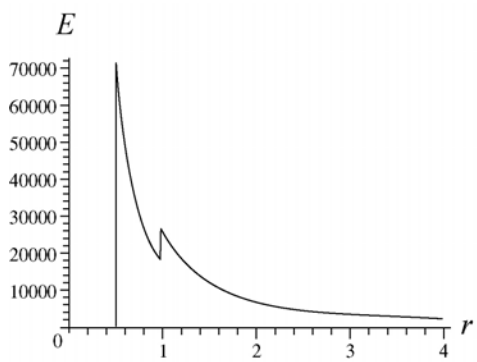
$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\,\text{C}}{0.500\ \text{m}} + \frac{1.00\times10^{-6}\,\text{C}}{1.00\ \text{m}}\right)$$
$$= 4.50\times10^4\ \text{V}.$$

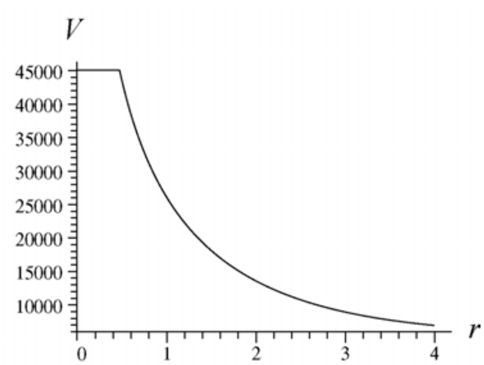
(h) For $R_2 > R_1 > r$,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\,\text{C}}{0.500\ \text{m}} + \frac{1.00\times10^{-6}\,\text{C}}{1.00\ \text{m}}\right)$$
$$= 4.50\times10^4\ \text{V}.$$

(i) At $r = 0$, the potential remains constant, $V = 4.50\times10^4$ V.

(j) The electric field and the potential as a function of r are depicted below:





67. (a) The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(3.0\times10^{-8}\,\mathrm{C}\right)\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)}{\left(0.15\,\mathrm{m}\right)^2} = 1.2\times10^{4}\,\mathrm{N/C}.$$

(b) $V = RE = (0.15\ \mathrm{m})(1.2 \times 10^4\ \mathrm{N/C}) = 1.8 \times 10^3\ \mathrm{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\mathrm{V},$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\mathrm{m})(-500\,\mathrm{V})}{-1800\,\mathrm{V} + 500\,\mathrm{V}} = 5.8\times10^{-2}\,\mathrm{m}.$$

68. (a) We use Eq. 24-18 to find the potential: $V_{\text{wall}} - V = -\int_r^R E\,dr$, or

$$0 - V = -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \quad \Rightarrow \quad -V = -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right).$$

Consequently, $V = \rho(R^2 - r^2)/4\varepsilon_0$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1\times10^{-3}\ \text{C/m}^3}{4\left(8.85\times10^{-12}\ \text{C/V}\cdot\text{m}\right)}\left((0.05\,\text{m})^2 - 0\right) = -7.8\times10^4\ \text{V}.$$

Thus, the difference is $|V_{\text{center}}| = 7.8\times10^4\ \text{V}$.

69. The electric potential energy in the presence of the dipole is

$$U = qV_{\text{dipole}} = \frac{qp\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{(-e)(ed)\cos\theta}{4\pi\varepsilon_0 r^2} \; .$$

Noting that $\theta_i = \theta_f = 0°$, conservation of energy leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 m d}\left(\frac{1}{25} - \frac{1}{49}\right)} = 7.0 \times 10^5 \text{ m/s} \; .$$

70. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 24-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)+\frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2+r^2}-z\right).$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = 0.200R$ and $z = 2.00R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = \frac{(6.20\times10^{-12}\,\text{C/m}^2)(0.130\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = 1.03\times10^{-2}\text{ V}.$$

71. (a) When the electron is released, its energy is $K + U = 3.0$ eV $- 6.0$ eV (the latter value is inferred from the graph along with the fact that $U = qV$ and $q = -e$). Because of the minus sign (of the charge) it is convenient to imagine the graph multiplied by a minus sign so that it represents potential energy in eV. Thus, the 2 V value shown at $x = 0$ would become –2 eV, and the 6 V value at $x = 4.5$ cm becomes –6 eV, and so on. The total energy (-3.0 eV) is constant and can then be represented on our (imagined) graph as a horizontal line at -3.0 V. This intersects the potential energy plot at a point we recognize as the turning point. Interpolating in the region between 1.0 cm and 4.0 cm, we find the turning point is at $x = 1.75$ cm $\approx$ 1.8 cm.

(b) There is no turning point towards the right, so the speed there is nonzero. Noting that the kinetic energy at $x = 7.0$ cm is $K = -3.0$ eV $-(-5.0$ eV$) = 2.0$ eV, we find the speed using energy conservation:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 8.4\times10^{5}\text{ m/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the electron follows immediately from $\vec{F} = q\vec{E}$, where $q = -e$ for the electron. In the region just to the left of

$x = 4.0$ cm, the electric field is $\vec{E} = (-133\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 2.1\times10^{-17}$ N.

(d) The force points in the $+x$ direction.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = +100$ V/m $\hat{\text{i}}$ and the force is $\vec{F} = (-1.6 \times 10^{-17}\text{ N})\ \hat{\text{i}}$. Thus, the magnitude of the force is $F = 1.6\times10^{-17}$ N.

(f) The minus sign indicates that $\vec{F}$ points in the $-x$ direction.

72. The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged sphere:

$$V_A = V_S = \frac{q}{4\pi\varepsilon_0 R}$$

where $q = 30 \times 10^{-9}$ C and $R = 0.030$ m. For points beyond the surface of the sphere, the potential follows Eq. 24-26:

$$V_B = \frac{q}{4\pi\varepsilon_0 r}$$

where $r = 0.050$ m.

(a) We see that

$$V_S - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) = 3.6 \times 10^3 \text{ V}.$$

(b) Similarly,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) = 3.6 \times 10^3 \text{ V}.$$

73. (a) Using $d = 2$ m, we find the potential at P:

$$V_P = \frac{2e}{4\pi\varepsilon_0 d} + \frac{-2e}{4\pi\varepsilon_0 (2d)} = \frac{e}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\,\text{C})}{2.00\ \text{m}} = 7.192\times10^{-10}\ \text{V}\ .$$

Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since $U = qV$, then the movable particle's contribution of the potential energy when it is at $r = \infty$ is zero, and its contribution to U_{system} when it is at P is

$$U = qV_P = 2(1.6\times10^{-19}\,\text{C})(7.192\times10^{-10}\ \text{V}) = 2.3014\times10^{-28}\ \text{J}\,.$$

Thus, the work done is approximately equal to $W_{\text{app}} = 2.30 \times 10^{-28}$ J.

(c) Now, combining the contribution to U_{system} from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{(4.00\ \text{m})^2 + (2.00\ \text{m})^2}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\,\text{C})^2}{\sqrt{20.0}\ \text{m}},$$

$$= -2.058\times10^{-28}\ \text{J}\ .$$

we obtain

$$U_{\text{system}} = W_{\text{app}} + U_{\text{fixed}} = 2.43 \times 10^{-29}\ \text{J}\ \ .$$

74. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_o}\ln\left(\frac{L+\sqrt{L^2+d^2}}{D+\sqrt{D^2+d^2}}\right) = \frac{2.0 \times 10^{-6}}{4\pi\varepsilon_o}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 2.18 \times 10^4 \text{ V}.$$

75. The work done results in a change of potential energy:

$$W = \Delta U = \frac{2q^2}{4\pi\varepsilon_0 d'} - \frac{2q^2}{4\pi\varepsilon_0 d} = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{1}{d'} - \frac{1}{d}\right)$$

$$= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(0.12\ \mathrm{C})^2\left(\frac{1}{1.7\ \mathrm{m}/2} - \frac{1}{1.7\ \mathrm{m}}\right) = 1.5\times10^8\ \mathrm{J}.$$

At a rate of $P = 0.83 \times 10^3$ Joules per second, it would take $W/P = 1.8 \times 10^5$ seconds or about 2.1 days to do this amount of work.

76. Using Eq. 24-18, we have

$$\Delta V = -\int_2^3 \frac{A}{r^4}\, dr = \frac{A}{3}\left(\frac{1}{2^3} - \frac{1}{3^3}\right) = A(0.029/\text{m}^3).$$

77. The radius of the cylinder (0.020 m, the same as r_B) is denoted R, and the field magnitude there (160 N/C) is denoted E_B. The electric field beyond the surface of the sphere follows Eq. 23-12, which expresses inverse proportionality with r:

$$\frac{|\vec{E}|}{E_B} = \frac{R}{r} \quad \text{for } r \geq R \; .$$

(a) Thus, if $r = r_C = 0.050$ m, we obtain

$$|\vec{E}| = (160 \text{ N/C})(0.020 \text{ m})/(0.050 \text{ m}) = 64 \text{ N/C} \, .$$

(b) Integrating the above expression (where the variable to be integrated, r, is now denoted ρ) gives the potential difference between V_B and V_C.

$$V_B - V_C = \int_R^r \frac{E_B R}{\rho} d\rho = E_B R \ln\left(\frac{r}{R}\right) = 2.9 \text{ V} \; .$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

78. (a) The potential would be

$$
\begin{aligned}
V_e &= \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2 \sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e \sigma_e k \\
&= 4\pi\left(6.37\times10^6 \text{ m}\right)\left(1.0\,\text{electron/m}^2\right)\left(-1.6\times10^{-9}\text{ C/electron}\right)\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right) \\
&= -0.12\,\text{V}.
\end{aligned}
$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\text{V}}{6.37\times10^6\,\text{m}} = -1.8\times10^{-8}\,\text{N/C},$$

or $|E| = 1.8\times10^{-8}$ N/C.

(c) The minus sign in E indicates that $\vec{E}$ is radially inward.

79. We note that the net potential (due to the "fixed" charges) is zero at the first location ("at ∞") being considered for the movable charge q (where $q = +2e$). Thus, with $D = 4.00$ m and $e = 1.60 \times 10^{-19}$ C, we obtain

$$V = \frac{+2e}{4\pi\varepsilon_0(2D)} + \frac{+e}{4\pi\varepsilon_0 D} = \frac{2e}{4\pi\varepsilon_0 D} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(1.60\times10^{-19}\,\mathrm{C})}{4.00\ \mathrm{m}} \\ = 7.192\times10^{-10}\ \mathrm{V}\ .$$

The work required is equal to the potential energy in the final configuration:

$$W_{\text{app}} = qV = (2e)(7.192 \times 10^{-10}\ \mathrm{V}) = 2.30 \times 10^{-28}\ \mathrm{J}.$$

80. Since the electric potential is a scalar quantity, this calculation is far simpler than it would be for the electric field. We are able to simply take half the contribution that would be obtained from a complete (whole) sphere. If it were a whole sphere (of the same density) then its charge would be $q_{\text{whole}} = 8.00\ \mu\text{C}$. Then

$$V = \frac{1}{2} V_{\text{whole}} = \frac{1}{2} \frac{q_{\text{whole}}}{4\pi\varepsilon_o r} = \frac{1}{2} \frac{8.00 \times 10^{-6}\ \text{C}}{4\pi\varepsilon_o (0.15\ \text{m})} = 2.40 \times 10^5\ \text{V}\ .$$

81. The net potential at point P (the place where we are to place the third electron) due to the fixed charges is computed using Eq. 24-27 (which assumes $V \to 0$ as $r \to \infty$):

$$V_P = \frac{-e}{4\pi\varepsilon_0 d} + \frac{-e}{4\pi\varepsilon_0 d} = -\frac{2e}{4\pi\varepsilon_0 d} \ .$$

Thus, with $d = 2.00 \times 10^{-6}$ m and $e = 1.60 \times 10^{-19}$ C, we find

$$V_P = -\frac{2e}{4\pi\varepsilon_0 d} = -\frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{ C})}{2.00\times10^{-6}\text{ m}} = -1.438\times10^{-3}\text{ V} \ .$$

Then the required "applied" work is, by Eq. 24-14,

$$W_{\text{app}} = (-e)\, V_P = 2.30 \times 10^{-22}\text{ J} \ .$$

82. The work done is equal to the change in the (total) electric potential energy U of the system, where

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_3 q_2}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

and the notation r_{13} indicates the distance between q_1 and q_3 (similar definitions apply to r_{12} and r_{23}).

(a) We consider the difference in U where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ (r_{13} doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in U where initially $r_{23} = a$ and $r_{13} = a$, and finally r_{23} is again equal to a and r_{13} is also again equal to a (and of course, r_{12} doesn't change in this case). Thus, we obtain $\Delta U = 0$.

83. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0\times10^{-6}\ \mathrm{C}\right)\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)}{0.10\,\mathrm{m}} = 3.6\times10^{5}\ \mathrm{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6\times10^{5}\ \mathrm{V}}{0.10\,\mathrm{m}} = 3.6\times10^{6}\ \mathrm{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

84. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance $r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2}$. The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V = \frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\text{C}\right)}{0.020\text{ m}} = 2.5\times10^6\,\text{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0\times10^{-6}\,\text{C}\right)\left(2.54\times10^6\,\text{V}\right) = 5.1\text{ J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U = W + \frac{q^2}{4\pi\varepsilon_0 d} = 5.1\text{ J} + \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-6}\,\text{C}\right)^2}{0.020\,\text{m}} = 6.9\text{ J}.$$

85. For a point on the axis of the ring the potential (assuming $V \to 0$ as $r \to \infty$) is

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2 + R^2}}$$

where $q = 16 \times 10^{-6}$ C and $R = 0.0300$ m. Therefore,

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z_B^2 + R^2}} - \frac{1}{R}\right)$$

where $z_B = 0.040$ m. The result is -1.92×10^6 V.

86. The potential energy of the two-charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}\right] = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-6}\,\mathrm{C}\right)\left(-4.00\times10^{-6}\,\mathrm{C}\right)}{\sqrt{(3.50+2.00)^2 + (0.500-1.50)^2}\ \mathrm{cm}}$$

$$= -1.93\ \mathrm{J}.$$

Thus, –1.93 J of work is needed.

87. The initial speed v_i of the electron satisfies $K_i = \frac{1}{2} m_e v_i^2 = e\Delta V$, which gives

$$v_i = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{J}\right)\left(625\ \text{V}\right)}{9.11\times10^{-31}\ \text{kg}}} = 1.48\times10^{7}\ \text{m/s}.$$

88. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Q_q}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad mv^2 = \frac{Qq}{4\pi\varepsilon_0 r},$$

and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{Qq}{8\pi\varepsilon_0 r}.$$

The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E = K + U = \frac{Qq}{8\pi\varepsilon_0 r} - \frac{Qq}{4\pi\varepsilon_0 r} = -\frac{Qq}{8\pi\varepsilon_0 r}.$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

89. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37 \times 10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so

$$V = ER = (100 \text{ V/m})\,(6.37 \times 10^6 \text{ m}) = 6.4 \times 10^8 \text{ V}.$$

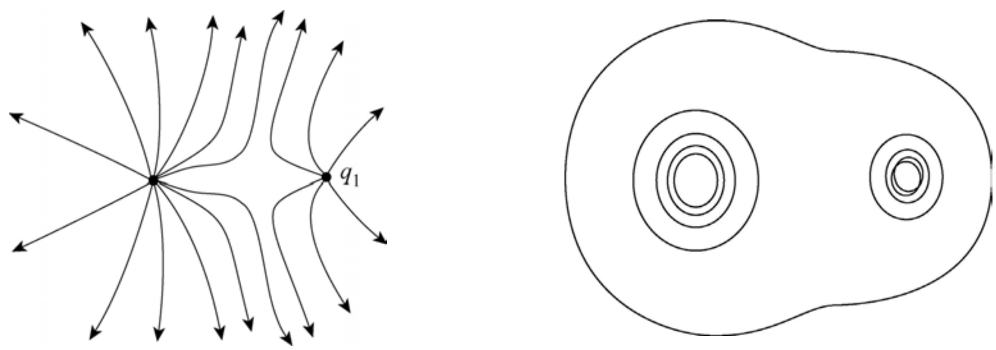
90. The net electric potential at point P is the sum of those due to the six charges:

$$V_P = \sum_{i=1}^{6} V_{Pi} = \sum_{i=1}^{6} \frac{q_i}{4\pi\varepsilon_0 r_i} = \frac{10^{-15}}{4\pi\varepsilon_0}\left[\frac{5.00}{\sqrt{d^2+(d/2)^2}} + \frac{-2.00}{d/2} + \frac{-3.00}{\sqrt{d^2+(d/2)^2}}\right.$$

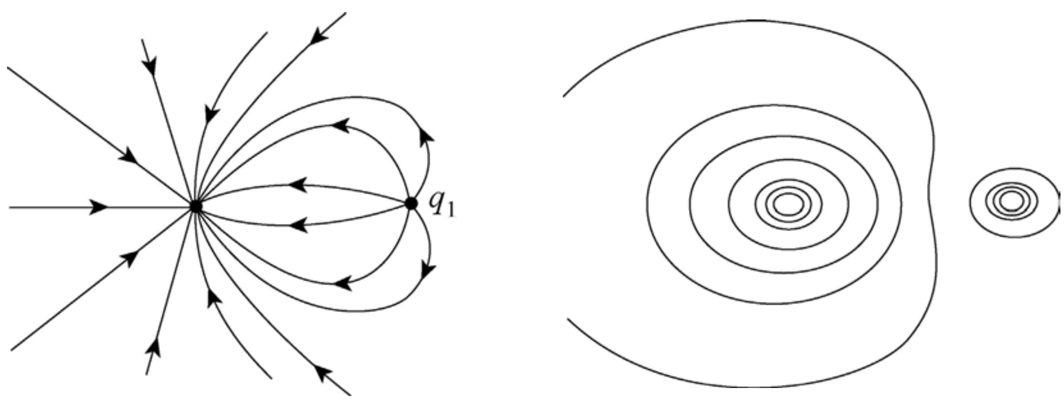
$$\left. + \frac{3.00}{\sqrt{d^2+(d/2)^2}} + \frac{-2.00}{d/2} + \frac{+5.00}{\sqrt{d^2+(d/2)^2}}\right] = \frac{9.4\times10^{-16}}{4\pi\varepsilon_0(2.54\times10^{-2})} = 3.34\times10^{-4}\ \text{V}.$$

91. In the sketches shown next, the lines with the arrows are field lines and those without are the equipotentials (which become more circular the closer one gets to the individual charges). In all pictures, q_2 is on the left and q_1 is on the right (which is reversed from the way it is shown in the textbook).

(a)



(b)



92. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside. Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3},$$

so

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R}.$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}\left(3R^2 - r^2\right).$$

(b) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R},$$

or $|\Delta V| = q/8\pi\varepsilon_0 R$.

93. (a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)},$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right).$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2\left(r_2^3 - r_1^3\right)}.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V &= V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr \\ &= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right). \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

Since $\rho = 3Q/4\pi\left(r_2^3 - r_1^3\right)$ this can also be written

$$V = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{3\left(r_2^2 - r_1^2\right)}{2\left(r_2^3 - r_1^3\right)},$$

or in terms of the charge density $V = \frac{\rho}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)$.

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

94. The distance r being looked for is that where the alpha particle has (momentarily) zero kinetic energy. Thus, energy conservation leads to

$$K_0 + U_0 = K + U \quad \Rightarrow \; (0.48 \times 10^{-12}\ \text{J}) + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r_0} = \ 0 \ + \ \frac{(2e)(92e)}{4\pi\varepsilon_0\, r} \quad .$$

If we set $r_0 = \infty$ (so $U_0 = 0$) then we obtain $r = 8.8 \times 10^{-14}$ m.

95. On the dipole axis $\theta = 0$ or π, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3} .$$

96. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$V(R) = V\big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50\times10^8\ \text{C}\right)}{0.160\text{m}}$$

$$= 8.43\times10^2\ \text{V}.$$

97. The potential difference is

$$\Delta V = E\Delta s = (1.92 \times 10^5 \text{ N/C})(0.0150 \text{ m}) = 2.90 \times 10^3 \text{ V}.$$

98. (a) Using Eq. 24-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.50\times10^{-8}\ \mathrm{C})}{30\ \mathrm{V}} = 4.5\,\mathrm{m}.$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

99. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$U_{\text{up-up}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e/3)(2e/3)}{r} = \frac{4ke}{9r}e = \frac{4\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)}{9\left(1.32\times10^{-15}\,\text{m}\right)}e$$

$$= 4.84\times10^5\,\text{eV} = 0.484\,\text{MeV}.$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(2e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r}\right] = 0.$$

100. (a) At the smallest center-to-center separation d_p the initial kinetic energy K_i of the proton is entirely converted to the electric potential energy between the proton and the nucleus. Thus,

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{eq_{\text{lead}}}{d_p} = \frac{82e^2}{4\pi\varepsilon_0 d_p}.$$

In solving for d_p using the eV unit, we note that a factor of e cancels in the middle line:

$$\begin{aligned} d_p &= \frac{82e^2}{4\pi\varepsilon_0 K_i} = k\frac{82e^2}{4.80\times10^6\,e\text{V}} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{82\left(1.6\times10^{-19}\,\text{C}\right)}{4.80\times10^6\,\text{V}} \\ &= 2.5\times10^{-14}\,\text{m} = 25\,\text{fm}\ . \end{aligned}$$

It is worth recalling that $1\ \text{V} = 1\ \text{N}\cdot\text{m/C}$, in making sense of the above manipulations.

(b) An alpha particle has 2 protons (as well as 2 neutrons). Therefore, using r'_{min} for the new separation, we find

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{lead}}}{d_\alpha} = 2\left(\frac{82e^2}{4\pi\varepsilon_0 d_\alpha}\right) = \frac{82e^2}{4\pi\varepsilon_0 d_p}$$

which leads to $d_\alpha / d_p = 2.00$.

101. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r=\sqrt{z^2+R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}}=\frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq=\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E=-\frac{\partial V}{\partial z}=-\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)=\frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

102. The electric potential energy is

$$U = k\sum_{i\neq j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0 d}\left(q_1q_2 + q_1q_3 + q_2q_4 + q_3q_4 + \frac{q_1q_4}{\sqrt{2}} + \frac{q_2q_3}{\sqrt{2}}\right)$$

$$= \frac{\left(8.99\times10^9\right)}{1.3}\left[(12)(-24)+(12)(31)+(-24)(17)+(31)(17)+\frac{(12)(17)}{\sqrt{2}}+\frac{(-24)(31)}{\sqrt{2}}\right](10^{-19})^2$$

$$= -1.2\times10^{-6}\ \text{J}.$$

103. (a) With $V = 1000$ V, we solve $V = q/4\pi\varepsilon_0 R,$ where $R = 0.010$ m for the net charge on the sphere, and find $q = 1.1 \times 10^{-9}$ C. Dividing this by e yields 6.95×10^9 electrons that entered the copper sphere. Now, half of the 3.7×10^8 decays per second resulted in electrons entering the sphere, so the time required is

$$\frac{6.95\times10^9}{(3.7\times10^8\,/\mathrm{s})/2} = 38\ \mathrm{s}$$

(b) We note that 100 keV is 1.6×10^{-14} J (per electron that entered the sphere). Using the given heat capacity, we note that a temperature increase of $\Delta T = 5.0$ K = 5.0 C° required 71.5 J of energy. Dividing this by 1.6×10^{-14} J, we find the number of electrons needed to enter the sphere (in order to achieve that temperature change); since this is half the number of decays, we multiply to 2 and find

$$N = 8.94 \times 10^{15} \text{ decays.}$$

We divide N by 3.7×10^8 to obtain the number of seconds. Converting to days, this becomes roughly 280 days.

104. The charges are equidistant from the point where we are evaluating the potential — which is computed using Eq. 24-27 (or its integral equivalent). Eq. 24-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{R}$$

$$= \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.52\times10^{-12}\ \text{C})}{0.0850\ \text{m}} = 0.956\ \text{V}.$$

105. Since the electric potential energy is not changed by the introduction of the third particle, we conclude that the net electric potential evaluated at P caused by the original two particles must be zero:

$$\frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} = 0 \ .$$

Setting $r_1 = 5d/2$ and $r_2 = 3d/2$ we obtain $q_1 = -5q_2/3$, or $q_1/q_2 = -5/3 \approx -1.7$.

106. (a) Clearly, the net voltage

$$V = \frac{q}{4\pi\varepsilon_0 \,|\, x \,|} + \frac{2q}{4\pi\varepsilon_0 \,|\, d - x \,|}$$

is not zero for any finite value of x.

(b) The electric field cancels at a point between the charges:

$$\frac{q}{4\pi\varepsilon_0 x^2} = \frac{2q}{4\pi\varepsilon_0 (d - x)^2}$$

which has the solution: $x = (\sqrt{2} - 1)\, d = 0.41$ m.

107. This can be approached more than one way, but the simplest is to observe that the net potential (using Eq. 24-27) due to $q_1 = +2e$ and $q_3 = -2e$ is zero at both the initial and final positions of the movable charge $q_2 = +5q$. This implies that no work is necessary to effect its change of position, which, in turn, implies there is no resulting change in potential energy of the configuration. Hence, the ratio is unity.

108. We use $E_x = -dV/dx$, where dV/dx is the local slope of the V vs. x curve depicted in Fig. 24-69. The results are:

(a) $E_x(ab) = -6.0$ V/m,

(b) $E_x(bc) = 0$,

(c) $E_x(cd) = 3.0$ V/m,

(d) $E_x(de) = 3.0$ V/m,

(e) $E_x(ef) = 15$ V/m,

(f) $E_x(fg) = 0$,

(g) $E_x(gh) = -3.0$ V/m.

Since these values are constant during their respective time-intervals, their graph consists of several disconnected line-segments (horizontal) and is not shown here.

109. (a) We denote the surface charge density of the disk as σ_1 for $0 < r < R/2$, and as σ_2 for $R/2 < r < R$. Thus the total charge on the disk is given by

$$
\begin{aligned}
q &= \int_{\text{disk}} dq = \int_0^{R/2} 2\pi\sigma_1 r\,dr + \int_{R/2}^{R} 2\pi\sigma_2 r\,dr = \frac{\pi}{4}R^2\left(\sigma_1 + 3\sigma_2\right) \\
&= \frac{\pi}{4}\left(2.20\times10^{-2}\text{ m}\right)^2\left[1.50\times10^{-6}\text{ C/m}^2 + 3\left(8.00\times10^{-7}\text{ C/m}^2\right)\right] \\
&= 1.48\times10^{-9}\text{ C}.
\end{aligned}
$$

(b) We use Eq. 24-36:

$$
\begin{aligned}
V(z) &= \int_{\text{disk}} dV = k\left[\int_0^{R/2} \frac{\sigma_1(2\pi R')dR'}{\sqrt{z^2+R'^2}} + \int_{R/2}^{R} \frac{\sigma_2(2\pi R')dR'}{\sqrt{z^2+R'^2}}\right] \\
&= \frac{\sigma_1}{2\varepsilon_0}\left(\sqrt{z^2+\frac{R^2}{4}} - z\right) + \frac{\sigma_2}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - \sqrt{z^2+\frac{R^2}{4}}\right).
\end{aligned}
$$

Substituting the numerical values of σ_1, σ_2, R and z, we obtain $V(z) = 7.95\times10^2$ V.

110. The net potential (at point A or B) is computed using Eq. 24-27. Thus, using k for $1/4\pi\varepsilon_0$, the difference is

$$V_A - V_B = \left(\frac{ke}{d} + \frac{k(-5e)}{5d}\right) - \left(\frac{ke}{2d} + \frac{k(-5e)}{2d}\right) = \frac{2ke}{d}$$

$$= \frac{2(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\,\mathrm{C})}{5.60\times10^{-6}\ \mathrm{m}} = 5.14\times10^{-4}\ \mathrm{V}.$$

111. We denote $q = 25 \times 10^{-9}$ C, $y = 0.6$ m, $x = 0.8$ m, with V = the net potential (assuming $V \to 0$ as $r \to \infty$). Then,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{y} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{x}, \quad V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{y}$$

leads to

$$\Delta V = V_B - V_A = \frac{2}{4\pi\varepsilon_0}\frac{q}{x} - \frac{2}{4\pi\varepsilon_0}\frac{q}{y} = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{y}\right)$$

$$= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(25\times10^{-9}\,\mathrm{C})\left(\frac{1}{0.80\ \mathrm{m}} - \frac{1}{0.60\ \mathrm{m}}\right) = -187\ \mathrm{V}\ .$$

112. (a) The total electric potential energy consists of three equal terms:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} + \frac{q_2 q_3}{4\pi\varepsilon_0 r} + \frac{q_1 q_3}{4\pi\varepsilon_0 r}$$

where $q_1 = q_2 = q_3 = q = -\frac{e}{3}$, and $r = 2.82\times10^{-15}$ m as given in the problem. The result is

$$U = \frac{3q^2}{4\pi\varepsilon_0 r} = \frac{3\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-1.60\times10^{-19}\,\mathrm{C}/3\right)^2}{2.82\times10^{-15}\,\mathrm{m}} = 2.72\times10^{-14}\ \mathrm{J}.$$

(b) Dividing by the square of the speed of light (roughly 3.0×10^8 m/s), we obtain

$$m = \frac{U}{c^2} = \frac{2.72\times10^{-14}\ \mathrm{J}}{(3.0\times10^8\ \mathrm{m/s})^2} = 3.02\times10^{-31}\ \mathrm{kg}$$

which is about a third of the correct electron mass value.

113. A positive charge q is a distance $r - d$ from P, another positive charge q is a distance r from P, and a negative charge $-q$ is a distance $r + d$ from P. Sum the individual electric potentials created at P to find the total:

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right].$$

We use the binomial theorem to approximate $1/(r - d)$ for r much larger than d:

$$\frac{1}{r-d} = (r-d)^{-1} \approx (r)^{-1} - (r)^{-2}(-d) = \frac{1}{r} + \frac{d}{r^2}.$$

Similarly,

$$\frac{1}{r+d} \approx \frac{1}{r} - \frac{d}{r^2}.$$

Only the first two terms of each expansion were retained. Thus,

$$V \approx \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{d}{r^2} + \frac{1}{r} - \frac{1}{r} + \frac{d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{2d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{2d}{r}\right].$$

114. (a) The net potential is

$$V = V_1 + V_2 = \frac{q_1}{4\pi\varepsilon_o r_1} + \frac{q_2}{4\pi\varepsilon_o r_2}$$

where $r_1 = \sqrt{x^2 + y^2}$ and $r_2 = \sqrt{(x-d)^2 + y^2}$. The distance d is 8.6 nm. To find the locus of points resulting in $V = 0$, we set V_1 equal to the (absolute value of) V_2 and square both sides. After simplifying and rearranging we arrive at an equation for a circle:

$$y^2 + \left(x + \frac{9d}{16}\right)^2 = \frac{225}{256}\, d^2 .$$

From this form, we recognize that the center of the circle is $-9d/16 = -4.8$ nm.

(b) Also from this form, we identify the radius as the square root of the right-hand side: $R = 15d/16 = 8.1$ nm.

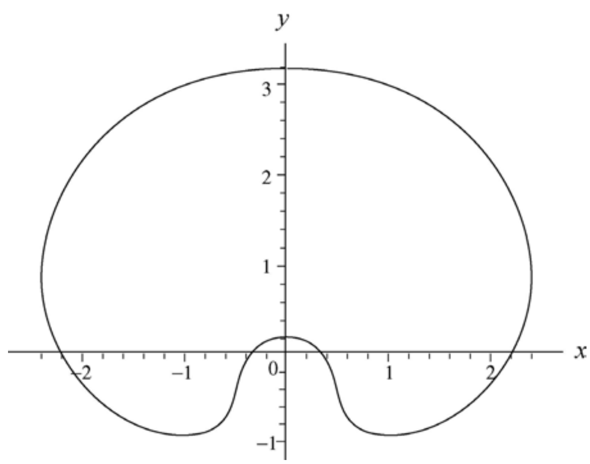
(c) If one uses a graphing program with "implicitplot" features, it is certainly possible to set V = 5 volts in the expression (shown in part (a)) and find its (or one of its) equipotential curves in the xy plane. In fact, it will look very much like a circle. Algebraically, attempts to put the expression into any standard form for a circle will fail, but that can be a frustrating endeavor. Perhaps the easiest way to show that it is not truly a circle is to find where its "horizontal diameter" D_x and its "vertical diameter" D_y (not hard to do); we find $D_x = 2.582$ nm and $D_y = 2.598$ nm. The fact that $D_x \neq D_y$ is evidence that it is not a true circle.

115. The (implicit) equation for the pair (x,y) in terms of a specific V is

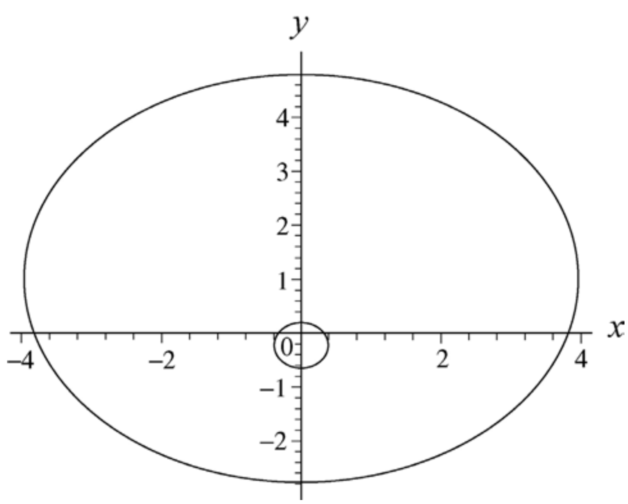
$$V = \frac{q_1}{4\pi\varepsilon_o\sqrt{x^2+y^2}} + \frac{q_2}{4\pi\varepsilon_o\sqrt{x^2+(y-d)^2}}$$

where $d = 0.50$ m. The values of q_1 and q_2 are given in the problem.

(a) We set $V = 5.0$ V and plotted (using MAPLE's implicit plotting routine) those points in the xy plane which (when plugged into the above expression for V) yield 5.0 volts. The result is



(b) In this case, the same procedure yields these two equipotential lines:



(c) One way to search for the "crossover" case (from a single equipotential line, to two) is to "solve" for a point on the y axis (chosen here to be an absolute distance ξ below q_1 – that is, the point is at a negative value of y, specifically at $y = -\xi$) in terms of V (or more conveniently, in terms of the parameter $\eta = 4\pi\varepsilon_o V \times 10^{10}$). Thus, the above expression for V becomes simply

$$\eta = \frac{-12}{\xi} + \frac{25}{d+\xi} \ .$$

This leads to a quadratic equation with the (formal) solution

$$\xi = \frac{13 - d\eta \pm \sqrt{d^2\eta^2 + 169 - 74\, d\eta}}{2\,\eta} \ .$$

Clearly there is the possibility of having two solutions (implying two intersections of equipotential lines with the –y axis) when the square root term is nonzero. This suggests that we explore the special case where the square root term is zero; that is,

$$\sqrt{d^2\eta^2 + 169 - 74\, d\eta} = 0 \ .$$

Squaring both sides, using the fact that $d = 0.50$ m and recalling how we have defined the parameter η, this leads to a "critical value" of the potential (corresponding to the crossover case, between one and two equipotentials):

$$\eta_{\text{critical}} = \frac{37 - 20\sqrt{3}}{d} \quad \Rightarrow \quad V_{\text{critical}} = \frac{\eta_{\text{critical}}}{4\pi\varepsilon_o \times 10^{10}} = 4.2 \text{ V}.$$

116. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

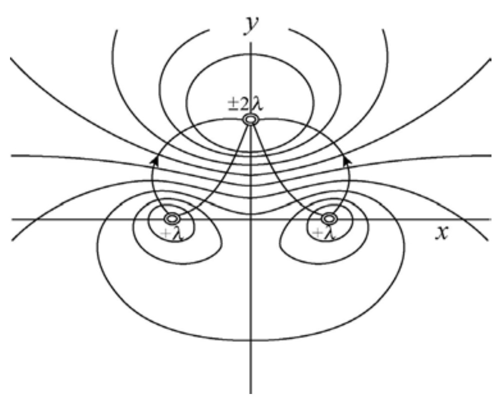
which integrates, using Eq. 24-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right).$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which intersects the plane of our figure, shown next, at the xy coordinate origin, placed midway between the bottom two line charges — that is, the midpoint of the bottom side of the equilateral triangle) at a distance $r_f = a$ from each of the bottom wires (and a distance $a\sqrt{3}$ from the topmost wire). Thus, each side of the triangle is of length $2a$. Skipping some steps, we arrive at an expression for the net potential created by the three wires (where we have set $V_o = 0$):

$$V_{\text{net}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\left(x^2+\left(y-a\sqrt{3}\right)^2\right)^2}{\left((x+a)^2+y^2\right)\left((x-a)^2+y^2\right)}\right)$$

which forms the basis of our contour plot shown below. On the same plot we have shown four electric field lines, which have been sketched (as opposed to rigorously calculated) and are not meant to be as accurate as the equipotentials. The ±2λ by the top wire in our figure should be –2λ (the ± typo is an artifact of our plotting routine).



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$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right).$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which will be the z axis described in this problem) at a distance $r_f = a$ from the wire. In the "end-view" presented here, the wires and the z axis appear as points as they intersect the xy plane. The potential due to the wire on the left (intersecting the plane at $x = -a$) is

$$V_{\text{negative wire}} = V_o + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right),$$

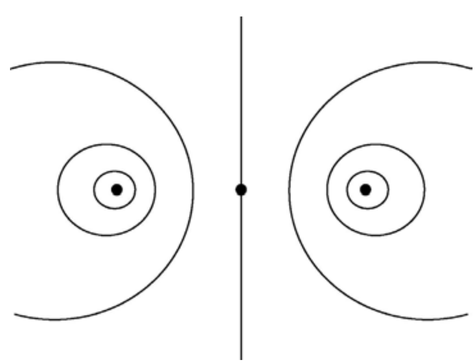
and the potential due to the wire on the right (intersecting the plane at $x = +a$) is

$$V_{\text{positive wire}} = V_o + \frac{(+\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right).$$

Since potential is a scalar quantity, the net potential at point P is the addition of $V_{-\lambda}$ and $V_{+\lambda}$ which simplifies to

$$V_{\text{net}} = 2V_0 + \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

where we have set the potential along the z axis equal to zero ($V_o = 0$) in the last step (which we are free to do). This is the expression used to obtain the equipotentials shown next. The center dot in the figure is the intersection of the z axis with the xy plane, and the dots on either side are the intersections of the wires with the plane.



118. The electric field (along the radial axis) is the (negative of the) derivative of the voltage with respect to r. There are no other components of $\vec{E}$ in this case, so (noting that the derivative of a constant is zero) we conclude that the magnitude of the field is

$$E = -\frac{dV}{dr} = -\frac{Ze}{4\pi\varepsilon_o}\left(\frac{d\,r^{-1}}{dr} + 0 + \frac{1}{2R^3}\frac{d\,r^2}{dr}\right) = \frac{Ze}{4\pi\varepsilon_o}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

for $r \le R$. This agrees with the Rutherford field expression shown in exercise 37 (in the textbook). We note that he has designed his voltage expression to be zero at $r = R$. Since the zero point for the voltage of this system (in an otherwise empty space) is arbitrary, then choosing $V = 0$ at $r = R$ is certainly permissible.

Chapter 25

1. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}.$$

(b) The capacitance is independent of q; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}.$$

2. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\text{ F})(120\text{ V}) = 3.0 \times 10^{-3}\text{ C}.$$

3. For a given potential difference *V*, the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where *d* is the depth from which the electrons come in the plate, and *n* is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by $q = CV$ (Eq. 25-1). Combining the two expressions leads to

$$\frac{C}{A} = ne\frac{d}{V}.$$

With $d/V = d_s/V_s = 5.0\times10^{-14}$ m/V and $n = 8.49\times10^{28}/\text{m}^3$ (see, for example, Sample Problem 25-1), we obtain

$$\frac{C}{A} = (8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\,\text{C})(5.0\times10-14\ \text{m/V}) = 6.79\times10^{-4}\,\text{F/m}^2.$$

4. We use $C = A\varepsilon_0/d$.

(a) The distance between the plates is

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\ \text{m}.$$

(b) Since d is much less than the size of an atom ($\sim 10^{-10}$ m), this capacitor cannot be constructed.

5. (a) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus,

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\ \text{F/m}\right)\pi\left(8.2\times10^{-2}\,\text{m}\right)^2}{1.3\times10^{-3}\,\text{m}} = 1.44\times10^{-10}\,\text{F} = 144\,\text{pF}.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus,

$$q = (1.44 \times 10^{-10}\ \text{F})(120\ \text{V}) = 1.73 \times 10^{-8}\ \text{C} = 17.3\ \text{nC}.$$

6. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm} - 38.0\,\text{mm})} = 84.5\,\text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm} - 38.0\,\text{mm})}{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 191\,\text{cm}^2.$$

7. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With $R = 2.00$ mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\,\text{m}\right) = 2.80\times10^{-13}\,\text{F}$.

8. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00\,\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$

9. The equivalent capacitance is

$$C_{\text{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0\,\mu\text{F} + 5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 3.16\,\mu\text{F}.$$

10. The equivalent capacitance is given by $C_{eq} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{eq} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\mathrm{C}}{(110\,\mathrm{V})(1.00\times10^{-6}\,\mathrm{F})} = 9.09\times10^{3}.$$

11. The charge that passes through meter *A* is

$$q = C_{\text{eq}}V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\,\text{C}.$$

12. (a) The potential difference across C_1 is $V_1 = 10.0$ V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 1.00 \times 10^{-4}\ \text{C}.$$

(b) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2C}{C+C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C+C_{eq}} = \frac{CV_1}{C+1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\,\mu\text{F})\left(\frac{10.0\,\text{V}}{5}\right) = 2.00\times10^{-5}\,\text{C}.$$

13. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{eq}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\,\text{V}.$$

Thus $\Delta V_1 = 100.0\text{ V} - 21.1\text{ V} = 78.9\text{ V}$ and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0\ \mu\text{F})(78.9\text{ V}) = 7.89 \times 10^{-4}\text{ C}.$$

14. The two 6.0 μF capacitors are in parallel and are consequently equivalent to $C_{eq} = 12\ \mu\text{F}$. Thus, the total charge stored (before the squeezing) is

$$q_{\text{total}} = C_{eq}V = (12\,\mu\text{F})\,(10.0\text{V}) = 120\ \mu\text{C}.$$

(a) and (b) As a result of the squeezing, one of the capacitors is now 12 μF (due to the inverse proportionality between C and d in Eq. 25-9) which represents an increase of $6.0\ \mu\text{F}$ and thus a charge increase of

$$\Delta q_{\text{total}} = \Delta C_{eq}V = (6.0\,\mu\text{F})\,(10.0\text{V}) = 60\ \mu\text{C}\ .$$

15. The charge initially on the charged capacitor is given by $q = C_1V_0$, where $C_1 = 100$ pF is the capacitance and $V_0 = 50$ V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where $V = 35$ V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substituting C_1V_0 for q and C_1V for q_1, we obtain $q_2 = C_1\,(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V}C_1 = \frac{50\,\text{V} - 35\,\text{V}}{35\,\text{V}}\left(100\,\text{pF}\right) = 43\,\text{pF}.$$

16. We note that the voltage across C_3 is $V_3 = (12\text{ V} - 2\text{ V} - 5\text{ V}) = 5\text{ V}$. Thus, its charge is $q_3 = C_3 V_3 = 4\ \mu\text{C}$.

(a) Therefore, since C_1, C_2 and C_3 are in series (so they have the same charge), then

$$C_1 = \frac{4\ \mu\text{C}}{2\text{ V}} = 2.0\ \mu\text{F}\ .$$

(b) Similarly, $C_2 = 4/5 = 0.80\ \mu\text{F}$.

17. (a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to C' = 2(3.00 μF) = 6.00 μF). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C+C'} = \frac{(6.00\,\mu\text{F})\,(6.00\,\mu\text{F})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 3.00\,\mu\text{F}.$$

(b) Let V = 20.0 V be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\,\mu\text{F})\,(20.0\,\text{V})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 10.0\,\text{V}.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1$= (3.00 μF)(10.0 V) = 3.00 × 10^{-5} C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1$ = 20.0 V – 10.0 V = 10.0 V.

(f) The charge carried by C_2 is $q_2 = C_2V_2$ = (2.00 μF)(10.0 V) = 2.00 × 10^{-5} C.

(g) Since this voltage difference V_2 is divided equally between C_3 and the other 4.00-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2$ = 10.0 V/2 = 5.00 V.

(h) Thus, $q_3 = C_3V_3$ = (4.00 μF)(5.00 V) = 2.00 × 10^{-5} C.

18. We determine each capacitance from the slope of the appropriate line in the graph. Thus, C_1 = (12 μC)/(2.0 V) = 6.0 μF. Similarly, C_2 = 4.0 μF and C_3 = 2.0 μF. The total equivalent capacitance is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)},$$

or

$$C_{123} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3} = \frac{(6.0\ \mu\text{F})(4.0\ \mu\text{F} + 2.0\ \mu\text{F})}{6.0\ \mu\text{F} + 4.0\ \mu\text{F} + 2.0\ \mu\text{F}} = \frac{36}{12}\ \mu\text{F} = 3.0\ \mu\text{F}.$$

This implies that the charge on capacitor 1 is $q_1 = (3.0\ \mu\text{F})(6.0\ \text{V}) = 18\ \mu\text{C}$. The voltage across capacitor 1 is therefore V_1 = (18 μC)/(6.0 μF) = 3.0 V. From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be 6.0 V – 3.0 V = 3.0 V. Consequently, the charge on capacitor 2 is (4.0 μF)(3.0 V) = 12 μC.

19. (a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\text{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance. The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1V = \left(1.0\times10^{-6}\ \text{F}\right)\left(100\ \text{V}\right) = 1.0\times10^{-4}\ \text{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2V = \left(3.0\times10^{-6}\ \text{F}\right)\left(100\ \text{V}\right) = 3.0\times10^{-4}\ \text{C},$$

so the net charge on the combination is 3.0×10^{-4} C – 1.0×10^{-4} C = 2.0×10^{-4} C. The potential difference is

$$V_{ab} = \frac{2.0\times10^{-4}\ \text{C}}{4.0\times10^{-6}\ \text{F}} = 50\ \text{V}.$$

(b) The charge on capacitor 1 is now $q_1 = C_1V_{ab} = (1.0 \times 10^{-6}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-5}$ C.

(c) The charge on capacitor 2 is now $q_2 = C_2V_{ab} = (3.0 \times 10^{-6}\ \text{F})(50\ \text{V}) = 1.5 \times 10^{-4}$ C.

20. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\text{bat}} = 100\ \mu\text{C}$, and q_1, q_2 and q_3 are the charges on C_1, C_2 and C_3 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3 \ .$$

Since the parallel pair C_2 and C_3 are identical, it is clear that $q_2 = q_3$. They are in parallel with C_1 so that $V_1 = V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = (q_3 / 2) + q_3 + q_3 = 5q_3 / 2$$

which yields $q_3 = 2Q/5 = 2(100\ \mu\text{C})/5 = 40\ \mu\text{C}$ and consequently $q_1 = q_3/2 = 20\ \mu\text{C}$.

21. Eq. 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_o E_1 A_1 + \varepsilon_o E_2 A_2 = 3.6 \text{ pC}$$

where we have been careful to convert cm^2 to m^2 by dividing by 10^4.

22. Using Equation 25-14, the capacitances are

$$C_1 = \frac{2\pi\varepsilon_0 L_1}{\ln(b_1/a_1)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.050\ \text{m})}{\ln(15\ \text{mm}/5.0\ \text{mm})} = 2.53\ \text{pF}$$

$$C_2 = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(10\ \text{mm}/2.5\ \text{mm})} = 3.61\ \text{pF} \ .$$

Initially, the total equivalent capacitance is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.53\ \text{pF})(3.61\ \text{pF})}{2.53\ \text{pF} + 3.61\ \text{pF}} = 1.49\ \text{pF} \ ,$$

and the charge on the positive plate of each one is (1.49 pF)(10 V) = 14.9 pC. Next, capacitor 2 is modified as described in the problem, with the effect that

$$C_2' = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2'/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(25\ \text{mm}/2.5\ \text{mm})} = 2.17\ \text{pF} \ .$$

The new total equivalent capacitance is

$$C_{12}' = \frac{C_1 C_2'}{C_1 + C_2'} = \frac{(2.53\ \text{pF})(2.17\ \text{pF})}{2.53\ \text{pF} + 2.17\ \text{pF}} = 1.17\ \text{pF}$$

and the new charge on the positive plate of each one is (1.17 pF)(10 V) = 11.7 pC. Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is 14.9 pC – 11.7 pC = 3.2 pC.

(a) This charge, divided by *e* gives the number of electrons that pass point *P*. Thus,

$$N = \frac{3.2\times10^{-12}\,\text{C}}{1.6\times10^{-19}\,\text{C}} = 2.0\times10^{7} \ .$$

(b) These electrons move rightwards in the figure (that is, away from the battery) since the positive plates (the ones closest to point *P*) of the capacitors have suffered a *decease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have "returned" to the positive plates (making them less positive).

23. (a) and (b) We note that the charge on C_3 is $q_3 = 12\ \mu\text{C} - 8.0\ \mu\text{C} = 4.0\ \mu\text{C}$. Since the charge on C_4 is $q_4 = 8.0\ \mu\text{C}$, then the voltage across it is $q_4/C_4 = 2.0$ V. Consequently, the voltage V_3 across C_3 is 2.0 V $\Rightarrow C_3 = q_3/V_3 = 2.0\ \mu\text{F}$.

Now C_3 and C_4 are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with C_2; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown C_1. We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is $(12\ \mu\text{C})/V_{\text{battery}} = 4\mu\text{F}/3$. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \quad \Rightarrow \quad C_1 = 4.0\ \mu\text{F}\ .$$

24. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of $(n - 1)$ identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.25\times10^{-4}\ \text{m}^2)}{3.40\times10^{-3}\ \text{m}} = 2.28\times10^{-12}\,\text{F}.$$

25. We note that the total equivalent capacitance is $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6\ \mu\text{F}$.

(a) Thus, the charge that passed point a is $C_{123}\, V_{\text{batt}} = (6\ \mu\text{F})(12\ \text{V}) = 72\ \mu\text{C}$. Dividing this by the value $e = 1.60 \times 10^{-19}$ C gives the number of electrons: 4.5×10^{14}, which travel to the left – towards the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is $C_{12} = C_1 + C_2 = 12\ \mu\text{F}$. Thus, the voltage across the pair (which is the same as the voltage across C_1 and C_2 individually) is

$$\frac{72\ \mu\text{C}}{12\ \mu\text{F}} = 6\ \text{V} .$$

Thus, the charge on C_1 is $q_1 = (4\ \mu\text{F})(6\ \text{V}) = 24\ \mu\text{C}$, and dividing this by e gives $N_1 = q_1 / e = 1.5 \times 10^{14}$, the number of electrons that have passed (upward) though point b.

(c) Similarly, the charge on C_2 is $q_2 = (8\ \mu\text{F})(6\ \text{V}) = 48\ \mu\text{C}$, and dividing this by e gives $N_2 = q_2 / e = 3.0 \times 10^{14}$, the number of electrons which have passed (upward) though point c.

(d) Finally, since C_3 is in series with the battery, its charge is the same that passed through the battery (the same as passed through the switch). Thus, 4.5×10^{14} electrons passed rightward though point d. By leaving the rightmost plate of C_3, that plate is then the positive plate of the fully charged capacitor – making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point b.

(f) As stated in (c), the electrons travel up through point c.

26. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_2+C_3}{C_2C_3}.$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1}=\frac{q_2}{C_{\text{eq}}},\ q_1+q_2=C_1V_0$$

for q_1 and q_2, we obtain

$$q_1=\frac{C_1^2V_0}{C_{\text{eq}}+C_1}=\frac{C_1^2V_0}{\dfrac{C_2C_3}{C_2+C_3}+C_1}=\frac{C_1^2\left(C_2+C_3\right)V_0}{C_1C_2+C_1C_3+C_2C_3}.$$

With $V_0 = 12.0$ V, $C_1= 4.00\ \mu$F, $C_2= 6.00\ \mu$F and $C_3 =3.00\ \mu$F, we find $C_{\text{eq}} = 2.00\ \mu$F and $q_1 = 32.0\ \mu$C.

(b) The charge on capacitors 2 is

$$q_2=C_1V_0-q_1=(4.00\mu\text{F})(12.0\text{V})-32.0\mu\text{C}=16.0\mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3=C_1V_0-q_1=(4.00\mu\text{F})(12.0\text{V})-32.0\mu\text{C}=16.0\mu\text{C}.$$

27. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1 C_3 V}{C_1 + C_3} = \frac{(1.00\,\mu\text{F})\,(3.00\,\mu\text{F})\,(12.0\text{V})}{1.00\,\mu\text{F}+3.00\,\mu\text{F}} = 9.00\,\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2 C_4 V}{C_2 + C_4} = \frac{(2.00\,\mu\text{F})\,(4.00\,\mu\text{F})\,(12.0\text{V})}{2.00\,\mu\text{F}+4.00\,\mu\text{F}} = 16.0\,\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\,\mu\text{C}.$

(d) $q_4 = q_2 = 16.0\,\mu\text{C}.$

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}\,V = \frac{(3.00\,\mu\text{F}+4.00\,\mu\text{F})(12.0\text{V})}{1.00\,\mu\text{F}+2.00\,\mu\text{F}+3.00\,\mu\text{F}+4.00\,\mu\text{F}} = 8.40\text{V}.$$

Thus, $q_1 = C_1V_1 = (1.00\ \mu\text{F})(8.40\ \text{V}) = 8.40\ \mu\text{C}$.

(f) Similarly, $q_2 = C_2V_1 = (2.00\ \mu\text{F})(8.40\ \text{V}) = 16.8\ \mu\text{C}$.

(g) $q_3 = C_3(V - V_1) = (3.00\ \mu\text{F})(12.0\ \text{V} - 8.40\ \text{V}) = 10.8\ \mu\text{C}$.

(h) $q_4 = C_4(V - V_1) = (4.00\ \mu\text{F})(12.0\ \text{V} - 8.40\ \text{V}) = 14.4\ \mu\text{C}$.

28. Initially the capacitors C_1, C_2, and C_3 form a combination equivalent to a single capacitor which we denote C_{123}. This obeys the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)} .$$

Hence, using $q = C_{123}V$ and the fact that $q = q_1 = C_1 V_1$, we arrive at

$$V_1 = \frac{q_1}{C_1} = \frac{q}{C_1} = \frac{C_{123}}{C_1}V = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V .$$

(a) As $C_3 \to \infty$ this expression becomes $V_1 = V$. Since the problem states that V_1 approaches 10 volts in this limit, so we conclude $V = 10$ V.

(b) and (c) At $C_3 = 0$, the graph indicates $V_1 = 2.0$ V. The above expression consequently implies $C_1 = 4C_2$. Next we note that the graph shows that, at $C_3 = 6.0$ μF, the voltage across C_1 is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0\ \mu\text{F}}{C_1 + C_2 + 6.0\ \mu\text{F}} = \frac{C_2 + 6.0\ \mu\text{F}}{4C_2 + C_2 + 6.0\ \mu\text{F}}$$

which leads to $C_2 = 2.0$ μF. We conclude, too, that $C_1 = 8.0$ μF.

29. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\,\text{F} + 4.0\times10^{-6}\,\text{F}\right)\left(300\,\text{V}\right)^2 = 0.27\,\text{J}.$$

30. (a) The capacitance is

$$C=\frac{\varepsilon_0 A}{d}=\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(40\times10^{-4}\ \text{m}^2\right)}{1.0\times10^{-3}\,\text{m}}=3.5\times10^{-11}\,\text{F}=35\,\text{pF}.$$

(b) $q = CV = (35\ \text{pF})(600\ \text{V}) = 2.1\times10^{-8}\ \text{C} = 21\ \text{nC}$.

(c) $U=\frac{1}{2}CV^2=\frac{1}{2}(35\,\text{pF})(21\,\text{nC})^2=6.3\times10^{-6}\,\text{J}=6.3\,\mu\text{J}$.

(d) $E = V/d = 600\ \text{V}/1.0\times10^{-3}\ \text{m} = 6.0\times10^5\ \text{V/m}$.

(e) The energy density (energy per unit volume) is

$$u=\frac{U}{Ad}=\frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\,\text{m}\right)}=1.6\ \text{J/m}^3.$$

31. The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where V is the potential difference across its plates. We convert the given value of the energy to Joules. Since $1\text{ J} = 1\text{ W}\cdot\text{s}$, we multiply by ($10^3$ W/kW)(3600 s/h) to obtain $10\text{ kW}\cdot\text{h} = 3.6\times10^7\text{ J}$. Thus,

$$C = \frac{2U}{V^2} = \frac{2\left(3.6\times10^7\text{ J}\right)}{\left(1000\text{ V}\right)^2} = 72\text{ F}.$$

32. Let $\mathcal{V} = 1.00\ \text{m}^3$. Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(150\ \text{V/m})^2\left(1.00\ \text{m}^3\right) = 9.96\times10^{-8}\ \text{J}.$$

33. The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4}.$$

(a) At $r = 1.00\times10^{-3}\,\text{m}$, with $e = 1.60\times10^{-19}\,\text{C}$ and $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$, we have $u = 9.16\times10^{-18}\ \text{J/m}^3$.

(b) Similarly, at $r = 1.00\times10^{-6}\,\text{m}$, $u = 9.16\times10^{-6}\ \text{J/m}^3$.

(c) At $r = 1.00\times10^{-9}\,\text{m}$, $u = 9.16\times10^{6}\ \text{J/m}^3$.

(d) At $r = 1.00\times10^{-12}\,\text{m}$, $u = 9.16\times10^{18}\ \text{J/m}^3$.

(e) From the expression above $u \propto r^{-4}$. Thus, for $r \to 0$, the energy density $u \to \infty$.

34. (a) The potential difference across C_1 (the same as across C_2) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}+15.0\,\mu\text{F}} = 50.0\,\text{V}.$$

Also, $V_3 = V - V_1 = V - V_2 = 100$ V – 50.0 V = 50.0 V. Thus,

$$q_1 = C_1 V_1 = (10.0\,\mu\text{F})(50.0\,\text{V}) = 5.00\times10^{-4}\,\text{C}$$
$$q_2 = C_2 V_2 = (5.00\,\mu\text{F})(50.0\,\text{V}) = 2.50\times10^{-4}\,\text{C}$$
$$q_3 = q_1 + q_2 = 5.00\times10^{-4}\,\text{C} + 2.50\times10^{-4}\,\text{C} = 7.50\times10^{-4}\,\text{C}.$$

(b) The potential difference V_3 was found in the course of solving for the charges in part (a). Its value is $V_3 = 50.0$ V.

(c) The energy stored in C_3 is $U_3 = C_3 V_3^2/2 = (15.0\,\mu\text{F})(50.0\,\text{V})^2/2 = 1.88\times10^{-2}\,\text{J}$.

(d) From part (a), we have $q_1 = 5.00\times10^{-4}\,\text{C}$, and

(e) $V_1 = 50.0$ V, as shown in (a).

(f) The energy stored in C_1 is

$$U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(50.0\,\text{V})^2 = 1.25\times10^{-2}\,\text{J}.$$

(g) Again, from part (a), $q_2 = 2.50\times10^{-4}\,\text{C}$.

(h) $V_2 = 50.0$ V, as shown in (a).

(i) The energy stored in C_2 is $U_2 = \frac{1}{2}C_2 V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(50.0\,\text{V})^2 = 6.25\times10^{-3}\,\text{J}$.

35. (a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\varepsilon_0 A/d_i$, the charge is $q = CV = \varepsilon_0 AV_i/d_i$. After the plates are pulled apart, their separation is d_f and the potential difference is V_f. Then $q = \varepsilon_0 AV_f/2d_f$ and

$$V_f = \frac{d_f}{\varepsilon_0 A} q = \frac{d_f}{\varepsilon_0 A}\frac{\varepsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i.$$

With $d_i = 3.00\times10^{-3}\,\text{m}$, $V_i = 6.00$ V and $d_f = 8.00\times10^{-3}\,\text{m}$, we have $V_f = 16.0\text{ V}$.

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.50\times10^{-4}\text{ m}^2)(6.00\text{ V})^2}{2(3.00\times10^{-3}\text{ m})} = 4.51\times10^{-11}\text{ J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 AV_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f/d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}$ J.

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52\times10^{-11}\text{ J}.$$

36. (a) The charge q_3 in the figure is $q_3 = C_3V = (4.00\ \mu\text{F})(100\ \text{V}) = 4.00\times10^{-4}\ \text{C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\,\text{J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1+C_2} = \frac{(10.0\ \mu\text{F})(5.00\ \mu\text{F})(100\ \text{V})}{10.0\ \mu\text{F} + 5.00\ \mu\text{F}} = 3.33\times10^{-4}\,\text{C}.$$

(e) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}$ C/10.0 μF = 33.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\,\text{J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\,\text{C}$.

(h) $V_2 = V - V_1 = 100$ V – 33.3 V = 66.7 V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\,\text{J}$.

37. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain U_{total} = 0.095 J.

38. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\ \text{m})(0.10\ \text{m}) + \pi(0.20\ \text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\ \mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge q is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0\times10^{-7}\ \text{C})^2}{2(35\times10^{-12}\ \text{F})} = 3.6\times10^{-3}\ \text{J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

39. We use $E = q / 4\pi\varepsilon_0 R^2 = V / R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\ \text{V}}{0.050\ \text{m}}\right)^2 = 0.11\ \text{J/m}^3.$$

40. (a) We use $C = \varepsilon_0 A/d$ to solve for d:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.35\ \mathrm{m^2})}{50\times10^{-12}\,\mathrm{F}} = 6.2\times10^{-2}\ \mathrm{m}.$$

(b) We use $C \propto \kappa$. The new capacitance is

$$C' = C(\kappa/\kappa_{\mathrm{air}}) = (50\ \mathrm{pf})(5.6/1.0) = 2.8\times10^2\ \mathrm{pF}.$$

41. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4\times 10^{-6}\,\text{J})}{(7.4\times 10^{-12}\,\text{F})(652\text{V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

42. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A / 2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6 \text{ pF}/1.3 \text{ pF}) = 4.0.$$

43. The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\ \text{F/m})}{\ln[(0.60\ \text{mm})/(0.10\ \text{mm})]} = 8.1\times10^{-11}\,\text{F/m} = 81\ \text{pF/m}.$$

44. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21 \times 10^{-11} \text{ F} ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64 \times 10^{-11} \text{ F} .$$

This leads to $q_1 = C_1V_1 = 8.00 \times 10^{-10}$ C and $q_2 = C_2V_2 = 2.66 \times 10^{-10}$ C. The addition of these gives the desired result: $q_{\text{tot}} = 1.06 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.

45. The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Thus,

$$A = \frac{CV}{\kappa\varepsilon_0 E}.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\text{F})(4.0\times10^{3}\,\text{V})}{2.8(8.85\times10^{-12}\,\text{F/m})(18\times10^{6}\ \text{V/m})} = 0.63\ \text{m}^2.$$

46. (a) We use Eq. 25-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\text{ m})}{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\text{ cm}/3.6\text{ cm})} = 0.73\text{ nF}.$$

(b) The breakdown potential is (14 kV/mm) (3.8 cm – 3.6 cm) = 28 kV.

47. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^3 \text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5} \text{ J} .$$

48. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right)$$

$$= \frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(5.56\times10^{-4}\ \mathrm{m^2})}{5.56\times10^{-3}\ \mathrm{m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times10^{-12}\ \mathrm{F}.$$

49. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A},$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A}.$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right] = \frac{qd}{2\varepsilon_0 A}\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

With $A = 7.89\times10^{-4}\,\text{m}^2$, $d = 4.62\times10^{-3}\,\text{m}$, $\kappa_1 = 11.0$ and $\kappa_2 = 12.0$, the capacitance is

$$C = \frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.89\times10^{-4}\ \text{m}^2)}{4.62\times10^{-3}\ \text{m}}\frac{(11.0)(12.0)}{11.0+12.0} = 1.73\times10^{-11}\,\text{F}.$$

50. Let $C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d$, $C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d$, and $C_3 = \varepsilon_0 A\kappa_3/2d$. Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A\kappa_1}{4d} + \frac{(\varepsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} = \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right).$$

With $A = 1.05\times10^{-3}\,\text{m}^2$, $d = 3.56\times10^{-3}\,\text{m}$, $\kappa_1 = 21.0$, $\kappa_2 = 42.0$ and $\kappa_3 = 58.0$, the capacitance is

$$C = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.05\times10^{-3}\ \text{m}^2)}{4(3.56\times10^{-3}\ \text{m})}\left(21.0 + \frac{2(42.0)(58.0)}{42.0+58.0}\right) = 4.55\times10^{-11}\,\text{F}.$$

51. (a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\varepsilon_0 A / C$ and

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\ \text{V})(100\times 10^{-12}\ \text{F})}{5.4(8.85\times 10^{-12}\ \text{F/m})(100\times 10^{-4}\ \text{m}^2)} = 1.0\times 10^4\ \text{V/m}.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-9}$ C.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times 10^{-9}\,\text{C} - (8.85\times 10^{-12}\ \text{F/m})(100\times 10^{-4}\,\text{m}^2)(1.0\times 10^4\ \text{V/m}) \\ &= 4.1\times 10^{-9}\,\text{C} = 4.1\,\text{nC}. \end{aligned}$$

52. (a) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(115\times10^{-4}\ \mathrm{m^2}\right)(2.61)}{(2.61)(0.0124\,\mathrm{m}-0.00780\,\mathrm{m})+(0.00780\,\mathrm{m})} = 13.4\,\mathrm{pF}.$$

(b) $q = CV = (13.4 \times 10^{-12}\ \mathrm{F})(85.5\ \mathrm{V}) = 1.15\ \mathrm{nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\ \mathrm{C}}{\left(8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)\left(115\times10^{-4}\ \mathrm{m^2}\right)} = 1.13\times10^{4}\ \mathrm{N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^{4}\ \mathrm{N/C}}{2.61} = 4.33\times10^{3}\ \mathrm{N/C}.$$

53. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant κ. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{23.5}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}}\cdot\frac{(0.0120\ \mathrm{m})(0.0170\ \mathrm{m})}{0.0170\ \mathrm{m} - 0.0120\ \mathrm{m}} = 0.107\ \mathrm{nF}.$$

(b) The charge on the positive plate is $q = CV = (0.107\ \mathrm{nF})(73.0\ \mathrm{V}) = 7.79\ \mathrm{nC}$.

(c) Let the charge on the inner conductor be $-q$. Immediately adjacent to it is the induced charge q'. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V = \left(\frac{23.5-1.00}{23.5}\right)(7.79\ \mathrm{nC}) = 7.45\ \mathrm{nC}.$$

54. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.4\times10^{-6}\ \text{V/m}\right)\left(100\times10^{-4}\,\text{m}^2\right)} = 7.2.$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = \left(8.9\times10^{-7}\ \text{C}\right)\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$$

55. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.12\ \mathrm{m^2})}{1.2\times10^{-2}\ \mathrm{m}} = 89\ \mathrm{pF}.$$

(b) Working through Sample Problem 25-7 algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.12\,\mathrm{m^2})(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\mathrm{m})+(4.0\times10^{-3}\,\mathrm{m})} = 1.2\times10^{2}\,\mathrm{pF}.$$

(c) Before the insertion, $q = C_0V$ (89 pF)(120 V) = 11 nC.

(d) Since the battery is disconnected, q will remain the same after the insertion of the slab, with q = 11 nC.

(e) $E = q/\varepsilon_0 A = 11\times10^{-9}\ \mathrm{C}/(8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}})(0.12\ \mathrm{m^2}) = 10\ \mathrm{kV/m}.$

(f) $E' = E/\kappa$ = (10 kV/m)/4.8 = 2.1 kV/m.

(g) The potential difference across the plates is

$V = E(d-b) + E'b$ = (10 kV/m)(0.012 m – 0.0040 m)+ (2.1 kV/m)(0.40 × 10^{-3} m) = 88 V.

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(11\times10^{-9}\,\mathrm{C})^2}{2}\left(\frac{1}{89\times10^{-12}\,\mathrm{F}}-\frac{1}{120\times10^{-12}\,\mathrm{F}}\right) = -1.7\times10^{-7}\,\mathrm{J}.$$

56. (a) Eq. 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^{3}\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

57. Initially the capacitors C_1, C_2, and C_3 form a series combination equivalent to a single capacitor which we denote C_{123}. Solving the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1C_2C_3},$$

we obtain C_{123} = 2.40 μF. With V = 12.0 V, we then obtain $q = C_{123}V$ = 28.8 μC. In the final situation, C_2 and C_4 are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{C_1C_{24} + C_{24}C_3 + C_1C_3}{C_1C_{24}C_3}$$

and find C_{1234} = 3.00 μF. Therefore, the final charge is $q = C_{1234}V$ = 36.0 μC.

(a) This represents a change (relative to the initial charge) of Δq = 7.20 μC.

(b) The capacitor C_{24} which we imagined to replace the parallel pair C_2 and C_4 is in series with C_1 and C_3 and thus also has the final charge q =36.0 μC found above. The voltage across C_{24} would be

$$V_{24} = \frac{q}{C_{24}} = \frac{36.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 3.00\ \text{V}.$$

This is the same voltage across each of the parallel pair. In particular, V_4 = 3.00 V implies that $q_4 = C_4 V_4$ = 18.0 μC.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

58. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

(a) $100\ \mu\text{J} = \frac{1}{2} C_1 (10\ \text{V})^2 \Rightarrow C_1 = 2.0\ \mu\text{F}$

(b) $300\ \mu\text{J} = \frac{1}{2} C_2 (10\ \text{V})^2 \Rightarrow C_2 = 6.0\ \mu\text{F}$.

59. Initially, the total equivalent capacitance is $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0\ \mu F$, and the charge on the positive plate of each one is $(3.0\ \mu F)(10\ V) = 30\ \mu C$. Next, the capacitor (call is C_1) is squeezed as described in the problem, with the effect that the new value of C_1 is 12 μF (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0\ \mu F,$$

and the new charge on the positive plate of each one is $(4.0\ \mu F)(10\ V) = 40\ \mu C$.

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is $40\ \mu C - 30\ \mu C = 10\ \mu C$.

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series): 20 μC.

60. (a) We reduce the parallel group C_2, C_3 and C_4, and the parallel pair C_5 and C_6, obtaining equivalent values $C' = 12\ \mu\text{F}$ and $C'' = 12\ \mu\text{F}$, respectively. We then reduce the series group C_1, C' and C'' to obtain an equivalent capacitance of $C_{eq} = 3\ \mu\text{F}$ hooked to the battery. Thus, the charge stored in the system is $q_{sys} = C_{eq}V_{bat} = 36\ \mu\text{C}$.

(b) Since $q_{sys} = q_1$ then the voltage across C_1 is

$$V_1 = \frac{q_1}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\ \text{V} \ .$$

The voltage across the series-pair C' and C'' is consequently $V_{bat} - V_1 = 6.0$ V. Since $C' = C''$, we infer $V' = V'' = 6.0/2 = 3.0$ V, which, in turn, is equal to V_4, the potential across C_4. Therefore,

$$q_4 = C_4V_4 = (4.0\ \mu\text{F})(3.0\ \text{V}) = 12\ \mu\text{C} \ .$$

61. The pair C_3 and C_4 are in parallel and consequently equivalent to 30 μF. Since this numerical value is identical to that of the others (with which it is in series, with the battery), we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4, producing a charge

$$q_4 = C_4 V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C} \ .$$

62. (a) The potential across C_1 is 10 V, so the charge on it is

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 100\ \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to 6.00 μF, with 10.0 V across it. Thus, a charge of 60.0 μC is on it -- and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\ \mu\text{C}}{10\ \mu\text{F}} = 6.00\ \text{V}$$

which leaves 10.0 V – 6.00 V = 4.00 V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by C_2 and the capacitor directly below it (in series with it). Therefore, with 2.00 V across C_2 we find

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(2.00\ \text{V}) = 20.0\ \mu\text{C}\ .$$

63. The pair C_1 and C_2 are in parallel, as are the pair C_3 and C_4; they reduce to equivalent values 6.0 μF and 3.0 μF, respectively. These are now in series and reduce to 2.0 μF, across which we have the battery voltage. Consequently, the charge on the 2.0 μF equivalence is (2.0 μF)(12 V) = 24 μC. This charge on the 3.0 μF equivalence (of C_3 and C_4) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V}\ .$$

Finally, this voltage on capacitor C_4 produces a charge (2.0 μF)(8.0 V) = 16 μC.

64. (a) Here D is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with C and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4C)}{C + 4.4C} = 0.82\,C = 0.82\,(50\,\mu\text{F}) = 41\,\mu\text{F}$$

where we have used the fact that $C = 50\ \mu$F.

(b) Now, B is the point which is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the C capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5C)}{C + 5.5C} = 0.85C = 42\ \mu\text{F}\ .$$

65. (a) The equivalent capacitance is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 2.40\,\mu\text{F}\ .$$

(b) $q_1 = C_{eq}V = (2.40\ \mu\text{F})(200\ \text{V}) = 4.80 \times 10^{-4}$ C.

(c) $V_1 = q_1/C_1 = 4.80 \times 10^{-4}\ \text{C}/6.00\ \mu\text{F} = 80.0$ V.

(d) $q_2 = q_1 = 4.80 \times 10^{-4}$ C.

(e) $V_2 = V - V_1 = 200\ \text{V} - 80.0\ \text{V} = 120$ V.

66. (a) Now $C_{eq} = C_1 + C_2 = 6.00\ \mu F + 4.00\ \mu F = 10.0\ \mu F$.

(b) $q_1 = C_1V = (6.00\ \mu F)(200\ V) = 1.20 \times 10^{-3}$ C.

(c) V_1=200 V.

(d) $q_2 = C_2V = (4.00\ \mu F)(200\ V) = 8.00 \times 10^{-4}$ C.

(e) $V_2 = V_1 = 200$ V.

67. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors C and 60 μF). We do expect charge to be conserved. Thus, if Q is the charge originally stored on C and q_1, q_2 are the charges on the parallel pair after "settling down," then

$$Q = q_1 + q_2 \quad \Rightarrow \quad C(100\,\text{V}) = C(40\,\text{V}) + (60\,\mu\text{F})(40\,\text{V})$$

which leads to the solution $C = 40$ μF.

68. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\varepsilon_0 L r},$$

and the energy density at that point is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral

$$U_R = \int u\, d\mathcal{V}.$$

Now, $d\mathcal{V} = 2\pi r L dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} 2\pi r L dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{R}{a}.$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{b}{a}.$$

We want the ratio U_R/U_b to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$, $\ln(R/a) = \ln\left(\sqrt{b/a}\right)$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

69. (a) Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances (d = 0.00300 m) between the plates, then the field in A is equal to the field in B:

$$\left|\vec{E}\right| = \frac{V}{d} = 2.00 \times 10^5 \text{ V/m} .$$

(b) $|\vec{E}| = 2.00\times10^5$ V/m . See the note in part (a).

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma = \frac{q}{A} = \varepsilon_0\left|\vec{E}\right| = 1.77 \times 10^{-6} \text{ C/m}^2 .$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma = \kappa\varepsilon_0\left|\vec{E}\right| = 4.60 \times 10^{-6} \text{ C/m}^2 .$$

(e) Although the discussion in the textbook (§25-8) is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors which have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor B has a relatively large charge but only produces the field that A produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = \left(1.77 \times 10^{-6}\right) - \left(4.60 \times 10^{-6}\right) = -2.83 \times 10^{-6} \text{ C/m}^2 .$$

70. (a) The equivalent capacitance is $C_{eq} = C_1C_2/(C_1 + C_2)$. Thus the charge q on each capacitor is

$$q = q_1 = q_2 = C_{eq}V = \frac{C_1C_2V}{C_1+C_2} = \frac{(2.00\mu\text{F})(8.00\mu\text{F})(300\,\text{V})}{2.00\mu\text{F}+8.00\mu\text{F}} = 4.80\times10^{-4}\,\text{C}.$$

(b) The potential difference is $V_1 = q/C_1 = 4.80 \times 10^{-4}$ C/2.0 μF = 240 V.

(c) As noted in part (a), $q_2 = q_1 = 4.80\times10^{-4}\,\text{C}$.

(d) $V_2 = V - V_1$ = 300 V – 240 V = 60.0 V.

Now we have $q'_1/C_1 = q'_2/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q'_1 + q'_2 = 2q$. We solve for q'_1, q'_2 and V:

(e) $$q'_1 = \frac{2C_1q}{C_1+C_2} = \frac{2(2.00\mu\text{F})(4.80\times10^{-4}C)}{2.00\mu\,\text{F}+8.00\mu\,\text{F}} = 1.92\times10^{-4}\,\text{C}.$$

(f) $$V_1' = \frac{q_1'}{C_1} = \frac{1.92\times10^{-4}C}{2.00\,\mu\text{F}} = 96.0\,\text{V}.$$

(g) $q'_2 = 2q - q_1 = 7.68\times10^{-4}C.$

(h) $V_2' = V_1' = 96.0\,\text{V}.$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = 0$,

(j) $V_1 = 0$,

(k) $q_2 = 0$,

(l) and $V_2 = V_1 = 0$.

71. We use $U = \frac{1}{2}CV^2$. As V is increased by ΔV, the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus, $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\% .$$

72. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

73. We may think of this as two capacitors in series C_1 and C_2, the former with the $\kappa_1 = 3.00$ material and the latter with the $\kappa_2 = 4.00$ material. Upon using Eq. 25-9, Eq. 25-27 and then reducing C_1 and C_2 to an equivalent capacitance (connected directly to the battery) with Eq. 25-20, we obtain

$$C_{eq} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d} = 1.52 \times 10^{-10}\ \text{F} \quad .$$

Therefore, $q = C_{eq}V = 1.06 \times 10^{-9}$ C.

74. (a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d-b) = 0.708$ pF.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} = \frac{5.00}{5.00-2.00} = 1.67.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\varepsilon_0 A} = -5.44 \text{ J}.$$

(d) Since $W < 0$ the slab is sucked in.

75. (a) $C' = \varepsilon_0 A/(d - b) = 0.708$ pF, the same as part (a) in Problem 25-74.

(b) The ratio of the stored energy is now

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)} = \frac{d-b}{d} = \frac{5.00-2.00}{5.00} = 0.600.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C' - C)V^2 = \frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b} - \frac{1}{d}\right)V^2 = \frac{\varepsilon_0 AbV^2}{2d(d-b)} = 1.02\times10^{-9}\,\text{J}.$$

(d) In Problem 25-74 where the capacitor is disconnected from the battery and the slab is sucked in, F is certainly given by $-dU/dx$. However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

76. (a) Put five such capacitors in series. Then, the equivalent capacitance is 2.0 μF/5 = 0.40 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now C_{eq} = 3(0.40 μF) = 1.2 μF. With each capacitor taking a 200-V potential difference the equivalent capacitor can withstand 1000 V.

77. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0\text{ V} .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2V_2 = (20\,\mu\text{F})(2\text{ V}) = 60\,\mu\text{C} ,$$

which means a total of 90 μC of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of 90 μC of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45$ μC. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3\text{ V} .$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

78. One way to approach this is to note that – since they are identical – the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is n times that value, and the problem requires the total be equal to 25×10^{-6} J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6}$$

leads to $n = 4$.

Chapter 26

1. (a) The charge that passes through any cross section is the product of the current and time. Since $t = 4.0$ min = (4.0 min)(60 s/min) = 240 s,

$$q = it = (5.0\ \text{A})(240\ \text{s}) = 1.2\times 10^3\ \text{C}.$$

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200\ \text{C})/(1.60\times 10^{-19}\ \text{C}) = 7.5\times 10^{21}.$$

2. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where r is the radius of the sphere. This means $\Delta q = 4\pi\varepsilon_0 r\,\Delta V$. Now, $\Delta q = (i_{in} - i_{out})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\Delta t = \frac{\Delta q}{i_{in} - i_{out}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{in} - i_{out}} = \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times10^9\text{ F/m})(1.0000020\text{ A} - 1.0000000\text{ A})}$$

$$= 5.6\times10^{-3}\text{ s}.$$

3. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6}\ \text{A}}{(30\ \text{m/s})(50 \times 10^{-2}\ \text{m})} = 6.7 \times 10^{-6}\ \text{C/m}^2 .$$

4. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4\left(1.2\times10^{-10}\ \text{A}\right)}{\pi\left(2.5\times10^{-3}\ \text{m}\right)^2} = 2.4\times10^{-5}\ \text{A/m}^2.$$

(b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4\times10^{-5}\ \text{A/m}^2}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)} = 1.8\times10^{-15}\ \text{m/s}.$$

5. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is $J = i / A = i / \pi r^2$, so

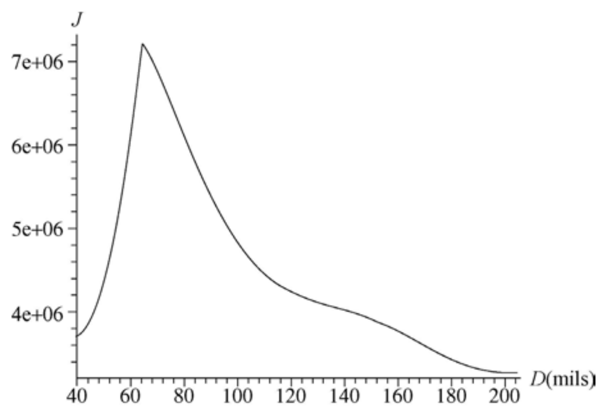
$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50 \text{ A}}{\pi\left(440 \times 10^4 \text{ A/m}^2\right)}} = 1.9 \times 10^{-4} \text{ m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4} \text{ m}) = 3.8 \times 10^{-4}$ m.

6. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with D = 64 mil = 0.0016 m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6$ A/m^2. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



7. (a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\ \text{m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C},$$

and the drift speed is 1.0×10^5 m/s. Thus,

$$J = \left(2 \times 10^{14} / \text{m}\right)\left(3.2 \times 10^{-19}\ \text{C}\right)\left(1.0 \times 10^5\ \text{m/s}\right) = 6.4\ \text{A/m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

8. (a) Circular area depends, of course, on r^2, so the horizontal axis of the graph in Fig. 26-24(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of π. The fact that the current increases linearly in the graph means that $i/A = J$ = constant. Thus, the answer is "yes, the current density is uniform."

(b) We find $i/(\pi r^2) = (0.005\text{ A})/(\pi \times 4 \times 10^{-6}\text{ m}^2) = 398 \approx 4.0 \times 10^2\text{ A/m}^2$.

9. We use $v_d = J/ne = i/Ane$. Thus,

$$t = \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} = \frac{(0.85\,\mathrm{m})\left(0.21\times10^{-14}\,\mathrm{m}^2\right)\left(8.47\times10^{28}/\mathrm{m}^3\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)}{300\,\mathrm{A}}$$
$$= 8.1\times10^2\,\mathrm{s} = 13\,\mathrm{min}.$$

10. (a) Since 1 cm^3 = 10^{-6} m^3, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6}\ \text{m}^3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(470\times10^{3}\ \text{m/s}\right) = 6.54\times10^{-7}\ \text{A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi\left(6.37\times10^{6}\ \text{m}\right)^2\left(6.54\times10^{-7}\ \text{A/m}^2\right) = 8.34\times10^{7}\ \text{A}.$$

11. We note that the radial width Δr = 10 μm is small enough (compared to r = 1.20 mm) that we can make the approximation

$$\int Br2\pi r dr \approx Br2\pi r\Delta r$$

Thus, the enclosed current is $2\pi Br^2\Delta r$ = 18.1 μA. Performing the integral gives the same answer.

12. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 26-4,

$$i = \int |\vec{J}|\, dA = \int_{9R/10}^{R} (kr^2) 2\pi r dr = \frac{1}{2} k\pi \left(R^4 - 0.656 R^4 \right)$$

where k = 3.0 × 10^8 and SI units understood. Therefore, if R = 0.00200 m, we obtain $i = 2.59 \times 10^{-3}$ A .

13. (a) The current resulting from this non-uniform current density is

$$i=\int_{\text{cylinder}} J_a\, dA=\frac{J_0}{R}\int_0^R r\cdot 2\pi r dr=\frac{2}{3}\pi R^2 J_0=\frac{2}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2)$$
$$=1.33\ \text{A}$$

(b) In this case,

$$i=\int_{\text{cylinder}} J_b\, dA=\int_0^R J_0\left(1-\frac{r}{R}\right)2\pi r dr=\frac{1}{3}\pi R^2 J_0=\frac{1}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2)$$
$$=0.666\ \text{A}.$$

(c) The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, J_a has its maximum value near the surface of the wire.

14. We use $R/L = \rho/A = 0.150\ \Omega/\text{km}$.

(a) For copper $J = i/A = (60.0\ \text{A})(0.150\ \Omega/\text{km})/(1.69 \times 10^{-8}\ \Omega{\cdot}\text{m}) = 5.32 \times 10^{5}\ \text{A/m}^2$.

(b) We denote the mass densities as ρ_m. For copper,

$$(m/L)_c = (\rho_m A)_c = (8960\ \text{kg/m}^3)\ (1.69 \times 10^{-8}\ \Omega{\cdot}\ \text{m})/(0.150\ \Omega/\text{km}) = 1.01\ \text{kg/m}.$$

(c) For aluminum $J = (60.0\ \text{A})(0.150\ \Omega/\text{km})/(2.75 \times 10^{-8}\ \Omega{\cdot}\text{m}) = 3.27 \times 10^{5}\ \text{A/m}^2$.

(d) The mass density of aluminum is

$$(m/L)_a = (\rho_m A)_a = (2700\ \text{kg/m}^3)(2.75 \times 10^{-8}\ \Omega{\cdot}\text{m})/(0.150\ \Omega/\text{km}) = 0.495\ \text{kg/m}.$$

15. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\text{ m})(4.0\text{ A})}{(2.0\text{ V})(1.0\times10^{-6}\text{ m}^2)} = 2.0\times10^{6}\ /\Omega\cdot\text{m}.$$

16. (a) $i = V/R = 23.0\ \text{V}/15.0 \times 10^{-3}\ \Omega = 1.53 \times 10^{3}\ \text{A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4\left(1.53\times10^{-3}\ \text{A}\right)}{\pi\left(6.00\times10^{-3}\ \text{m}\right)^2} = 5.41\times10^{7}\ \text{A}/\text{m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0\times10^{-3}\,\Omega)\pi(6.00\times10^{-3}\ \text{m})^2}{4(4.00\ \text{m})} = 10.6\times10^{-8}\ \Omega\cdot\text{m}.$$

(d) The material is platinum.

17. The resistance of the wire is given by $R = \rho L / A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area. In this case,

$$A = \pi r^2 = \pi\left(0.50 \times 10^{-3}\ \text{m}\right)^2 = 7.85 \times 10^{-7}\ \text{m}^2.$$

Thus,

$$\rho = \frac{RA}{L} = \frac{\left(50 \times 10^{-3}\,\Omega\right)\left(7.85 \times 10^{-7}\,\text{m}^2\right)}{2.0\,\text{m}} = 2.0 \times 10^{-8}\,\Omega \cdot \text{m}.$$

18. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R.$$

19. The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12\text{ m}) = 188.5\text{ m}.$$

If r_w is the radius of the wire itself, then its cross-sectional area is

$$A = \pi r^2 w = \pi(0.65 \times 10^{-3}\text{ m})^2 = 1.33 \times 10^{-6}\text{ m}^2.$$

According to Table 26-1, the resistivity of copper is $\rho = 1.69 \times 10^{-8}\,\Omega\cdot\text{m}$. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(1.69 \times 10^{-8}\,\Omega\cdot\text{m}\right)\left(188.5\text{ m}\right)}{1.33 \times 10^{-6}\text{ m}^2} = 2.4\,\Omega.$$

20. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\ \text{A})(2000\ \Omega) = 100\ \text{V}$.

21. Since the mass density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3 L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0,$$

where R_0 is the original resistance. Thus, $R = 9(6.0\ \Omega) = 54\ \Omega$.

22. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, J_1: J_2: J_3 are in the ratio 2.5/4/1.5 (see Fig. 26-25). Now the currents in the rods must be the same (they are "in series") so

$$J_1 A_1 = J_3 A_3 , \quad J_2 A_2 = J_3 A_3 .$$

Since $A = \pi r^2$ this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2 , \quad 2.5r_1^2 = 1.5r_3^2 .$$

Thus, with r_3 = 2 mm, the latter relation leads to r_1 = 1.55 mm.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to r_2 = 1.22 mm.

23. The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)}.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0.$$

24. The cross-sectional area is $A = \pi r^2 = \pi(0.002 \text{ m})^2$. The resistivity from Table 26-1 is $\rho = 1.69 \times 10^{-8}\ \Omega\cdot\text{m}$. Thus, with $L = 3$ m, Ohm's Law leads to $V = iR = i\rho L/A$, or

$$12 \times 10^{-6}\ \text{V} = i\,(1.69 \times 10^{-8}\ \Omega\cdot\text{m})(3.0\ \text{m})/\ \pi(0.002\ \text{m})^2$$

which yields $i = 0.00297$ A or roughly 3.0 mA.

25. The resistance at operating temperature T is $R = V/i = 2.9 \text{ V}/0.30 \text{ A} = 9.67\ \Omega$. Thus, from $R - R_0 = R_0\alpha\,(T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \left(\frac{1}{4.5\times10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) = 1.8\times10^{3}\ °\text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

26. Let $r = 2.00$ mm be the radius of the kite string and $t = 0.50$ mm be the thickness of the water layer. The cross-sectional area of the layer of water is

$$A = \pi\left[(r+t)^2 - r^2\right] = \pi[(2.50\times10^{-3}\text{ m})^2 - (2.00\times10^{-3}\text{ m})^2] = 7.07\times10^{-6}\text{ m}^2.$$

Using Eq. 26-16, the resistance of the wet string is

$$R = \frac{\rho L}{A} = \frac{(150\,\Omega\cdot\text{m})(800\text{ m})}{7.07\times10^{-6}\text{ m}^2} = 1.698\times10^{10}\,\Omega.$$

The current through the water layer is

$$i = \frac{V}{R} = \frac{1.60\times10^{8}\,\text{V}}{1.698\times10^{10}\,\Omega} = 9.42\times10^{-3}\,\text{A}.$$

27. First we find the resistance of the copper wire to be

$$R = \frac{\rho L}{A} = \frac{\left(1.69\times10^{-8}\ \Omega\cdot\text{m}\right)\left(0.020\ \text{m}\right)}{\pi(2.0\times10^{-3}\ \text{m})^2} = 2.69\times10^{-5}\ \Omega\,.$$

With potential difference $V = 3.00\ \text{nV}$, the current flowing through the wire is

$$i = \frac{V}{R} = \frac{3.00\times10^{-9}\,\text{V}}{2.69\times10^{-5}\,\Omega} = 1.115\times10^{-4}\,\text{A}\,.$$

Therefore, in 3.00 ms, the amount of charge drifting through a cross section is

$$\Delta Q = i\Delta t = (1.115\times10^{-4}\,\text{A})(3.00\times10^{-3}\ \text{s}) = 3.35\times10^{-7}\text{C}\ .$$

28. The absolute values of the slopes (for the straight-line segments shown in the graph of Fig. 26-27(b)) are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m}) \ .$$

We note that the current densities are the same since the values of i and A are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2\,(3.00 \times 10^7 (\Omega\cdot\text{m})^{-1}) = 6.00 \times 10^7 \ (\Omega\cdot\text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (3.00 \times 10^7 (\Omega\cdot\text{m})^{-1})/4 = 7.50 \times 10^6 \ (\Omega\cdot\text{m})^{-1}$.

29. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115\ \text{V}}{(10\ \text{m})\left(1.4\times 10^{4}\ \text{A/m}^2\right)} = 8.2\times 10^{-4}\,\Omega\cdot\text{m}.$$

30. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) The magnitude of the current density is

$$J = \sigma E = (2.70 \times 10^{-14} / \Omega\cdot\text{m})\ (120\ \text{V/m}) = 3.24 \times 10^{-12}\ \text{A/m}^2.$$

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{\left(2.70\times10^{-14}/\Omega\cdot\text{m}\right)\left(120\ \text{V/m}\right)}{\left[\left(620+550\right)/\text{cm}^3\right]\left(1.60\times10^{-19}\ \text{C}\right)} = 1.73\ \text{cm/s}.$$

31. (a) The current in the block is $i = V/R = 35.8 \text{ V}/935\ \Omega = 3.83 \times 10^{-2}$ A.

(b) The magnitude of current density is

$$J = i/A = (3.83 \times 10^{-2} \text{ A})/(3.50 \times 10^{-4} \text{ m}^2) = 109 \text{ A/m}^2.$$

(c) $v_d = J/ne = (109 \text{ A/m}^2)/[(5.33 \times 10^{22}/\text{m}^3)\ (1.60 \times 10^{-19} \text{ C})] = 1.28 \times 10^{-2}$ m/s.

(d) $E = V/L = 35.8 \text{ V}/0.158 \text{ m} = 227$ V/m.

32. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be

$$R = (1.00\ \Omega)\ (25\ \text{ft}/1000\ \text{ft})(4)^2 = 0.40\ \Omega.$$

33. (a) The current in each strand is $i = 0.750\ \text{A}/125 = 6.00 \times 10^{-3}$ A.

(b) The potential difference is $V = iR = (6.00 \times 10^{-3}\ \text{A})\,(2.65 \times 10^{-6}\ \Omega) = 1.59 \times 10^{-8}$ V.

(c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\ \Omega/125 = 2.12 \times 10^{-8}\ \Omega$.

34. We follow the procedure used in Sample Problem 26-5.

Since the current spreads uniformly over the hemisphere, the current density at any given radius r from the striking point is $J = I/2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance r is

$$E = \rho_w J = \frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30\ \Omega\cdot\text{m}$ is the resistivity of water. The potential difference between a point at radial distance D and a point at $D + \Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right) = -\frac{\rho_w I}{2\pi}\frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R}\frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0\ \Omega\cdot\text{m})(7.80\times10^4\ \text{A})}{2\pi(4.00\times10^3\ \Omega)}\frac{0.70\ \text{m}}{(35.0\ \text{m})(35.0\ \text{m}+0.70\ \text{m})} = 5.22\times10^{-2}\ \text{A}.$$

35. (a) The current i is shown in Fig. 26-30 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x. The direction of $\vec{J}$ is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 26-11, we then find an expression for the electric field at each value of x, and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x, so (with c_1 and c_2 to be determined later) we may write

$$r = c_1 + c_2 x.$$

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b-a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b-a}{L}\right)x.$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}x\right)^{-2}.$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx = \frac{i\rho}{\pi}\frac{L}{b-a}\left(a + \frac{b-a}{L}x\right)^{-1}\Bigg|_0^L \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab} = \frac{i\rho L}{\pi ab}. \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} = \frac{(731\ \Omega\cdot\text{m})(1.94\times10^{-2}\ \text{m})}{\pi(2.00\times10^{-3}\ \text{m})(2.30\times10^{-3}\ \text{m})} = 9.81\times10^{5}\ \Omega$$

Note that if $b = a$, then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

36. The number density of conduction electrons in copper is $n = 8.49 \times 10^{28}$ /m^3. The electric field in section 2 is (10.0 μV)/(2.00 m) = 5.00 μV/m. Since ρ = 1.69 × 10^{-8} Ω·m for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude J_2 = (5.00 μV/m)/(1.69 × 10^{-8} Ω·m) = 296 A/m^2 in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1(4\pi R^2) = J_2(\pi R^2)$$

(see Eq. 26-5). This leads to J_1 = 74 A/m^2. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 5.44 \times 10^{-9} \text{ m/s}$$

37. From Eq. 26-25, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem 26-6, which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

38. Since $P = iV$, the charge is

$$q = it = Pt/V = (7.0\text{ W})\,(5.0\text{ h})\,(3600\text{ s/h})/9.0\text{ V} = 1.4 \times 10^{4}\text{ C}.$$

39. (a) Electrical energy is converted to heat at a rate given by $P = V^2 / R,$ where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\ \text{V})^2}{14\ \Omega} = 1.0 \times 10^3\ \text{W} = 1.0\ \text{kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW} \cdot \text{h}) = \text{US\$0.25}$.

40. (a) Referring to Fig. 26-32, the electric field would point down (towards the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction-electrons would be "drifting" upward in the strip.

(b) Eq. 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19}$ C) 1.9×10^{-18} J for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don't (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or 1.9×10^{-18} J.

41. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\ \text{V}}{3.00\ \text{V}} \right)^2 (0.540\ \text{W}) = 0.135\ \text{W}.$$

42. The resistance is $R = P/i^2 = (100 \text{ W})/(3.00 \text{ A})^2 = 11.1\ \Omega$.

43. (a) The power dissipated, the current in the heater, and the potential difference across the heater are related by $P = iV$. Therefore,

$$i = \frac{P}{V} = \frac{1250\text{ W}}{115\text{ V}} = 10.9\text{ A}.$$

(b) Ohm's law states $V = iR$, where R is the resistance of the heater. Thus,

$$R = \frac{V}{i} = \frac{115\text{ V}}{10.9\text{ A}} = 10.6\ \Omega.$$

(c) The thermal energy E generated by the heater in time $t = 1.0$ h = 3600 s is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.50\times10^{6}\text{ J}.$$

44. The slope of the graph is $P = 5.0 \times 10^{-4}$ W. Using this in the $P = V^2/R$ relation leads to $V = 0.10$ Vs.

45. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0\,\text{A})(120\,\text{V}) = 1.20\,\text{kW}.$$

Dividing this into the 180 kJ necessary to cook the three hot-dogs leads to the result $t = 150$ s.

46. The mass of the water over the length is

$$m = \rho AL = (1000 \text{ kg/m}^3)(15\times10^{-5} \text{ m}^2)(0.12 \text{ m}) = 0.018 \text{ kg},$$

and the energy required to vaporize the water is

$$Q = Lm = (2256 \text{ kJ / kg})(0.018 \text{ kg}) = 4.06\times10^4 \text{ J}.$$

The thermal energy is supplied by Joule heating of the resistor:

$$Q = P\Delta t = I^2 R\Delta t.$$

Since the resistance over the length of water is

$$R = \frac{\rho_w L}{A} = \frac{(150\ \Omega\cdot\text{m})(0.120 \text{ m})}{15\times10^{-5} \text{ m}^2} = 1.2\times10^5\ \Omega,$$

the average current required to vaporize water is

$$I = \sqrt{\frac{Q}{R\Delta t}} = \sqrt{\frac{4.06\times10^4 \text{ J}}{(1.2\times10^5\ \Omega)(2.0\times10^{-3} \text{ s})}} = 13.0 \text{ A}.$$

47. (a) From $P = V^2/R$ we find $R = V^2/P = (120\ \text{V})^2/500\ \text{W} = 28.8\ \Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500\ \text{W}}{(1.60\times 10^{-19}\ \text{C})(120\ \text{V})} = 2.60\times 10^{19}\ /\ \text{s}.$$

48. The slopes of the lines yield P_1 = 8 mW and P_2 = 4 mW. Their sum (by energy conservation) must be equal to that supplied by the battery: P_{batt} = (8 + 4) mW = 12 mW.

49. (a) From $P = V^2/R = AV^2 / \rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60\times10^{-6}\ \text{m}^2)(75.0\ \text{V})^2}{(5.00\times10^{-7}\ \Omega\cdot\text{m})(500\ \text{W})} = 5.85\ \text{m}.$$

(b) Since $L \propto V^2$ the new length should be

$$L' = L\left(\frac{V'}{V}\right)^2 = (5.85\ \text{m})\left(\frac{100\ \text{V}}{75.0\ \text{V}}\right)^2 = 10.4\ \text{m}.$$

50. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m^4 and $R = 0.00300$ m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 210$ W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5$ J.

51. (a) Assuming a 31-day month, the monthly cost is

$$(100\text{ W})(24\text{ h/day})(31\text{day/month})\,(6\text{ cents/kW}\cdot\text{h}) = 446\text{ cents} = \text{US\$4.46}.$$

(b) $R = V^2/P = (120\text{ V})^2/100\text{ W} = 144\ \Omega$.

(c) $i = P/V = 100\text{ W}/120\text{ V} = 0.833\text{ A}$.

52. (a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho|\vec{J}| = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(\frac{2.00\,\text{A}}{2.00\times10^{-6}\,\text{m}^2}\right) = 1.69\times10^{-2}\,\text{V/m}.$$

(b) Using L = 4.0 m, the resistance is found from Eq. 26-16: $R = \rho L/A$ = 0.0338 Ω. The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00\text{ A})^2(0.0338\ \Omega) = 0.135\text{ W}.$$

Assuming a steady rate, the thermal energy generated in 30 minutes is (0.135 J/s)(30 × 60s) = 2.43×10^2 J.

53. (a) We use Eq. 26-16 to compute the resistances:

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0\times10^{-6}\ \Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00050\text{ m})^2} = 2.55\ \Omega.$$

The voltage follows from Ohm's law: $|V_1 - V_2| = V_C = iR_C = (2.0\text{ A})(2.55\ \Omega) = 5.1\text{V}$.

(b) Similarly,

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0\times10^{-6}\ \Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00025\text{ m})^2} = 5.09\ \Omega$$

and $|V_2 - V_3| = V_D = iR_D = (2.0\text{ A})(5.09\ \Omega) = 10.2\text{ V} \approx 10\text{ V}$.

(c) The power is calculated from Eq. 26-27: $P_C = i^2 R_C = 10\text{ W}$.

(d) Similarly, $P_D = i^2 R_D = 20\text{ W}$.

54. From $P = V^2 / R$, we have $R = (5.0 \text{ V})^2/(200 \text{ W}) = 0.125\ \Omega$. To meet the conditions of the problem statement, we must therefore set

$$\int_0^L 5.00x\ dx = 0.125\ \Omega$$

Thus,

$$\frac{5}{2}L^2 = 0.125 \quad \Rightarrow \quad L = 0.224 \text{ m}.$$

55. (a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\Delta t}{2e} = \frac{(0.25\times10^{-6}\,\mathrm{A})(3.0\,\mathrm{s})}{2(1.6\times10^{-19}\,\mathrm{C})} = 2.3\times10^{12}\,.$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is,

$$i = 2eN/t = 2eNv/L.$$

Thus $N = iL/2ev$. To find the particle speed, we note the kinetic energy of a particle is

$$K = 20\,\mathrm{MeV} = (20\times10^{6}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV}) = 3.2\times10^{-12}\,\mathrm{J}\,.$$

Since $K = \frac{1}{2}mv^2$,then the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67\times10^{-27}\text{ kg}) = 6.68\times10^{-27}$ kg, so

$$v = \sqrt{\frac{2(3.2\times10^{-12}\,\mathrm{J})}{6.68\times10^{-27}\,\mathrm{kg}}} = 3.1\times10^{7}\,\mathrm{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25\times10^{-6})(20\times10^{-2}\ \mathrm{m})}{2(1.60\times10^{-19}\ \mathrm{C})(3.1\times10^{7}\ \mathrm{m/s})} = 5.0\times10^{3}.$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is 20 MeV = 3.2×10^{-12} J. We note, too, that the initial potential energy is $U_i = qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Consequently,

$$K_f = U_i = 2eV \quad\Rightarrow\quad V = \frac{K_f}{2e} = \frac{3.2\times10^{-12}\,\mathrm{J}}{2(1.60\times10^{-19}\,\mathrm{C})} = 1.0\times10^{7}\,\mathrm{V}.$$

56. (a) Current is the transport of charge; here it is being transported "in bulk" due to the volume rate of flow of the powder. From Chapter 14, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho A v$ where ρ is the charge per unit volume. If the cross-section is that of a circle, then $i = \rho\pi R^2 v$.

(b) Recalling that a Coulomb per second is an Ampere, we obtain

$$i = \left(1.1\times10^{-3}\ \mathrm{C/m^3}\right)\pi\left(0.050\ \mathrm{m}\right)^2\left(2.0\ \mathrm{m/s}\right) = 1.7\times10^{-5}\ \mathrm{A}.$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 68 of Chapter 24. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P = \vec{F}\cdot\vec{v}$ makes it clear that $P = 0$ if $\vec{F}\perp\vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 68 of Chapter 24, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 26-26:

$$P = iV = \left(1.7\times10^{-5}\ \mathrm{A}\right)\left(7.8\times10^{4}\ \mathrm{V}\right) = 1.3\ \mathrm{W}.$$

(e) Recalling that a Joule per second is a Watt, we obtain (1.3 W)(0.20 s) = 0.27 J for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

57. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\,\Delta V$. The percentage change is roughly

$$\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%.$$

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

58. (a) The current is

$$i=\frac{V}{R}=\frac{V}{\rho L/A}=\frac{\pi Vd^2}{4\rho L}=\frac{\pi(1.20\ \text{V})[(0.0400\,\text{in.})(2.54\times10^{-2}\text{m/in.})]^2}{4(1.69\times10^{-8}\,\Omega\cdot\text{m})(33.0\text{m})}=1.74\ \text{A}.$$

(b) The magnitude of the current density vector is

$$|\vec{J}|=\frac{i}{A}=\frac{4i}{\pi d^2}=\frac{4(1.74\ \text{A})}{\pi[(0.0400\,\text{in.})(2.54\times10^{-2}\ \text{m/in.})]^2}=2.15\times10^{6}\ \text{A/m}^2.$$

(c) $E = V/L = 1.20\ \text{V}/33.0\ \text{m} = 3.63\times10^{-2}\ \text{V/m}$.

(d) $P = Vi = (1.20\ \text{V})(1.74\ \text{A}) = 2.09\ \text{W}$.

59. Let R_H be the resistance at the higher temperature (800°C) and let R_L be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now

$$R_L = R_H + \alpha R_H \Delta T,$$

where ΔT is the temperature difference $T_L - T_H = -600$ C° $= -600$ K. Thus,

$$P_L = \frac{R_H}{R_H + \alpha R_H \Delta T} P_H = \frac{P_H}{1+\alpha\Delta T} = \frac{500\ \text{W}}{1+(4.0\times10^{-4}/\text{K})(-600\ \text{K})} = 660\ \text{W}.$$

60. We denote the copper rod with subscript c and the aluminum rod with subscript a.

(a) The resistance of the aluminum rod is

$$R = \rho_a \frac{L}{A} = \frac{(2.75\times 10^{-8}\,\Omega\cdot\text{m})(1.3\,\text{m})}{(5.2\times 10^{-3}\,\text{m})^2} = 1.3\times 10^{-3}\,\Omega.$$

(b) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter d of the copper rod:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4(1.69\times 10^{-8}\,\Omega\cdot\text{m})(1.3\text{ m})}{\pi(1.3\times 10^{-3}\,\Omega)}} = 4.6\times 10^{-3}\text{ m}.$$

61. (a) Since

$$\rho = \frac{RA}{L} = \frac{R(\pi d^2/4)}{L} = \frac{(1.09\times10^{-3}\,\Omega)\pi(5.50\times10^{-3}\ \text{m})^2/4}{1.60\ \text{m}} = 1.62\times10^{-8}\ \Omega\cdot\text{m},$$

the material is silver.

(b) The resistance of the round disk is

$$R = \rho\frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62\times10^{-8}\,\Omega\cdot\text{m})(1.00\times10^{-3}\,\text{m})}{\pi(2.00\times10^{-2}\,\text{m})^2} = 5.16\times10^{-8}\,\Omega.$$

62. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L / A}} = \sqrt{\frac{P}{\rho L A}} = \sqrt{\frac{1.0\,\mathrm{W}}{\pi\left(3.5\times10^{-5}\Omega\cdot\mathrm{m}\right)\left(2.0\times10^{-2}\ \mathrm{m}\right)\left(5.0\times10^{-3}\ \mathrm{m}\right)^2}}$$

$$= 1.3\times10^{5}\ \mathrm{A/m^2}.$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi\left(5.0\times10^{-3}\ \mathrm{m}\right)^2\left(1.3\times10^{5}\ \mathrm{A/m^2}\right)} = 9.4\times10^{-2}\ \mathrm{V}.$$

63. We use $P = i^2 R = i^2 \rho L/A$, or $L/A = P/i^2\rho$.

(a) The new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}}.$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875}L_{\text{old}} = 1.37L_{\text{old}} \quad \Rightarrow \quad \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37 .$$

(b) Similarly, we note that $(LA)_{\text{new}} = (LA)_{\text{old}}$, and

$$A_{\text{new}} = \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}} \quad \Rightarrow \quad \frac{A_{\text{new}}}{A_{\text{old}}} = 0.730 .$$

64. The horsepower required is

$$P = \frac{iV}{0.80} = \frac{(10\text{A})(12\ \text{V})}{(0.80)(746\ \text{W/hp})} = 0.20\ \text{hp}.$$

65. We find the current from Eq. 26-26: $i = P/V = 2.00$ A. Then, from Eq. 26-1 (with constant current), we obtain

$$\Delta q = i\Delta t = 2.88 \times 10^4 \text{ C} \quad .$$

66. We find the drift speed from Eq. 26-7:

$$v_d = \frac{|\vec{J}|}{ne} = \frac{2.0\times10^6\ \text{A/m}^2}{(8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\,\text{C})} = 1.47\times10^{-4}\ \text{m/s}\,.$$

At this (average) rate, the time required to travel L = 5.0 m is

$$t = \frac{L}{v_d} = \frac{5.0\ \text{m}}{1.47\times10^{-4}\ \text{m/s}} = 3.4\times10^4\ \text{s}.$$

67. We find the rate of energy consumption from Eq. 26-28:

$$P = \frac{V^2}{R} = \frac{(90\ \text{V})^2}{400\ \Omega} = 20.3\ \text{W}$$

Assuming a steady rate, the energy consumed is $(20.3\ \text{J/s})(2.00 \times 3600\ \text{s}) = 1.46 \times 10^5\ \text{J}$.

68. We use Eq. 26-28:

$$R = \frac{V^2}{P} = \frac{(200\ \mathrm{V})^2}{3000\ \mathrm{W}} = 13.3\ \Omega .$$

69. The rate at which heat is being supplied is $P = iV$ = (5.2 A)(12 V) = 62.4 W. Considered on a one-second time-frame, this means 62.4 J of heat are absorbed the liquid each second. Using Eq. 18-16, we find the heat of transformation to be

$$L = \frac{Q}{m} = \frac{62.4\ \text{J}}{21\times10^{-6}\ \text{kg}} = 3.0\times10^{6}\ \text{J/kg}.$$

70. (a) The current is 4.2×10^{18} e divided by 1 second. Using $e = 1.60 \times 10^{-19}$ C we obtain 0.67 A for the current.

(b) Since the electric field points away from the positive terminal (high potential) and towards the negative terminal (low potential), then the current density vector (by Eq. 26-11) must also point towards the negative terminal.

71. Combining Eq. 26-28 with Eq. 26-16 demonstrates that the power is inversely proportional to the length (when the voltage is held constant, as in this case). Thus, a new length equal to 7/8 of its original value leads to

$$P = \frac{8}{7}(2.0 \text{ kW}) = 2.4 \text{ kW} .$$

72. We use Eq. 26-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20°\,\mathrm{C} + \frac{1}{4.3\times10^{-3}\,/\,\mathrm{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57°\,\mathrm{C}.$$

We are assuming that $\rho/\rho_0 = R/R_0$.

73. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0 \times 10^{-3}\ \text{A})(80 \times 10^{3}\ \text{V}) = 560\ \text{W}.$$

74. (a) The potential difference between the two ends of the caterpillar is

$$V = iR = i\rho\frac{L}{A} = \frac{(12\text{ A})(1.69\times10^{-8}\,\Omega\cdot\text{m})(4.0\times10^{-2}\text{ m})}{\pi(5.2\times10^{-3}\text{ m}/2)^2} = 3.8\times10^{-4}\text{ V}.$$

(b) Since it moves in the direction of the electron drift which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$t = \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} = \frac{\pi(1.0\times10^{-2}\text{ m})(5.2\times10^{-3}\text{ m})^2(8.47\times10^{28}/\text{m}^3)(1.60\times10^{-19}\text{ C})}{4(12\text{ A})}$$

$$= 238\text{ s} = 3\text{ min }58\text{ s}.$$

75. (a) In Eq. 26-17, we let $\rho = 2\rho_0$ where ρ_0 is the resistivity at $T_0 = 20°C$:

$$\rho - \rho_0 = 2\rho_0 - \rho_0 = \rho_0 \alpha (T - T_0),$$

and solve for the temperature T:

$$T = T_0 + \frac{1}{\alpha} = 20° \text{C} + \frac{1}{4.3 \times 10^{-3} / \text{K}} \approx 250° \text{C}.$$

(b) Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. It is worth noting that this agrees well with Fig. 26-10.

76. Since 100 cm = 1 m, then 10^4 cm^2 = 1 m^2. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(3.00 \times 10^{-7}\,\Omega \cdot \text{m}\right)\left(10.0 \times 10^{3}\ \text{m}\right)}{56.0 \times 10^{-4}\ \text{m}^2} = 0.536\ \Omega.$$

Chapter 27

1. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\ \text{V})^2 (2.0\,\text{min})(60\,\text{s/min})}{1.0\,\Omega + 5.0\,\Omega} = 80\ \text{J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\ \text{V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\ \text{min})(60\ \text{s/min}) = 67\ \text{J}.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

2. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\,\mathrm{A\cdot h})(12.0\,\mathrm{V})}{100\,\mathrm{W}} = 14.4\,\mathrm{h}.$$

3. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time Δt = 6.0 min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0 \text{ A})(6.0 \text{ V})\ (6.0 \text{ min})\ (60 \text{ s/min}) = 1.1 \times 10^4 \text{ J}.$$

We note the conversion of time from minutes to seconds.

4. (a) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/2.0 \text{ W} \cdot \text{h})$ ($0.80) = $\$3.2 \times 10^2$.

(b) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/10^3 \text{ W} \cdot \text{h})$ ($0.06) = $0.048 = 4.8 cents.

5. (a) The potential difference is $V = \varepsilon + ir = 12\text{ V} + (50\text{ A})(0.040\ \Omega) = 14\text{ V}$.

(b) $P = i^2r = (50\text{ A})^2(0.040\ \Omega) = 1.0\times10^2\text{ W}$.

(c) $P' = iV = (50\text{ A})(12\text{ V}) = 6.0\times10^2\text{ W}$.

(d) In this case $V = \varepsilon - ir = 12\text{ V} - (50\text{ A})(0.040\ \Omega) = 10\text{ V}$.

(e) $P_r = i^2r = (50\text{ A})^2(0.040\ \Omega) = 1.0\times10^2\text{ W}$.

6. The current in the circuit is

$$i = (150\text{ V} - 50\text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20\text{ A}.$$

So from $V_Q + 150\text{ V} - (2.0\ \Omega)i = V_P$, we get $V_Q = 100\text{ V} + (2.0\ \Omega)(20\text{ A}) - 150\text{ V} = -10\text{ V}$.

7. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0$. We solve for i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\text{ V} - 6.0\text{ V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\text{ A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2 R$.

(b) For R_1, $P_1 = i^2 R_1 = (0.50\text{ A})^2(4.0\ \Omega) = 1.0\text{ W}$,

(c) and for R_2, $P_2 = i^2 R_2 = (0.50\text{ A})^2\ (8.0\ \Omega) = 2.0\text{ W}$.

If i is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\text{ A})(12\text{ V}) = 6.0\text{ W}$

(e) and for ε_2, $P_2 = i\varepsilon_2 = (0.50\text{ A})(6.0\text{ V}) = 3.0\text{ W}$.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

8. (a) The loop rule leads to a voltage-drop across resistor 3 equal to 5.0 V (since the total drop along the upper branch must be 12 V). The current there is consequently $i = (5.0\ \text{V})/(200\ \Omega) = 25$ mA. Then the resistance of resistor 1 must be $(2.0\ \text{V})/i = 80\ \Omega$.

(b) Resistor 2 has the same voltage-drop as resistor 3; its resistance is 200 Ω.

9. (a) Since $R_{eq} < R$, the two resistors ($R = 12.0\ \Omega$ and R_x) must be connected in parallel:

$$R_{eq} = 3.00\,\Omega = \frac{R_x R}{R + R_x} = \frac{R_x\left(12.0\,\Omega\right)}{12.0\,\Omega + R_x}.$$

We solve for R_x: $R_x = R_{eq}R/(R - R_{eq}) = (3.00\ \Omega)(12.0\ \Omega)/(12.0\ \Omega - 3.00\ \Omega) = 4.00\ \Omega$.

(b) As stated above, the resistors must be connected in parallel.

10. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e(12.0\text{V}) = 12.0 \text{ eV}.$$

(b) $P = iV = neV = (3.40 \times 10^{18}/\text{s})(1.60 \times 10^{-19}\text{ C})(12.0\text{ V}) = 6.53\text{ W}.$

11. Since the potential differences across the two paths are the same, $V_1 = V_2$ (V_1 for the left path, and V_2 for the right path), we have

$$i_1 R_1 = i_2 R_2 ,$$

where $i = i_1 + i_2 = 5000\text{ A}$. With $R = \rho L / A$ (see Eq. 26-16), the above equation can be rewritten as

$$i_1 d = i_2 h \quad \Rightarrow \quad i_2 = i_1 (d/h) .$$

With $d/h = 0.400$, we get $i_1 = 3571\text{ A}$ and $i_2 = 1429\text{ A}$. Thus, the current through the person is $i_1 = 3571\text{ A}$, or approximately 3.6 kA.

12. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\text{ V} - 2.0\text{ V}}{1.0 \times 10^{-3}\text{ A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9 \times 10^2\,\Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\text{ A})^2(9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\text{ W}$.

13. (a) If i is the current and ΔV is the potential difference, then the power absorbed is given by $P = i\,\Delta V$. Thus,

$$\Delta V = \frac{P}{i} = \frac{50\ \text{W}}{1.0\ \text{A}} = 50\ \text{V}.$$

Since the energy of the charge decreases, point A is at a higher potential than point B; that is, $V_A - V_B = 50$ V.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \varepsilon$, where ε is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus,

$$\varepsilon = V_A - V_B - iR = 50\ \text{V} - (1.0\ \text{A})(2.0\ \Omega) = 48\ \text{V}.$$

(c) A positive value was obtained for ε, so it is toward the left. The negative terminal is at B.

14. The part of R_0 connected in parallel with R is given by $R_1 = R_0x/L$, where L = 10 cm. The voltage difference across R is then $V_R = \varepsilon R'/R_{eq}$, where $R' = RR_1/(R + R_1)$ and $R_{eq} = R_0(1 - x/L) + R'$. Thus

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L)+RR_1/(R+R_1)}\right)^2 = \frac{100R(\varepsilon x/R_0)^2}{(100\,R/R_0 + 10x - x^2)^2},$$

where x is measured in cm.

15. (a) We denote L = 10 km and α = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\,\Omega\,.$$

16. Line 1 has slope $R_1 = 6.0\ \text{k}\Omega$. Line 2 has slope $R_2 = 4.0\ \text{k}\Omega$. Line 3 has slope $R_3 = 2.0\ \text{k}\Omega$. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2) = 2.4\ \text{k}\Omega$. That in series with R_3 gives an equivalence of $R_{123} = R_{12} + R_3 = 2.4\ \text{k}\Omega + 2.0\ \text{k}\Omega = 4.4\ \text{k}\Omega$. The current through the battery is therefore $i = \varepsilon / R_{123} = (6\ \text{V})/(4.4\ \text{k}\Omega)$ and the voltage drop across R_3 is $(6\ \text{V})(2\ \text{k}\Omega)/(4.4\ \text{k}\Omega) = 2.73\ \text{V}$. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2. Then Ohm's law gives the current through R_2: $(6\ \text{V} - 2.73\ \text{V})/(4\ \text{k}\Omega) = 0.82\ \text{mA}$.

17. (a) The parallel set of three identical $R_2 = 18\ \Omega$ resistors reduce to $R = 6.0\ \Omega$, which is now in series with the $R_1 = 6.0\ \Omega$ resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R = 12\ \Omega$. Thus, the current through R' is $(12\text{V})/R' = 1.0$ A, which is the current through R. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, $i_1 = 0.333$ A.

(b) The direction of i_1 is clearly rightward.

(c) We use Eq. 26-27: $P = i^2R' = (1.0\ \text{A})^2(12\ \Omega) = 12$ W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

18. (a) For each wire, $R_{\text{wire}} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{\text{wire}} = (1.69 \times 10^{-8}\,\Omega\cdot\text{m})(0.200\text{ m})/\pi(0.00100\text{ m})^2 = 0.0011\ \Omega.$$

The total resistive load on the battery is therefore

$$R_{\text{tot}} = 2R_{\text{wire}} + R = 2(0.0011\ \Omega) + 6.00\ \Omega = 6.0022\ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{\text{tot}}} = \frac{12.0\text{ V}}{6.0022\Omega} = 1.9993\text{ A}.$$

The voltage across the $R = 6.00\ \Omega$ resistor is therefore

$$V = iR = (1.9993\text{ A})(6.00\ \Omega) = 11.996\text{ V} \approx 12.0\text{ V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{\text{wire}} = iR_{\text{wire}} = (1.9993\text{ A})(0.0011\ \Omega) = 2.15\text{ mV}.$$

(c) $P = i^2R = (1.9993\text{ A})(6.00\ \Omega)^2 = 23.98\text{ W} \approx 24.0\text{ W}.$

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.

19. Let the emf be V. Then $V = iR = i'(R + R')$, where $i = 5.0$ A, $i' = 4.0$ A and $R' = 2.0\ \Omega$. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\text{ A})(2.0\,\Omega)}{5.0\text{ A} - 4.0\text{ A}} = 8.0\ \Omega.$$

20. (a) Here we denote the battery emf's as V_1 and V_2. The loop rule gives

$$V_2 - ir_2 + V_1 - ir_1 - iR = 0 \quad \Rightarrow \quad i = \frac{V_2 + V_1}{r_1 + r_2 + R} .$$

The terminal voltage of battery 1 is V_{1T} and (see Fig. 27-4(a)) is easily seen to be equal to $V_1 - ir_1$; similarly for battery 2. Thus,

$$V_{1T} = V_1 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R}, \quad V_{2T} = V_2 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R} .$$

The problem tells us that V_1 and V_2 each equal 1.20 V. From the graph in Fig. 27-36(b) we see that $V_{2T} = 0$ and $V_{1T} = 0.40$ V for $R = 0.10\ \Omega$. This supplies us (in view of the above relations for terminal voltages) with simultaneous equations, which, when solved, lead to $r_1 = 0.20\ \Omega$.

(b) The simultaneous solution also gives $r_2 = 0.30\ \Omega$.

21. To be as general as possible, we refer to the individual emf's as ε_1 and ε_2 and wait until the latter steps to equate them ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\varepsilon_1 + \varepsilon_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

(a) The current in the circuit is

$$i = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R},$$

and the requirement of zero terminal voltage leads to $\varepsilon_1 = ir_1$, or

$$R = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{\varepsilon_1} = \frac{(12.0\text{ V})(0.016\,\Omega) - (12.0\text{ V})(0.012\,\Omega)}{12.0\text{ V}} = 0.0040\ \Omega$$

Note that $R = r_1 - r_2$ when we set $\varepsilon_1 = \varepsilon_2$.

(b) As mentioned above, this occurs in battery 1.

22. (a) Let the emf of the solar cell be ε and the output voltage be V. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10\ \text{V} = \varepsilon - (0.10\ \text{V}/500\ \Omega)r$$
$$0.15\ \text{V} = \varepsilon - (0.15\ \text{V}/1000\ \Omega)r.$$

We solve for ε and r.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\,\text{V}}{(1000\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

23. We note that two resistors in parallel, R_1 and R_2, are equivalent to

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \; R_{12} = \frac{R_1 R_2}{R_1 + R_2}.$$

This situation (Figure 27-38) consists of a parallel pair which are then in series with a single $R_3 = 2.50\ \Omega$ resistor. Thus, the situation has an equivalent resistance of

$$R_{eq} = R_3 + R_{12} = 2.50\Omega + \frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega + 4.00\Omega} = 4.50\Omega.$$

24. Let the resistances of the two resistors be R_1 and R_2, with $R_1 < R_2$. From the statements of the problem, we have

$$R_1R_2/(R_1 + R_2) = 3.0\ \Omega \text{ and } R_1 + R_2 = 16\ \Omega.$$

So R_1 and R_2 must be 4.0 Ω and 12 Ω, respectively.

(a) The smaller resistance is $R_1 = 4.0\ \Omega$.

(b) The larger resistance is $R_2 = 12\ \Omega$.

25. The potential difference across each resistor is V = 25.0 V. Since the resistors are identical, the current in each one is $i = V/R = (25.0\ \text{V})/(18.0\ \Omega) = 1.39$ A. The total current through the battery is then $i_{\text{total}} = 4(1.39\ \text{A}) = 5.56$ A. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus

$$i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\ \text{V})/(18.0\ \Omega) = 5.56\ \text{A}.$$

26. (a) R_{eq} $(FH) = (10.0\ \Omega)(10.0\ \Omega)(5.00\ \Omega)/[(10.0\ \Omega)(10.0\ \Omega) + 2(10.0\ \Omega)(5.00\ \Omega)] = 2.50\ \Omega$.

(b) R_{eq} $(FG) = (5.00\ \Omega)\ R/(R + 5.00\ \Omega)$, where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} $(FG) = (5.00\ \Omega)(8.33\ \Omega)/(5.00\ \Omega + 8.33\ \Omega) = 3.13\ \Omega$.

27. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0\,.$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0\ \text{V}}{100\,\Omega} = 0.050\ \text{A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0\,.$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0\ \text{V} - 5.0\ \text{V} - 4.0\ \text{V}}{50\,\Omega} = -0.060\ \text{A}\,,$$

or $|\,i_2\,| = 0.060\ \text{A}.$ The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0\ \text{V} + 5.0\ \text{V} = 9.0\ \text{V}.$$

28. The currents i_1, i_2 and i_3 are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3)-\varepsilon_2 R_3}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{ V})(10\,\Omega+5.0\,\Omega)-(1.0\text{ V})(5.0\,\Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\,\Omega)} = 0.275\text{ A}\ ,$$

$$i_2 = \frac{\varepsilon_1 R_3-\varepsilon_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{ V})(5.0\,\Omega)-(1.0\text{ V})(10\,\Omega+5.0\,\Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\,\Omega)} = 0.025\text{ A}\ ,$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}\ .$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\text{ A})\,(10\ \Omega) = +0.25\text{ V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -\,(-\,0.250\text{ A})\,(5.0\ \Omega) - 1.0\text{ V} = +0.25\text{ V}.$$

29. Let r be the resistance of each of the narrow wires. Since they are in parallel the resistance R of the composite is given by

$$\frac{1}{R} = \frac{9}{r},$$

or $R = r/9$. Now $r = 4\rho\ell/\pi d^2$ and $R = 4\rho\ell/\pi D^2$, where ρ is the resistivity of copper. $A = \pi d^2/4$ was used for the cross-sectional area of a single wire, and a similar expression was used for the cross-sectional area of the thick wire. Since the single thick wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2} \Rightarrow D = 3d.$$

30. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of “voltage going through” a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm’s law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery which means more current is in R_3, implying its voltage-drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A which implies the voltage-drop across R_3 is 8.0 V. The loop rule then tells us that voltage-drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

31. First, we note V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40\text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8\text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = 16.8\text{ V}/(16.0\ \Omega) = 1.05\text{ A}$.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.05\text{ A} + 1.40\text{ A} = 2.45\text{ A},$$

so its voltage is $V_2 = (2.00\ \Omega)(2.45\text{ A}) = 4.90\text{ V}$.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 21.7\text{ V}$ (implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 10.85\text{ A}$).

The junction rule now gives the current in R_1 as $i_1 = i_2 + i_3 = 2.45\text{ A} + 10.85\text{ A} = 13.3\text{ A}$, implying that the voltage across it is $V_1 = (13.3\text{ A})(2.00\ \Omega) = 26.6\text{ V}$. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6\text{ V} + 21.7\text{ V} = 48.3\text{ V}.$$

32. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$10.0\text{ V} - i_1R_1 - (i_1 + i_2)\,R_3 = 0$$

$$5.00\text{ V} - i_2R_2 - (i_1 + i_2)\,R_3 = 0.$$

(a) Solving, we find $i_2 = 0$, and

(b) $i_3 = i_1 + i_2 = 1.25$ A (downward, as was assumed in writing the equations as we did).

33. (a) We reduce the parallel pair of identical 2.0 Ω resistors (on the right side) to $R' =$ 1.0 Ω, and we reduce the series pair of identical 2.0 Ω resistors (on the upper left side) to $R'' =$ 4.0 Ω. With R denoting the 2.0 Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series which are equivalent to

$$R + R' + R'' = 7.0\ \Omega$$

across which the voltage is 7.0 V (by the loop rule, this is 12 V – 5.0 V), implying that the current is 1.0 A (clockwise). Thus, the voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is 12 – 1 = 11 V. Noting the orientation of the battery, we conclude $V_1 = -11\ \text{V}$.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is 5.0 + 4.0 = 9.0 V. Noting the orientation of the battery, we conclude V_2 = –9.0 V. This can be verified by considering the voltage across R and the value we obtained for V_1.

34. (a) The voltage across $R_3 = 6.0\ \Omega$ is $V_3 = iR_3 = (6.0\ \text{A})(6.0\ \Omega) = 36\ \text{V}$. Now, the voltage across $R_1 = 2.0\ \Omega$ is

$$(V_A - V_B) - V_3 = 78 - 36 = 42\ \text{V},$$

which implies the current is $i_1 = (42\ \text{V})/(2.0\ \Omega) = 21\ \text{A}$. By the junction rule, then, the current in $R_2 = 4.0\ \Omega$ is

$$i_2 = i_1 - i = 21\ \text{A} - 6.0\ \text{A} = 15\ \text{A}.$$

The total power dissipated by the resistors is (using Eq. 26-27)

$$i_1^2(2.0\ \Omega) + i_2^2(4.0\ \Omega) + i^2(6.0\ \Omega) = 1998\ \text{W} \approx 2.0\ \text{kW} \ .$$

By contrast, the power supplied (externally) to this section is $P_A = i_A\,(V_A - V_B)$ where $i_A = i_1 = 21$ A. Thus, $P_A = 1638$ W. Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is $(1998 - 1638)\text{W} = 3.6\times10^2$ W.

35. The voltage difference across R_3 is $V_3 = \varepsilon R' /(R' + 2.00\ \Omega)$, where

$$R' = (5.00\ \Omega R)/(5.00\ \Omega + R_3).$$

Thus,

$$P_3 = \frac{V_3^2}{R_3} = \frac{1}{R_3}\left(\frac{\varepsilon R'}{R' + 2.00\ \Omega}\right)^2 = \frac{1}{R_3}\left(\frac{\varepsilon}{1 + 2.00\ \Omega / R'}\right)^2 = \frac{\varepsilon^2}{R_3}\left[1 + \frac{(2.00\ \Omega)(5.00\ \Omega + R)}{(5.00\ \Omega) R_3}\right]^{-2}$$

$$\equiv \frac{\varepsilon^2}{f(R_3)}$$

where we use the equivalence symbol $\equiv$ to define the expression $f(R_3)$. To maximize P_3 we need to minimize the expression $f(R_3)$. We set

$$\frac{df(R_3)}{dR_3} = -\frac{4.00\ \Omega^2}{R_3^{\ 2}} + \frac{49}{25} = 0$$

to obtain $R_3 = \sqrt{(4.00\ \Omega^2)(25)/49} = 1.43\ \Omega.$

36. (a) For typing convenience, we denote the emf of battery 2 as V_2 and the emf of battery 1 as V_1. The loop rule (examining the left-hand loop) gives $V_2 + i_1 R_1 - V_1 = 0$. Since V_1 is held constant while V_2 and i_1 vary, we see that this expression (for large enough V_2) will result in a negative value for i_1 – so the downward sloping line (the line that is dashed in Fig. 27-47(b)) must represent i_1. It appears to be zero when $V_2 = 6$ V. With $i_1 = 0$, our loop rule gives $V_1 = V_2$ which implies that $V_1 = 6.0$ V.

(b) At $V_2 = 2$ V (in the graph) it appears that $i_1 = 0.2$ A. Now our loop rule equation (with the conclusion about V_1 found in part (a)) gives $R_1 = 20\ \Omega$.

(c) Looking at the point where the upward-sloping i_2 line crosses the axis (at $V_2 = 4$ V), we note that $i_1 = 0.1$ A there and that the loop rule around the right-hand loop should give

$$V_1 - i_1 R_1 = i_1 R_2$$

when $i_1 = 0.1$ A and $i_2 = 0$. This leads directly to $R_2 = 40\ \Omega$.

37. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\,\text{V} - 2.0\,\text{V}}{4(1.0\,\Omega) + 2.0\,\Omega} = 0.33\,\text{A}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\text{ A})(2.0\ \Omega) + 4.0\text{ V} = 3.3\text{ V}$.

38. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$
$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0.$$

Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) The direction is downward. See the results in part (a).

(c) $i_2 = 0.0109$ A . See the results in part (a).

(d) The direction is rightward. See the results in part (a).

(e) $i_3 = 0.0273$ A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +3.82 V.

39. (a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2 R_2 - i_1 R_1 = 0$$
$$\varepsilon - 2 i_1 R_1 - (i_1 - i_2) R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A, $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore, $V_A - V_B = i_1 R_1 = 5.25$ V.

(b) It follows also that $V_B - V_C = i_3 R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1 R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2 R_2 = 6.75$ V.

40. (a) Resistors R_2, R_3 and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega) + (50.0\Omega)(75.0\Omega) + (50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\ \text{V}/(118.8\ \Omega) = 5.05 \times 10^{-2}\ \text{A}$.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0\text{V} - (5.05 \times 10^{-2}\ \text{A})(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}\ \text{A}$.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\ \text{A})(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}\ \text{A}$.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\ \text{A} - 2(1.90 \times 10^{-2}\ \text{A}) = 1.25 \times 10^{-2}\ \text{A}$.

41. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\varepsilon - ir - 2iR = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{r + 2R}.$$

The power dissipated in R is

$$P = (2i)^2 R = \frac{4\varepsilon^2 R}{(r + 2R)^2}.$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\varepsilon^2}{(r+2R)^3} - \frac{16\varepsilon^2 R}{(r+2R)^3} = \frac{4\varepsilon^2 (r - 2R)}{(r+2R)^3}.$$

The derivative vanishes (and P is a maximum) if $R = r/2$. With $r = 0.300\ \Omega$, we have $R = 0.150\ \Omega$.

(b) We substitute $R = r/2$ into $P = 4\varepsilon^2 R/(r + 2R)^2$ to obtain

$$P_{\max} = \frac{4\varepsilon^2 (r/2)}{[r + 2(r/2)]^2} = \frac{\varepsilon^2}{2r} = \frac{(12.0\ \text{V})^2}{2(0.300\ \Omega)} = 240\ \text{W}.$$

42. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\varepsilon = ir + (2i)R$ with $r = 0.200\ \Omega$ and $R = 2.00r$, we get

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.400\Omega)} = 24.0\text{ A}.$$

(b) When connected in series $2\varepsilon - i_R r - i_R r - i_R R = 0$, or $i_R = 2\varepsilon/(2r + R)$. The result is

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.400\Omega} = 30.0\text{ A}.$$

(c) In series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.100\Omega)} = 60.0\text{ A}.$$

(e) For series connection, we have

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.100\Omega} = 48.0\text{ A}.$$

(f) In parallel arrangement, since $R < r$.

43. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is upward. The junction rule produces

$$i_1 + i_2 + i_3 = 0 .$$

The loop rule applied to the left-hand loop produces

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

and applied to the right-hand loop produces

$$\varepsilon_2 - i_2 R_2 + i_3 R_3 = 0 .$$

We substitute $i_3 = -i_2 - i_1$, from the first equation, into the other two to obtain

$$\varepsilon_1 - i_1 R_1 - i_2 R_3 - i_1 R_3 = 0$$

and

$$\varepsilon_2 - i_2 R_2 - i_2 R_3 - i_1 R_3 = 0 .$$

Solving the above equations yield

$$i_1 = \frac{\varepsilon_1 (R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(3.00\text{ V})(2.00\,\Omega + 5.00\,\Omega) - (1.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = 0.421\text{ A}.$$

$$i_2 = \frac{\varepsilon_2 (R_1 + R_3) - \varepsilon_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(1.00\text{ V})(4.00\,\Omega + 5.00\,\Omega) - (3.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.158\text{ A}.$$

$$i_3 = -\frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} = -\frac{(1.00\text{ V})(4.00\,\Omega) + (3.00\text{ V})(2.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.263\text{ A}.$$

Note that the current i_3 in R_3 is actually downward and the current i_2 in R_2 is to the right. The current i_1 in R_1 is to the right.

(a) The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (0.421\text{ A})^2 (4.00\,\Omega) = 0.709\text{ W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\text{ A})^2 (2.00\,\Omega) = 0.0499\text{ W} \approx 0.050$ W.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (-0.263\text{ A})^2 (5.00\,\Omega) = 0.346\text{ W}$.

(d) The power supplied by ε_1 is $i_3\varepsilon_1$ = (0.421 A)(3.00 V) = 1.26 W.

(e) The power "supplied" by ε_2 is $i_2\varepsilon_2$ = (–0.158 A)(1.00 V) = –0.158 W. The negative sign indicates that ε_2 is actually absorbing energy from the circuit.

44. (a) When $R_3 = 0$ all the current passes through R_1 and R_3 and avoids R_2 altogether. Since that value of the current (through the battery) is 0.006 A (see Fig. 27-54(b)) for $R_3 = 0$ then (using Ohm's law)

$$R_1 = (12\text{ V})/(0.006\text{ A}) = 2.0\times10^3\ \Omega.$$

(b) When $R_3 = \infty$ all the current passes through R_1 and R_2 and avoids R_3 altogether. Since that value of the current (through the battery) is 0.002 A (stated in problem) for $R_3 = \infty$ then (using Ohm's law)

$$R_2 = (12\text{ V})/(0.002\text{ A}) - R_1 = 4.0\times10^3\ \Omega.$$

45. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{eq}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR}.$$

Since the problem requires $R_{eq} = 10\ \Omega = R$, we must select $n = m$. Next we make use of Eq. 27-16. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{total} = n^2P$, where $P = 1.0\ \text{W}$ is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{total} \geq 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

46. The equivalent resistance in Fig. 27-55 (with n parallel resistors) is

$$R_{\text{eq}} = R + \frac{R}{n} = \left(\frac{n+1}{n}\right)R \ .$$

The current in the battery in this case should be

$$i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1}\frac{V_{\text{battery}}}{R} .$$

If there were n +1 parallel resistors, then

$$i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n+1}{n+2}\frac{V_{\text{battery}}}{R} \ .$$

For the relative increase to be 0.0125 (= 1/80), we require

$$\frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n+1)/(n+2)}{n/(n+1)} - 1 = \frac{1}{80} \ .$$

This leads to the second-degree equation

$$n^2 + 2n - 80 = (n + 10)(n - 8) = 0.$$

Clearly the only physically interesting solution to this is n = 8. Thus, there are eight resistors in parallel (as well as that resistor in series shown towards the bottom) in Fig. 27-55.

47. (a) The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}.$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{\left(r_A^2 - r_C^2\right)\rho_C\, i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}.$$

The denominators are the same and each has the value

$$\begin{aligned}\left(b^2 - a^2\right)\rho_C + a^2\rho_A &= \left[\left(0.380\times10^{-3}\text{ m}\right)^2 - \left(0.250\times10^{-3}\text{ m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &\quad + \left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &= 3.10\times10^{-15}\,\Omega\cdot\text{m}^3.\end{aligned}$$

Thus,

$$i_C = \frac{\left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\,\Omega\cdot\text{m}\right)\left(2.00\text{ A}\right)}{3.10\times10^{-15}\,\Omega\cdot\text{m}^3} = 1.11\text{ A}.$$

(b) Similarly,

$$i_A = \frac{\left[\left(0.380\times10^{-3}\text{m}\right)^2 - \left(0.250\times10^{-3}\text{m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(2.00\text{A}\right)}{3.10\times10^{-15}\,\Omega\cdot\text{m}^3} = 0.893\text{A}.$$

(c) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi\, a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{(\pi)\left(0.250\times10^{-3}\text{ m}\right)^2\left(12.0\text{ V}\right)}{\left(1.11\text{ A}\right)\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)} = 126\text{ m}.$$

48. (a) We use $P = \varepsilon^2/R_{eq}$, where

$$R_{eq} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega)+(12.0\,\Omega)R+(4.00\,\Omega)R}.$$

Put $P = 60.0$ W and $\varepsilon = 24.0$ V and solve for R: $R = 19.5\ \Omega$.

(b) Since $P \propto R_{eq}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{eq,\,min} = 7.00\ \Omega$, $P_{max} = \varepsilon^2/R_{eq,\,min} = (24.0\text{ V})^2/7.00\ \Omega = 82.3$ W.

(e) Since $R_{eq,\,max} = 7.00\ \Omega + (12.0\ \Omega)(4.00\ \Omega)/(12.0\ \Omega + 4.00\ \Omega) = 10.0\ \Omega$,

$$P_{min} = \varepsilon^2/R_{eq,\,max} = (24.0\text{ V})^2/10.0\ \Omega = 57.6\text{ W}.$$

49. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop to obtain

$$\varepsilon - iR_2 - i_1R_1 - ir = 0.$$

We apply the loop rule to the right-hand loop to obtain

$$i_1R_1 - (i - i_1)R_V = 0.$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} i_1.$$

We substitute this into the first equation to obtain

$$\varepsilon - \frac{(R_2 + r)(R_1 + R_V)}{R_V} i_1 + R_1 i_1 = 0.$$

This has the solution

$$i_1 = \frac{\varepsilon R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}.$$

The reading on the voltmeter is

$$i_1R_1 = \frac{\varepsilon R_V R_1}{(R_2 + r)\,(R_1 + R_V) + R_1 R_V} = \frac{(3.0\,\text{V})\,(5.0\times10^3\,\Omega)\,(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0\times10^3\,\Omega) + (250\,\Omega)(5.0\times10^3\,\Omega)}$$
$$= 1.12\,\text{V}.$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1R_1 = \frac{\varepsilon R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\,\Omega + 300\,\Omega + 100\,\Omega} = 1.15\,\text{V}.$$

The fractional error is (1.12 – 1.15)/(1.15) = –0.030, or –3.0%.

50. (a) Since $i = \varepsilon/(r + R_{ext})$ and $i_{max} = \varepsilon/r$, we have $R_{ext} = R(i_{max}/i - 1)$ where $r = 1.50$ V/1.00 mA $= 1.50 \times 10^3\ \Omega$. Thus,

$$R_{ext} = (1.5\times10^3\ \Omega)(1/0.100-1) = 1.35\times10^4\ \Omega$$

(b) $R_{ext} = (1.5\times10^3\ \Omega)(1/0.500-1) = 1.5\times10^3\ \Omega$.

(c) $R_{ext} = (1.5\times10^3\ \Omega)(1/0.900-1) = 167\,\Omega$.

(d) Since $r = 20.0\ \Omega + R$, $R = 1.50 \times 10^3\ \Omega - 20.0\ \Omega = 1.48 \times 10^3\ \Omega$.

51. (a) The current in R_1 is given by

$$i_1 = \frac{\varepsilon}{R_1 + R_2R_3/(R_2+R_3)} = \frac{5.0\ \text{V}}{2.0\Omega + (4.0\Omega)(6.0\Omega)/(4.0\Omega+6.0\Omega)} = 1.14\ \text{A}.$$

Thus,

$$i_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - i_1R_1}{R_3} = \frac{5.0\ \text{V} - (1.14\ \text{A})(2.0\Omega)}{6.0\Omega} = 0.45\ \text{A}.$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$i_3 = \frac{\varepsilon}{R_3 + \left(R_2R_1/(R_2+R_1)\right)} = \frac{5.0\text{V}}{6.0\Omega + \left((2.0\Omega)(4.0\Omega)/(2.0\Omega+4.0\Omega)\right)} = 0.6818\text{A}$$

and

$$i_1 = \frac{5.0\ \text{V} - (0.6818\ \text{A})(6.0\Omega)}{2.0\Omega} = 0.45\ \text{A},$$

the same as before.

52. (a) $\varepsilon = V + ir = 12\text{ V} + (10.0\text{ A})(0.0500\ \Omega) = 12.5\text{ V}$.

(b) Now $\varepsilon = V' + (i_{\text{motor}} + 8.00\text{ A})r$, where

$$V' = i'_A R_{\text{light}} = (8.00\text{ A})(12.0\text{ V}/10\text{ A}) = 9.60\text{ V}.$$

Therefore,

$$i_{\text{motor}} = \frac{\varepsilon - V'}{r} - 8.00\,\text{A} = \frac{12.5\,\text{V} - 9.60\,\text{V}}{0.0500\,\Omega} - 8.00\,\text{A} = 50.0\,\text{A}.$$

53. Since the current in the ammeter is i, the voltmeter reading is

$$V' = V + i\,R_A = i\,(R + R_A),$$

or $R = V'/i - R_A = R' - R_A$, where $R' = V'/i$ is the apparent reading of the resistance. Now, from the lower loop of the circuit diagram, the current through the voltmeter is $i_V = \varepsilon/(R_{\text{eq}} + R_0)$, where

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_V} + \frac{1}{R_A + R} \quad \Rightarrow \quad R_{\text{eq}} = \frac{R_V\left(R + R_A\right)}{R_V + R + R_A} = \frac{\left(300\,\Omega\right)\left(85.0\,\Omega + 3.00\,\Omega\right)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega.$$

The voltmeter reading is then

$$V' = i_V R_{\text{eq}} = \frac{\varepsilon R_{\text{eq}}}{R_{\text{eq}} + R_0} = \frac{(12.0\text{ V})(68.0\ \Omega)}{68.0\ \Omega + 100\Omega} = 4.86\text{ V}.$$

(a) The ammeter reading is

$$i = \frac{V'}{R + R_A} = \frac{4.86\text{ V}}{85.0\ \Omega + 3.00\Omega} = 0.0552\text{ A}.$$

(b) As shown above, the voltmeter reading is $V' = 4.86$ V.

(c) $R' = V'/i = 4.86\text{ V}/(5.52 \times 10^{-2}\text{ A}) = 88.0\ \Omega$.

(d) Since $R = R' - R_A$, if R_A is decreased, the difference between R' and R decreases. In fact, when $R_A = 0$, $R' = R$.

54. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call i (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is iR). The resistor network can be reduced to an equivalence of

$$R_{\text{eq}} = \frac{(2R)(R)}{2R+R} + \frac{(R)(R)}{R+R} = \frac{7}{6}R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\varepsilon}{R_{\text{eq}}} \Rightarrow i = \frac{\varepsilon}{2R_{\text{eq}}} = \frac{\varepsilon}{2(7R/6)} = \frac{3\varepsilon}{7R}.$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$ and one of the bottom resistors), we have

$$\varepsilon - i_{2R}(2R) - iR = 0 \Rightarrow i_{2R} = \frac{\varepsilon - iR}{2R}.$$

Substituting $i = 3\varepsilon/7R$, this gives $i_{2R} = 2\varepsilon/7R$. The difference between i_{2R} and i is the current through the ammeter. Thus,

$$i_{\text{ammeter}} = i - i_{2R} = \frac{3\varepsilon}{7R} - \frac{2\varepsilon}{7R} = \frac{\varepsilon}{7R} \Rightarrow \frac{i_{\text{ammeter}}}{\varepsilon / R} = \frac{1}{7} = 0.143.$$

55. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\frac{R_1}{R_s}i_1 \;\Rightarrow\; R_x = \frac{R_2R_s}{R_1}.$$

56. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R}=\frac{1}{R'}-\frac{1}{R_V} \quad \Rightarrow R'=\frac{RR_V}{R+R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}} = R_A + R_0 + R' = R_A + R_0 + \dfrac{RR_V}{R+R_V}$.

(a) The ammeter reading is

$$i'=\frac{\varepsilon}{R_{\text{eq}}}=\frac{\varepsilon}{R_A+R_0+R_V\ R/(R+R_V)}=\frac{12.0\,\text{V}}{3.00\Omega+100\Omega+(300\Omega)\ (85.0\Omega)/(300\Omega+85.0\Omega)}$$
$$=7.09\times10^{-2}\ \text{A}.$$

(b) The voltmeter reading is

$$V=\varepsilon - i'(R_A + R_0) = 12.0\ \text{V} - (0.0709\ \text{A})\ (103.00\ \Omega) = 4.70\ \text{V}.$$

(c) The apparent resistance is $R' = V/i' = 4.70\ \text{V}/(7.09 \times 10^{-2}\ \text{A}) = 66.3\ \Omega$.

(d) If R_V is increased, the difference between R and R' decreases. In fact, $R' \to R$ as $R_V \to \infty$.

57. During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$. We require $q = 0.99q_{eq} = 0.99C\varepsilon$, so

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\,\tau$.

58. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln(q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus,

$$t_{1/3} = \tau \ln\left(\frac{q_0}{2q_0/3}\right) = \tau \ln\left(\frac{3}{2}\right) = 0.41\tau \quad \Rightarrow \quad \frac{t_{1/3}}{\tau} = 0.41.$$

(b) $t_{2/3} = \tau \ln\left(\dfrac{q_0}{q_0/3}\right) = \tau \ln 3 = 1.1\tau \quad \Rightarrow \quad \dfrac{t_{2/3}}{\tau} = 1.1.$

59. (a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu\text{s}$ we have $V(t) = 5.00$ V, so $5.00\ \text{V} = (12.0\ \text{V})(1 - e^{-1.30\ \mu\text{s}/RC})$, which gives

$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R = (2.41\ \mu\text{s})/(15.0\ \text{k}\Omega) = 161$ pF.

60. (a) $\tau = RC = (1.40 \times 10^6\ \Omega)(1.80 \times 10^{-6}\ \text{F}) = 2.52$ s.

(b) $q_o = \varepsilon C = (12.0\ \text{V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}$.

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC \ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\text{s}) \ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\,\text{s}.$$

61. Here we denote the battery emf as V. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{cap}$, or

$$Ve^{-t/RC} = V(1 - e^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC \ln 2$, or $t = 0.208$ ms.

62. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0 e^{-t/\tau}$, this means $V = V_0 e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\text{s}}{\ln\left[(1.00\,\text{V})/(100\,\text{V})\right]} = 2.17\,\text{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0 \text{ s})/(2.17 \text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = (100\,\text{V})e^{-7.83} = 3.96\times 10^{-2}\ \text{V}\,.$$

63. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0$ s, we find

$$R = \frac{t}{C \ln(V_0/V)} = \frac{2.0\,\text{s}}{(2.0 \times 10^{-6}\,\text{F}) \ln 4} = 7.2 \times 10^5\,\Omega.$$

64. (a) The initial energy stored in a capacitor is given by $U_C = q_0^2/2C$, where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2\left(1.0\times10^{-6}\text{ F}\right)\left(0.50\text{ J}\right)} = 1.0\times10^{-3}\text{ C}.$$

(b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau},$$

and the initial current is $i_0 = q_0/\tau$. The time constant is

$$\tau = RC = \left(1.0\times10^{-6}\text{ F}\right)\left(1.0\times10^{6}\ \Omega\right) = 1.0\text{ s}.$$

Thus $i_0 = \left(1.0\times10^{-3}\text{ C}\right)\big/\left(1.0\text{ s}\right) = 1.0\times10^{-3}\text{ A}$.

(c) We substitute $q = q_0\, e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}e^{-t/\tau} = \left(\frac{1.0\times10^{-3}\text{ C}}{1.0\times10^{-6}\text{ F}}\right)e^{-t/1.0\text{ s}} = \left(1.0\times10^{3}\text{ V}\right)e^{-1.0t},$$

where t is measured in seconds.

(d) We substitute $i = \left(q_0/\tau\right)e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau}e^{-t/\tau} = \frac{\left(1.0\times10^{-3}\text{ C}\right)\left(1.0\times10^{6}\ \Omega\right)}{1.0\text{ s}}e^{-t/1.0\text{ s}} = \left(1.0\times10^{3}\text{ V}\right)e^{-1.0t},$$

where t is measured in seconds.

(e) We substitute $i = \left(q_0/\tau\right)e^{-t/\tau}$ into $P = i^2R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2}e^{-2t/\tau} = \frac{\left(1.0\times10^{-3}\text{ C}\right)^2\left(1.0\times10^{6}\ \Omega\right)}{\left(1.0\text{ s}\right)^2}e^{-2t/1.0\text{ s}} = \left(1.0\text{ W}\right)e^{-2.0t},$$

where t is again measured in seconds.

65. At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0 \ ,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0 \ .$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

(a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2 \times 10^3 \text{ V})}{3(0.73 \times 10^6 \ \Omega)} = 1.1 \times 10^{-3} \text{ A} \ ,$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2 \times 10^3 \text{ V}}{3(0.73 \times 10^6 \ \Omega)} = 5.5 \times 10^{-4} \text{ A}$, and

(c) $i_3 = i_2 = 5.5 \times 10^{-4}$ A.

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields

$$\varepsilon - i_1 R_1 - i_1 R_2 = 0 \ .$$

(d) The solution is

$$i_1 = \frac{\varepsilon}{2R} = \frac{1.2 \times 10^3 \text{ V}}{2(0.73 \times 10^6 \ \Omega)} = 8.2 \times 10^{-4} \text{ A}.$$

(e) $i_2 = i_1 = 8.2 \times 10^{-4}$ A.

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0 \ .$$

We use the first equation to substitute for i_1 in the second and obtain $\varepsilon - 2i_2R - i_3R = 0$. Thus $i_2 = (\varepsilon - i_3R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3R) + (\varepsilon/2) - (i_3R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R}e^{-2t/3RC}.$$

The current in the center branch is

$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R}e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

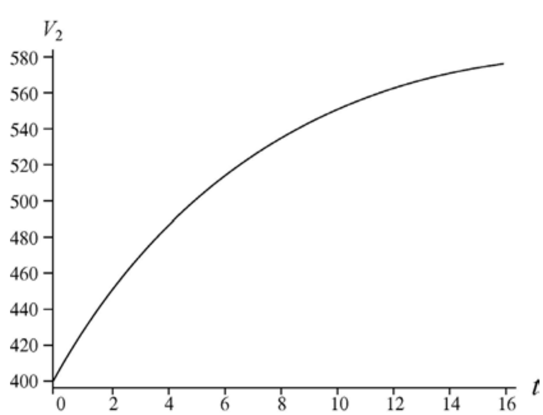
and the potential difference across R_2 is

$$V_2(t) = i_2R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right).$$

(g) For $t = 0, e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\text{ V}\right)/3 = 4.0\times10^2\text{ V}$.

(h) For $t = \infty, e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\text{ V}\right)/2 = 6.0\times10^2\text{ V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



66. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \varepsilon\left(1-e^{-t/RC}\right)$. We solve for R:

$$R = \frac{t}{C\ln\left[\varepsilon/\left(\varepsilon - V_L\right)\right]} = \frac{0.500\,\mathrm{s}}{\left(0.150\times10^{-6}\,\mathrm{F}\right)\ln\left[95.0\,\mathrm{V}/\left(95.0\,\mathrm{V}-72.0\,\mathrm{V}\right)\right]} = 2.35\times10^{6}\,\Omega$$

where we used $t = 0.500$ s given (implicitly) in the problem.

67. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2 \frac{\varepsilon}{R_1 + R_2} = (15.0\,\text{k}\Omega)\left(\frac{20.0\,\text{V}}{10.0\,\text{k}\Omega + 15.0\,\text{k}\Omega}\right) = 12.0\,\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11×10^{-4} A.

68. We apply Eq. 27-39 to each capacitor, demand their initial charges are in a ratio of 3:2 as described in the problem, and solve for the time. With

$$\tau_1 = R_1C_1 = (20.0\,\Omega)(5.00\times10^{-6}\text{ F}) = 1.00\times10^{-4}\text{ s}$$
$$\tau_2 = R_2C_2 = (10.0\,\Omega)(8.00\times10^{-6}\text{ F}) = 8.00\times10^{-5}\text{ s},$$

we obtain

$$t = \frac{\ln(3/2)}{\tau_2^{-1} - \tau_1^{-1}} = \frac{\ln(3/2)}{1.25\times10^{4}\text{ s}^{-1} - 1.00\times10^{4}\text{ s}^{-1}} = 1.62\times10^{-4}\text{s}.$$

69. (a) The charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where ε is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is

$$\tau = RC = (3.00 \times 10^6\ \Omega)(1.00 \times 10^{-6}\ \text{F}) = 3.00\ \text{s}.$$

At $t = 1.00$ s, $t/\tau = (1.00\ \text{s})/(3.00\ \text{s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau} = \frac{\left(1.00\times10^{-6}\ \text{F}\right)\left(4.00\,\text{V}\right)}{3.00\,\text{s}} e^{-0.333} = 9.55\times10^{-7}\ \text{C/s}.$$

(b) The energy stored in the capacitor is given by $U_C = \dfrac{q^2}{2C}$, and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Now

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right) = \left(1.00\times10^{-6}\right)\left(4.00\,\text{V}\right)\left(1 - e^{-0.333}\right) = 1.13\times10^{-6}\ \text{C},$$

so

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt} = \left(\frac{1.13\times10^{-6}\ \text{C}}{1.00\times10^{-6}\ \text{F}}\right)\left(9.55\times10^{-7}\ \text{C/s}\right) = 1.08\times10^{-6}\ \text{W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is 9.55×10^{-7} A, so

$$P = \left(9.55\times10^{-7}\ \text{A}\right)^2\left(3.00\times10^{6}\ \Omega\right) = 2.74\times10^{-6}\ \text{W}.$$

(d) The rate at which energy is delivered by the battery is

$$i\varepsilon = \left(9.55\times10^{-7}\ \text{A}\right)\left(4.00\,\text{V}\right) = 3.82\times10^{-6}\ \text{W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)\ (dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor R = 10 Ω (with current i). As if we were presented with a maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which — as in any good maze — winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\varepsilon = 40$ V.

(a) The current through R is then $i = \varepsilon/R = 4.0$ A.

(b) The direction is upward in the figure.

71. (a) In the process described in the problem, no charge is gained or lost. Thus, q = constant. Hence,

$$q = C_1V_1 = C_2V_2 \quad \Rightarrow \quad V_2 = V_1\frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3.0\times10^3\ \text{V}.$$

(b) Eq. 27-39, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0e^{-t/\tau} \;\Rightarrow\; t = RC\ln\left(\frac{V_0}{V}\right) = \left(300\times10^9\ \Omega\right)\left(10\times10^{-12}\ \text{F}\right)\ln\left(\frac{3000}{100}\right)$$

which yields t = 10 s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0e^{-t/RC}$ for R with the new values V_0 = 1400 V and t = 0.30 s. Thus,

$$R = \frac{t}{C\ln(V_0/V)} = \frac{0.30\,\text{s}}{\left(10\times10^{-12}\ \text{F}\right)\ln(1400/100)} = 1.1\times10^{10}\ \Omega\ .$$

72. (a) Since $R_{\text{tank}} = 140\ \Omega, i = 12\ \text{V}\big/\big(10\,\Omega + 140\,\Omega\big) = 8.0\times10^{-2}\ \text{A}$.

(b) Now, $R_{\text{tank}} = (140\ \Omega + 20\ \Omega)/2 = 80\ \Omega$, so $i = 12\ \text{V}/(10\ \Omega + 80\ \Omega) = 0.13\ \text{A}$.

(c) When full, $R_{\text{tank}} = 20\ \Omega$ so $i = 12\ \text{V}/(10\ \Omega + 20\ \Omega) = 0.40\ \text{A}$.

73. We use the result of Problem 27-63: $R = t/[C \ln(V_0/V)]$.

(a) Then, for $t_{\min} = 10.0\ \mu s$

$$R_{\min} = \frac{10.0\ \mu\text{s}}{(0.220\ \mu\text{F})\ln(5.00/0.800)} = 24.8\ \Omega.$$

(b) For $t_{\max} = 6.00$ ms,

$$R_{\max} = \left(\frac{6.00\ \text{ms}}{10.0\ \mu\text{s}}\right)(24.8\ \Omega) = 1.49\times10^{4}\ \Omega\ ,$$

where in the last equation we used $\tau = RC$.

74. (a) Using Eq. 27-4, we take the derivative of the power $P = i^2R$ with respect to R and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\varepsilon^2 R}{(R+r)^2}\right) = \frac{\varepsilon^2 (r-R)}{(R+r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\varepsilon^2 R}{(R+r)^2}\right|_{R=r} = \frac{\varepsilon^2}{4r}.$$

75. (a) The magnitude of the current density vector is

$$J_A = \frac{i}{A} = \frac{V}{(R_1 + R_2)A} = \frac{4V}{(R_1 + R_2)\pi D^2} = \frac{4(60.0\,\text{V})}{\pi(0.127\,\Omega + 0.729\,\Omega)(2.60\times10^{-3}\,\text{m})^2}$$

$$= 1.32\times10^7\ \text{A/m}^2\,.$$

(b) $V_A = V R_1/(R_1 + R_2) = (60.0\ \text{V})(0.127\ \Omega)/(0.127\ \Omega + 0.729\ \Omega) = 8.90\ \text{V}$.

(c) The resistivity of wire *A* is

$$\rho_A = \frac{R_A A}{L_A} = \frac{\pi R_A D^2}{4L_A} = \frac{\pi(0.127\,\Omega)(2.60\times10^{-3}\ \text{m})^2}{4(40.0\,\text{m})} = 1.69\times10^{-8}\,\Omega\cdot\text{m}\,.$$

So wire *A* is made of copper.

(d) $J_B = J_A = 1.32\times10^7\ \text{A/m}^2$.

(e) $V_B = V - V_A = 60.0\ \text{V} - 8.9\ \text{V} = 51.1\ \text{V}$.

(f) The resistivity of wire *B* is $\rho_B = 9.68\times10^{-8}\ \Omega\cdot\text{m}$, so wire *B* is made of iron.

76. Here we denote the battery emf as V. Eq. 27-30 leads to

$$i = \frac{\mathrm{V}}{R} - \frac{q}{RC} = \frac{12}{4} - \frac{8}{(4)(4)} = 2.5 \text{ A} .$$

77. The internal resistance of the battery is $r = (12\text{ V} - 11.4\text{ V})/50\text{ A} = 0.012\ \Omega < 0.020\ \Omega$, so the battery is OK. The resistance of the cable is

$$R = 3.0\text{ V}/50\text{ A} = 0.060\ \Omega > 0.040\ \Omega,$$

so the cable is defective.

78. The equivalent resistance of the series pair of $R_3 = R_4 = 2.0\ \Omega$ is $R_{34} = 4.0\ \Omega$, and the equivalent resistance of the parallel pair of $R_1 = R_2 = 4.0\ \Omega$ is $R_{12} = 2.0\ \Omega$. Since the voltage across R_{34} must equal that across R_{12}:

$$V_{34} = V_{12} \quad \Rightarrow \quad i_{34}R_{34} = i_{12}R_{12} \quad \Rightarrow \quad i_{34} = \frac{1}{2}i_{12}$$

This relation, plus the junction rule condition $I = i_{12} + i_{34} = 6.00\ \text{A}$ leads to the solution $i_{12} = 4.0\ \text{A}$. It is clear by symmetry that $i_1 = i_{12}/2 = 2.00\ \text{A}$.

79. (a) If S_1 is closed, and S_2 and S_3 are open, then $i_a = \varepsilon/2R_1 = 120\text{ V}/40.0\ \Omega = 3.00\text{ A}$.

(b) If S_3 is open while S_1 and S_2 remain closed, then

$$R_{\text{eq}} = R_1 + R_1\,(R_1 + R_2)\,/(2R_1 + R_2) = 20.0\ \Omega + (20.0\ \Omega) \times (30.0\ \Omega)/(50.0\ \Omega) = 32.0\ \Omega,$$

so $i_a = \varepsilon/R_{\text{eq}} = 120\text{ V}/32.0\ \Omega = 3.75\text{ A}$.

(c) If all three switches S_1, S_2 and S_3 are closed, then $R_{\text{eq}} = R_1 + R_1\,R'/(R_1 + R')$ where

$$R' = R_2 + R_1\,(R_1 + R_2)/(2R_1 + R_2) = 22.0\ \Omega,$$

i.e.,

$$R_{\text{eq}} = 20.0\ \Omega + (20.0\ \Omega)\,(22.0\ \Omega)/(20.0\ \Omega + 22.0\ \Omega) = 30.5\ \Omega,$$

so $i_a = \varepsilon/R_{\text{eq}} = 120\text{ V}/30.5\ \Omega = 3.94\text{ A}$.

80. (a) Reducing the bottom two series resistors to a single $R' = 4.00\ \Omega$ (with current i_1 through it), we see we can make a path (for use with the loop rule) that passes through R, the $\varepsilon_4 = 5.00$ V battery, the $\varepsilon_1 = 20.0$ V battery, and the $\varepsilon_3 = 5.00$ V. This leads to

$$i_1 = \frac{\varepsilon_1 + \varepsilon_3 + \varepsilon_4}{R'} = \frac{20.0\text{ V} + 5.00\text{ V} + 5.00\text{ V}}{40.0\,\Omega} = \frac{30.0\text{ V}}{4.0\,\Omega} = 7.50\text{ A}.$$

(b) The direction of i_1 is leftward.

(c) The voltage across the bottom series pair is $i_1 R' = 30.0$ V. This must be the same as the voltage across the two resistors directly above them, one of which has current i_2 through it and the other (by symmetry) has current $\frac{1}{2}i_2$ through it. Therefore,

$$30.0\text{ V} = i_2\,(2.00\,\Omega) + \tfrac{1}{2}\,i_2\,(2.00\,\Omega)$$

leads to $i_2 = 10.0$ A.

(d) The direction of i_2 is also leftward.

(e) We use Eq. 27-17: $P_4 = (i_1 + i_2)\varepsilon_4 = 87.5$ W.

(f) The energy is being supplied to the circuit since the current is in the "forward" direction through the battery.

81. The bottom two resistors are in parallel, equivalent to a $2.0R$ resistance. This, then, is in series with resistor R on the right, so that their equivalence is $R' = 3.0R$. Now, near the top left are two resistors ($2.0R$ and $4.0R$) which are in series, equivalent to $R'' = 6.0R$. Finally, R' and R'' are in parallel, so the net equivalence is

$$R_{eq} = \frac{(R')\,(R'')}{R' + R''} = 2.0R = 20\ \Omega$$

where in the final step we use the fact that $R = 10\ \Omega$.

82. (a) The four resistors R_1, R_2, R_3 and R_4 on the left reduce to

$$R_{\mathrm{eq}} = R_{12} + R_{34} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 7.0\Omega + 3.0\Omega = 10\Omega$$

With $\varepsilon = 30$ V across R_{eq} the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{\mathrm{eq}} = R_{56} + R_7 = \frac{R_5 R_6}{R_5 + R_6} + R_7 = \frac{(6.0\Omega)(2.0\Omega)}{6.0\Omega + 2.0\Omega} + 1.5\Omega = 3.0\Omega\,.$$

With $\varepsilon = 30$ V across R'_{eq} the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2}\, i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30\text{V} - i_4(1.5\ \Omega) - i_5(2.0\ \Omega) = 0$$

readily yields $i_5 = 7.5$ A.

83. (a) We analyze the lower left loop and find $i_1 = \varepsilon_1/R = (12.0\text{ V})/(4.00\ \Omega) = 3.00$ A.

(b) The direction of i_1 is downward.

(c) Letting $R = 4.00\ \Omega$, we apply the loop rule to the tall rectangular loop in the center of the figure (proceeding clockwise):

$$\varepsilon_2 + (+i_1 R) + (-i_2 R) + \left(-\frac{i_2}{2} R\right) + (-i_2 R) = 0 .$$

Using the result from part (a), we find $i_2 = 1.60$ A.

(d) The direction of i_2 is downward (as was assumed in writing the equation as we did).

(e) Battery 1 is supplying this power since the current is in the "forward" direction through the battery.

(f) We apply Eq. 27-17: The current through the $\varepsilon_1 = 12.0$ V battery is, by the junction rule, 3.00 A + 1.60 A = 4.60 A and $P = (4.60\text{ A})(12.0\text{ V}) = 55.2$ W.

(g) Battery 2 is supplying this power since the current is in the "forward" direction through the battery.

(h) $P = i_2(4.00\text{ V}) = 6.40$ W.

84. (a) We reduce the parallel pair of resistors (at the bottom of the figure) to a single R' $=1.00\ \Omega$ resistor and then reduce it with its series 'partner' (at the lower left of the figure) to obtain an equivalence of $R'' = 2.00\ \Omega + 1.00\Omega = 3.00\ \Omega$. It is clear that the current through R'' is the i_1 we are solving for. Now, we employ the loop rule, choose a path that includes R'' and all the batteries (proceeding clockwise). Thus, assuming i_1 goes leftward through R'', we have

$$5.00\text{ V} + 20.0\text{ V} - 10.0\text{ V} - i_1 R'' = 0$$

which yields $i_1 = 5.00$ A.

(b) Since i_1 is positive, our assumption regarding its direction (leftward) was correct.

(c) Since the current through the $\varepsilon_1 = 20.0$ V battery is "forward", battery 1 is supplying energy.

(d) The rate is $P_1 = (5.00\text{ A})(20.0\text{ V}) = 100$ W.

(e) Reducing the parallel pair (which are in parallel to the $\varepsilon_2 = 10.0$ V battery) to a single $R' = 1.00\ \Omega$ resistor (and thus with current $i' = (10.0\text{ V})/(1.00\ \Omega) = 10.0$ A downward through it), we see that the current through the battery (by the junction rule) must be $i = i' - i_1 = 5.00$ A *upward* (which is the "forward" direction for that battery). Thus, battery 2 is supplying energy.

(f) Using Eq. 27-17, we obtain $P_2 = 50.0$ W.

(g) The set of resistors that are in parallel with the $\varepsilon_3 = 5$ V battery is reduced to $R''' = 0.800\ \Omega$ (accounting for the fact that two of those resistors are actually reduced in series, first, before the parallel reduction is made), which has current $i''' = (5.00\text{ V})/(0.800\ \Omega) = 6.25$ A downward through it. Thus, the current through the battery (by the junction rule) must be $i = i''' + i_1 = 11.25$ A *upward* (which is the "forward" direction for that battery). Thus, battery 3 is supplying energy.

(h) Eq. 27-17 leads to $P_3 = 56.3$ W.

85. We denote silicon with subscript s and iron with i. Let $T_0 = 20°$. If

$$\begin{aligned} R(T) &= R_s(T) + R_i(T) = R_s(T_0)\left[1+\alpha(T-T_0)\right] + R_i(T_0)\left[1+\alpha_i(T-T_0)\right] \\ &= \left(R_s(T)_0\,\alpha_s + R_i(T_0)\alpha_i\right) + (\text{temperature independent terms}) \end{aligned}$$

is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\ \Omega$. We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i - \alpha_s} = \frac{(1000\Omega)\left(6.5\times10^{-3}\right)}{6.5\times10^{-3} + 70\times10^{-3}} = 85.0\Omega.$$

(b) $R_i(T_0) = 1000\ \Omega - 85.0\ \Omega = 915\ \Omega$.

86. Consider the lowest branch with the two resistors R_4 = 3.00 Ω and R_5 = 5.00 Ω. The voltage difference across R_5 is

$$V = i_5 R_5 = \frac{\varepsilon R_5}{R_4 + R_5} = \frac{(120\,\text{V})(5.00\,\Omega)}{3.00\,\Omega + 5.00\,\Omega} = 7.50\,\text{V}.$$

87. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\,\text{W})(0.10\,\Omega)} = 1.0\,\text{V}$.

(b) From $i = V/R = (\varepsilon - V)/r$ we find

$$r = R\left(\frac{\varepsilon - V}{V}\right) = (0.10\,\Omega)\left(\frac{1.5\,\text{V} - 1.0\,\text{V}}{1.0\,\text{V}}\right) = 0.050\,\Omega.$$

88. (a) $R_{eq}(AB) = 20.0\ \Omega/3 = 6.67\ \Omega$ (three 20.0 Ω resistors in parallel).

(b) $R_{eq}(AC) = 20.0\ \Omega/3 = 6.67\ \Omega$ (three 20.0 Ω resistors in parallel).

(c) $R_{eq}(BC) = 0$ (as B and C are connected by a conducting wire).

89. When S is open for a long time, the charge on C is $q_i = \varepsilon_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is

$$i = (\varepsilon_2 - \varepsilon_1)/(R_1 + R_2) = (3.0\text{ V} - 1.0\text{ V})/(0.20\ \Omega + 0.40\ \Omega) = 3.33\text{ A}.$$

The voltage difference V across the capacitor is then

$$V = \varepsilon_2 - iR_2 = 3.0\text{ V} - (3.33\text{ A})(0.40\ \Omega) = 1.67\text{ V}.$$

Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is

$$\Delta q = q_f - q_i = (V - \varepsilon_2)C = (1.67\text{ V} - 3.0\text{ V})(10\ \mu\text{F}) = -13\ \mu\text{C}.$$

90. From $V_a - \varepsilon_1 = V_c - ir_1 - iR$ and $i = (\varepsilon_1 - \varepsilon_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned}
V_a - V_c &= \varepsilon_1 - i(r_1 + R) = \varepsilon_1 - \left(\frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}\right)(r_1 + R) \\
&= 4.4\,\mathrm{V} - \left(\frac{4.4\,\mathrm{V} - 2.1\,\mathrm{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\
&= 2.5\,\mathrm{V}.
\end{aligned}$$

91. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\varepsilon R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}.$$

92. The current in the ammeter is given by

$$i_A = \varepsilon/(r + R_1 + R_2 + R_A).$$

The current in R_1 and R_2 without the ammeter is $i = \varepsilon/(r + R_1 + R_2)$. The percent error is then

$$\frac{\Delta i}{i} = \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} = \frac{0.10\,\Omega}{2.0\,\Omega + 5.0\,\Omega + 4.0\,\Omega + 0.10\,\Omega}$$

$$= 0.90\%.$$

93. The maximum power output is (120 V)(15 A) = 1800 W. Since 1800 W/500 W = 3.6, the maximum number of 500 W lamps allowed is 3.

94. In the steady state situation, there is no current going to the capacitors, so the resistors all have the same current. By the loop rule,

$$20.0 \text{ V} = (5.00\ \Omega)i + (10.0\ \Omega)i + (15.0\ \Omega)i$$

which yields $i = \frac{2}{3}$ A. Consequently, the voltage across the R_1 = 5.00 Ω resistor is (5.00 Ω)(2/3 A) = 10/3 V, and is equal to the voltage V_1 across the C_1 = 5.00 μF capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(5.00\times10^{-6}\text{ F})\left(\frac{10}{3}\text{V}\right)^2 = 2.78\times10^{-5}\text{ J}.$$

Similarly, the voltage across the R_2 = 10.0 Ω resistor is (10.0 Ω)(2/3 A) = 20/3 V and is equal to the voltage V_2 across the C_2 = 10.0 μF capacitor. Hence,

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10.0\times10^{-6}\text{ F})\left(\frac{20}{3}\text{V}\right)^2 = 2.22\times10^{-5}\text{ J}$$

Therefore, the total capacitor energy is $U_1 + U_2 = 2.50 \times 10^{-4}$ J.

95. (a) The charge q on the capacitor as a function of time is $q(t) = (\varepsilon C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\varepsilon/R)e^{-t/RC}$. The energy supplied by the emf is then

$$U = \int_0^\infty \varepsilon i\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC} dt = C\varepsilon^2 = 2U_C$$

where $U_C = \frac{1}{2}C\varepsilon^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2 R dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}C\varepsilon^2.$$

96. When connected in series, the rate at which electric energy dissipates is $P_s = \varepsilon^2/(R_1 + R_2)$. When connected in parallel, the corresponding rate is $P_p = \varepsilon^2(R_1 + R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\ \Omega$. We solve for R_2: $R_2 = 38\ \Omega$ or $260\ \Omega$.

(a) Thus, the smaller value of R_2 is $38\ \Omega$.

(b) The larger value of R_2 is $260\ \Omega$.

97. (a) The capacitor is *initially* uncharged, which implies (by the loop rule) that there is zero voltage (at $t = 0$) across the $R_2 = 10$ kΩ resistor, and that 30 V is across the R_1 =20 kΩ resistor. Therefore, by Ohm;s law, $i_{10} = (30 \text{ V})/(20 \text{ k}\Omega) = 1.5 \times 10^{-3}$ A.

(b) Similarly, $i_{20} = 0$.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the 20 kΩ and 10 kΩ resistors behave more like a series pair (having the same current), equivalent to 30 kΩ. The current through them, then, at long times, is

$$i = (30 \text{ V})/(30 \text{ k}\Omega) = 1.0 \times 10^{-3} \text{ A}.$$

98. Using the junction and the loop rules, we have

$$\begin{aligned} 20.0 - i_1R_1 - i_3R_3 &= 0 \\ 20.0 - i_1R_1 - i_2R_2 - 50 &= 0 \\ i_2 + i_3 &= i_1 \end{aligned}$$

Requiring no current through the battery 1 means that $i_1 = 0$, or $i_2 = i_3$. Solving the above equations with $R_1 = 10.0\Omega$ and $R_2 = 20.0\Omega$, we obtain

$$i_1 = \frac{40 - 3R_3}{20 + 3R_3} = 0 \quad \Rightarrow \quad R_3 = \frac{40}{3} = 13.3\Omega$$

99. With the unit Ω understood, the equivalent resistance for this circuit is

$$R_{eq} = \frac{20R_3 + 100}{R_3 + 10}.$$

Therefore, the power supplied by the battery (equal to the power dissipated in the resistors) is

$$P = \frac{V^2}{R_3} = V^2 \frac{R_3 + 10}{20R_3 + 100}$$

where $V = 12$ V. We attempt to extremize the expression by working through the $dP/dR_3 = 0$ condition and do not find a value of R_3 that satisfies it.

(a) We note, then, that the function is a monotonically decreasing function of R_3, with $R_3 = 0$ giving the maximum possible value (since $R_3 < 0$ values are not being allowed).

(b) With the value $R_3 = 0$, we obtain $P = 14.4$ W.

100. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the R = 4.0 Ω resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to 4.0 Ω) and ε denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3(\varepsilon - ir) - (2i)R = 0.$$

This yields i = 3.0 A. Consequently, i_R = 6.0 A.

(b) The terminal voltage of each battery is $\varepsilon - ir$ = 8.0 V.

(c) Using Eq. 27-17, we obtain $P = i\varepsilon$ = (3)(20) = 60 W.

(d) Using Eq. 26-27, we have $P = i^2 r$ = 36 W.

101. When all the batteries are connected in parallel, each supplies a current i; thus, $i_R = Ni$. Then from $\varepsilon = ir + i_R R = ir + Nir$, we get $i_R = N\varepsilon/[(N + 1)r]$. When all the batteries are connected in series, $i_r = i_R$ and

$$\varepsilon_{\text{total}} = N\varepsilon = Ni_r r + i_R R = Ni_R r + i_R r,$$

so $i_R = N\varepsilon/[(N + 1)r]$.

102. (a) Here we denote the battery emf as V. See Fig. 27-4(a): $V_T = V - ir$.

(b) Doing a least squares fit for the V_T versus i values listed, we obtain

$$V_T = 13.61 - 0.0599i$$

which implies $V = 13.6$ V.

(c) It also implies the internal resistance is 0.060 Ω.

103. (a) The loop rule (proceeding counterclockwise around the right loop) leads to $\varepsilon_2 - i_1R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.0600$ A.

(b) The direction of i_1 is downward.

(c) The loop rule (counterclockwise around the left loop) gives

$$(+\varepsilon_1)+(+i_1R_1)+(-i_2R_2)=0$$

where i_2 has been assumed leftward. This yields $i_3 = 0.180$ A.

(d) A positive value of i_3 implies that our assumption on the direction is correct, i.e., it flows leftward.

(e) The junction rule tells us that the current through the 12 V battery is 0.180 + 0.0600 = 0.240 A.

(f) The direction is upward.

104. (a) Since $P = \varepsilon^2/R_{eq}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 300$ W, $R_{eq} = R_1R_2/(R_1 + R_2) = (144\ \Omega)R_2/(144\ \Omega + R_2) = (120\ \text{V})^2/(300\ \text{W})$. We obtain $R_2 = 72\ \Omega$.

(c) For $P = 100$ W, $R_{eq} = R_1 = \varepsilon^2/P = (120\ \text{V})^2/100\ \text{W} = 144\ \Omega$;

105. (a) The six resistors to the left of ε_1 = 16 V battery can be reduced to a single resistor R = 8.0 Ω, through which the current must be $i_R = \varepsilon_1/R$ = 2.0 A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\,\text{V} - 8.0\,\text{V}}{3.0\,\Omega + 1.0\,\Omega} = 2.0\,\text{A}$$

in a direction that is "backward" relative to the ε_2 = 8.0 V battery. Thus, by the junction rule,

$$i_1 = i_R + i' = 4.0\,\text{A}\,.$$

(b) The direction of i_1 is upward (that is, in the "forward" direction relative to ε_1).

(c) The current i_2 derives from a succession of symmetric splittings of i_R (reversing the procedure of reducing those six resistors to find R in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2}i_R\right) = 0.50\,\text{A}\,.$$

(d) The direction of i_2 is clearly downward.

(e) Using our conclusion from part (a) in Eq. 27-17, we have

$$P = i_1\varepsilon_1 = (4.0\text{ A})(16\text{ V}) = 64\text{ W}.$$

(f) Using results from part (a) in Eq. 27-17, we obtain $P = i'\varepsilon_2$ = (2.0 A)(8.0 V) = 16 W.

(g) Energy is being supplied in battery 1.

(h) Energy is being absorbed in battery 2.

106. (a) R_2 and R_3 are in parallel; their equivalence is in series with R_1. Therefore,

$$R_{\mathrm{eq}} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 300\,\Omega.$$

(b) The current through the battery is $\varepsilon/R_{\mathrm{eq}} = 0.0200$ A, which is also the current through R_1. Hence, the voltage across R_1 is $V_1 = (0.0200\text{ A})(100\,\Omega) = 2.00$ V.

(c) From the loop rule,

$$\varepsilon - V_1 - i_3 R_3 = 0$$

which yields $i_3 = 6.67 \times 10^{-3}$ A.

107. (a) By symmetry, we see that i_3 is half the current that goes through the battery. The battery current is found by dividing ε by the equivalent resistance of the circuit, which is easily found to be 6.00 Ω. Thus,

$$i_3 = \frac{1}{2} i_{\text{bat}} = \frac{1}{2}\left(\frac{12\,\text{V}}{6.0\,\Omega}\right) = 1.00\,\text{A}$$

and is clearly downward (in the figure).

(b) We use Eq. 27-17: $P = i_{\text{bat}}\varepsilon = 24.0$ W.

108. (a) Dividing Eq. 27-39 by capacitance turns it into an equation that describes the dependence of the voltage on time: $V_C = V_0\ e^{-t/\tau}$;

(b) Taking logarithms of this equation produces a form amenable to a least squares fit:

$$\ln(V_C) = -\frac{1}{\tau}\ t + \ln(V_0)$$
$$\ln(V_C) = -1.2994\ t + 2.525$$

Thus, we have the emf equal to $V_0 = e^{2.525} = 12.49\text{ V} \approx 12\text{ V}$;

(c) This also tells us that the time constant is $\tau = 1/1.2994 = 0.77$ s.

(d) Since $\tau = RC$ then we find $C = 3.8\ \mu$F.

109. Here we denote the supply emf as V (understood to be in volts). The situation is much like that shown in Fig. 27-4, with r now interpreted as the resistance of the transmission line and R interpreted as the resistance of the "consumer" (the reason the circuit has been turned on in the first place – to supply power to some resistive load R). From Eq. 27-4 and Eq. 26-27 (remembering that we are asked to find the power dissipated in the *transmission line*) we obtain

$$P_{\text{line}} = \left(\frac{V}{R+r}\right)^2 r \,.$$

Now r is considered constant, certainly, but what about R? The load will not be the same in the two cases (where V = 110000 and V′ =110) because the problem requires us to consider the *total* power supplied to be constant, so

$$P_{\text{total}} = P'_{\text{total}} \quad \Rightarrow \quad \left(\frac{V}{R+r}\right)^2 (R+r) = \left(\frac{V'}{R'+r}\right)^2 (R'+r)$$

which implies

$$1 = \frac{V^2(R'+r)}{V'^2(R+r)} \quad \Rightarrow \quad \frac{R+r}{R'+r} = \frac{V^2}{V'^2} \,.$$

Now, as the problem directs, we take ratio of P_{line} to P'_{line} and obtain

$$\frac{P_{\text{line}}}{P'_{\text{line}}} = \frac{V^2(R'+r)^2}{V'^2(R+r)^2} = \frac{V^2}{V'^2}\left(\frac{V'}{V}\right)^4 = \frac{V'^2}{V^2} = 1.00 \times 10^{-6}$$

110. The power delivered by the motor is P = (2.00 V)(0.500 m/s) = 1.00 W. From $P = i^2R_{motor}$ and $\varepsilon = i(r + R_{motor})$ we then find $i^2r - i\varepsilon + P = 0$ (which also follows directly from the conservation of energy principle). We solve for i:

$$i = \frac{\varepsilon \pm \sqrt{\varepsilon^2 - 4rP}}{2r} = \frac{2.00\,\text{V} \pm \sqrt{(2.00\,\text{V})^2 - 4(0.500\,\Omega)(1.00\,\text{W})}}{2(0.500\,\Omega)}.$$

The answer is either 3.41 A or 0.586 A.

(a) The larger i is 3.41 A.

(b) We use $V = \varepsilon - ir$ = 2.00 V – i(0.500 Ω). We substitute value of i obtained in part (a) into the above formula to get V = 0.293 V.

(c) The smaller i is 0.586 A.

(d) The corresponding V is 1.71 V.

111. (a) Placing a wire (of resistance r) with current i running directly from point a to point b in Fig. 27-61 divides the top of the picture into a left and a right triangle. If we label the currents through each resistor with the corresponding subscripts (for instance, i_s goes toward the lower right through R_s and i_x goes toward the upper right through R_x), then the currents must be related as follows:

$$i_0 = i_1 + i_s\,,\ \ i_1 = i + i_2$$
$$i_s + i = i_x\ \ ,\ \ i_2 + i_x = i_0$$

where the last relation is not independent of the previous three. The loop equations for the two triangles and also for the bottom loop (containing the battery and point b) lead to

$$i_s R_s - i_1 R_1 - ir = 0$$
$$i_2 R_s - i_x R_x - ir = 0$$
$$\varepsilon - i_0 R_0 - i_s R_s - i_x R_x = 0.$$

We incorporate the current relations from above into these loop equations in order to obtain three well-posed "simultaneous" equations, for three unknown currents (i_s, i_1 and i):

$$i_s R_s - i_1 R_1 - ir = 0$$
$$i_1 R_2 - i_s R_x - i\left(r + R_x + R_2\right) = 0$$
$$\varepsilon - i_s\left(R_0 + R_s + R_x\right) - i_1 R_0 - iR_x = 0$$

The problem statement further specifies $R_1 = R_2 = R$ and $R_0 = 0$, which causes our solution for i to simplify significantly. It becomes

$$i = \frac{\varepsilon\left(R_s - R_x\right)}{2rR_s + 2R_x R_s + R_s R + 2rR_x + R_x R}$$

which is equivalent to the result shown in the problem statement.

(b) Examining the numerator of our final result in part (a), we see that the condition for $i = 0$ is $R_s = R_x$. Since $R_1 = R_2 = R$, this is equivalent to $R_x = R_s R_2/R_1$.

Chapter 28

1. (a) The force on the electron is

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}} + B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}\\ &= \left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\,\mathrm{m/s}\right)\left(-0.15\,\mathrm{T}\right) - \left(3.0\times10^{6}\,\mathrm{m/s}\right)\left(0.030\,\mathrm{T}\right)\right]\\ &= \left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.\end{aligned}$$

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}$.

2. (a) We use Eq. 28-3:

$$F_B = |q|\, vB \sin \phi = (+\,3.2 \times 10^{-19}\ \text{C})\ (550\ \text{m/s})\ (0.045\ \text{T})\ (\sin 52°) = 6.2 \times 10^{-18}\ \text{N}.$$

(b) $a = F_B/m = (6.2 \times 10^{-18}\ \text{N}) / (6.6 \times 10^{-27}\ \text{kg}) = 9.5 \times 10^{8}\ \text{m/s}^2$.

(c) Since it is perpendicular to $\vec{v}, \vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

3. (a) Eq. 28-3 leads to

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.60\times10^{-3}\,\text{T}\right)\sin 23.0°} = 4.00\times10^{5}\,\text{m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(4.00\times10^{5}\,\text{m/s}\right)^2 = 1.34\times10^{-16}\,\text{J},$$

which is equivalent to $K = (1.34 \times 10^{-16}\ \text{J}) / (1.60 \times 10^{-19}\ \text{J/eV}) = 835\ \text{eV}$.

4. The force associated with the magnetic field must point in the $\hat{\mathrm{j}}$ direction in order to cancel the force of gravity in the $-\hat{\mathrm{j}}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\mathrm{k}}$ direction (since $\hat{\mathrm{i}} \times \left(-\hat{\mathrm{k}}\right) = \hat{\mathrm{j}}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90°$). Therefore, with $m = 1.0 \times 10^{-2}$ kg, $v = 2.0 \times 10^{4}$ m/s and $q = 8.0 \times 10^{-5}$ C, we find

$$\vec{B} = B_z \hat{\mathrm{k}} = -\left(\frac{mg}{qv}\right)\hat{\mathrm{k}} = (-0.061\ \mathrm{T})\hat{\mathrm{k}}$$

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_x B_y - v_y B_x\right)\hat{\mathrm{k}} = q\left(v_x (3B_x) - v_y B_x\right)\hat{\mathrm{k}}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{\mathrm{k}}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \quad \Rightarrow \quad B_x = \frac{F_z}{q\left(3v_x - v_y\right)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s and $q = -1.6 \times 10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times 10^{-19}\ \text{N}}{(-1.6\times 10^{-19}\,\text{C})[3(2.0\ \text{m/s}) - 4.0\ \text{m}]} = -2.0\ \text{T}.$$

6. The magnetic force on the proton is

$$\vec{F} = q\vec{v} \times \vec{B}$$

where $q = +e$. Using Eq. 3-30 this becomes

$$(4 \times 10^{-17})\hat{\mathrm{i}} + (2 \times 10^{-17})\hat{\mathrm{j}} = e[(0.03v_y + 40)\hat{\mathrm{i}} + (20 - 0.03v_x)\hat{\mathrm{j}} - (0.02v_x + 0.01v_y)\hat{\mathrm{k}}]$$

with SI units understood. Equating corresponding components, we find

(a) $v_x = -3.5\times10^3$ m/s, and

(b) $v_y = 7.0\times10^3$ m/s.

7. Straight line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$. Note that $\vec{v} \perp \vec{B}$ so $\left|\vec{v} \times \vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\ \text{V}/(20\times10^{-3}\text{m})}{\sqrt{2\left(1.0\times10^{3}\text{V}\right)\left(1.60\times10^{-19}\text{C}\right)/\left(9.11\times10^{-31}\text{kg}\right)}} = 2.67\times10^{-4}\,\text{T}.$$

In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\ \text{T})\hat{\text{k}}$.

8. Letting $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$, we get $vB\sin\phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the sin ϕ factor is at its maximum value (which is 1, corresponding to $\phi = 90°$). So

$$v_{\min} = \frac{E}{B} = \frac{1.50\times10^{3}\ \text{V/m}}{0.400\ \text{T}} = 3.75\times10^{3}\ \text{m/s}.$$

9. We apply $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m_e \vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned}
\vec{E} &= \frac{m_e \vec{a}}{q} + \vec{B} \times \vec{v} \\
&= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)\left(2.00 \times 10^{12}\ \text{m/s}^2\right)\hat{\text{i}}}{-1.60 \times 10^{-19}\,\text{C}} + \left(400\,\mu\text{T}\right)\hat{\text{i}} \times \left[\left(12.0\,\text{km/s}\right)\hat{\text{j}} + \left(15.0\,\text{km/s}\right)\hat{\text{k}}\right] \\
&= \left(-11.4\hat{\text{i}} - 6.00\hat{\text{j}} + 4.80\hat{\text{k}}\right)\text{V/m}.
\end{aligned}$$

10. (a) The net force on the proton is given by

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} = \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}} \times \left(-2.50\times10^{-3}\,\text{T}\right)\hat{\text{i}}\right]$$
$$= \left(1.44\times10^{-18}\,\text{N}\right)\hat{\text{k}}.$$

(b) In this case

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \\ &= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(-4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}} \times \left(-2.50\text{ mT}\right)\hat{\text{i}}\right] \\ &= \left(1.60\times10^{-19}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

(c) In the final case,

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \\ &= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{i}} + \left(2000\,\text{m/s}\right)\hat{\text{j}} \times \left(-2.50\text{ mT}\right)\hat{\text{i}}\right] \\ &= \left(6.41\times10^{-19}\,\text{N}\right)\hat{\text{i}} + \left(8.01\times10^{-19}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

11. Since the total force given by $\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\text{ T})\sqrt{\frac{2\left(1.60\times10^{-19}\text{C}\right)\left(10\times10^{3}\text{V}\right)}{\left(9.99\times10^{-27}\text{kg}\right)}} = 6.8\times10^{5}\text{ V/m}.$$

12. (a) The force due to the electric field ($\vec{F} = q\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F} = q\vec{v} \times \vec{B}$) in that the latter vanishes at the speed is zero and the former is independent of speed. The graph (Fig.28-36) shows that the force (y-component) is negative at v = 0 (specifically, its value is –2.0 × 10^{-19} N there) which (because q = –e) implies that the electric field points in the +y direction. Its magnitude is

$$E = \frac{F_{\text{net},y}}{|q|} = \frac{2.0\times10^{-19}\ \text{N}}{1.6\times10^{-19}\,\text{C}} = 1.25\ \text{N/C} = 1.25\ \text{V/m}\,.$$

(b) We are told that the x and z components of the force remain zero throughout the motion, implying that the electron continues to move along the x axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v$ = 2.50 × 10^{-2} T.

For $\vec{F} = q\vec{v} \times \vec{B}$ to be in the opposite direction of $\vec{F} = q\vec{E}$ we must have $\vec{v} \times \vec{B}$ in the <u>opposite</u> direction from $\vec{E}$ which points in the +y direction, as discussed in part (a). Since the velocity is in the +x direction, then (using the right-hand rule) we conclude that the magnetic field must point in the +z direction ($\hat{\text{i}} \times \hat{\text{k}} = -\hat{\text{j}}$). In unit-vector notation, we have $\vec{B} = (2.50\times10^{-2}\ \text{T})\hat{\text{k}}$.

13. We use Eq. 28-12 to solve for V:

$$V = \frac{iB}{nle} = \frac{(23\text{A})(0.65\text{ T})}{\left(8.47\times10^{28}/\text{m}^3\right)(150\,\mu\text{m})\left(1.6\times10^{-19}\text{C}\right)} = 7.4\times10^{-6}\text{ V}.$$

14. For a free charge q inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{\left|V_x - V_y\right|/d_{xy}}{B} = \frac{\left(3.90 \times 10^{-9}\,\mathrm{V}\right)}{\left(1.20 \times 10^{-3}\,\mathrm{T}\right)\left(0.850 \times 10^{-2}\,\mathrm{m}\right)} = 0.382\,\mathrm{m/s}.$$

15. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec{E}| = v|\vec{B}| = (20.0 \text{ m/s})(0.030 \text{ T}) = 0.600 \text{ V/m} .$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec{v} \times \vec{B}$. In summary,

$$\vec{E} = -(0.600 \text{ V/m})\hat{\text{k}}$$

which insures that $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$ vanishes.

(b) Eq. 28-9 yields $V = Ed = (0.600 \text{ V/m})(2.00 \text{ m}) = 1.20 \text{ V}$.

16. We note that $\vec{B}$ must be along the x axis because when the velocity is along that axis there is no induced voltage. Combining Eq. 28-7 and Eq. 28-9 leads to

$$d = \frac{V}{E} = \frac{V}{vB}$$

where one must interpret the symbols carefully to ensure that $\vec{d}, \vec{v}$ and $\vec{B}$ are mutually perpendicular. Thus, when the velocity if parallel to the y axis the absolute value of the voltage (which is considered in the same "direction" as $\vec{d}$) is 0.012 V, and

$$d = d_z = \frac{0.012 \text{ V}}{(3.0 \text{ m/s})(0.020 \text{ T})} = 0.20 \text{ m}.$$

On the other hand, when the velocity if parallel to the z-axis the absolute value of the appropriate voltage is 0.018 V, and

$$d = d_y = \frac{0.018 \text{ V}}{(3.0 \text{ m/s})(0.020 \text{ T})} = 0.30 \text{ m}.$$

Thus, our answers are

(a) d_x = 25 cm (which we arrive at "by elimination" – since we already have figured out d_y and d_z),

(b) d_y = 30 cm, and

(c) d_z = 20 cm .

17. (a) From $K = \frac{1}{2} m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20 \times 10^3 \text{eV})(1.60 \times 10^{-19} \text{ eV/J})}{9.11 \times 10^{-31} \text{kg}}} = 2.05 \times 10^7 \text{ m/s}.$$

(b) From $r = m_e v / qB$ we get

$$B = \frac{m_e v}{qr} = \frac{(9.11 \times 10^{-31} \text{kg})(2.05 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{C})(25.0 \times 10^{-2} \text{m})} = 4.67 \times 10^{-4} \text{ T}.$$

(c) The "orbital" frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07 \times 10^7 \text{ m/s}}{2\pi(25.0 \times 10^{-2} \text{m})} = 1.31 \times 10^7 \text{ Hz}.$$

(d) $T = 1/f = (1.31 \times 10^7 \text{ Hz})^{-1} = 7.63 \times 10^{-8}$ s.

18. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(350\ \mathrm{V}\right)}{9.11\times10^{-31}\,\mathrm{kg}}} = 1.11\times10^{7}\ \mathrm{m/s}.$$

(b) Eq. 28-16 gives

$$r = \frac{m_e v}{eB} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(1.11\times10^{7}\ \mathrm{m/s}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(200\times10^{-3}\,\mathrm{T}\right)} = 3.16\times10^{-4}\,\mathrm{m}.$$

19. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.30\times10^{6}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.350\ \text{m}\right)} = 2.11\times10^{-5}\ \text{T}.$$

20. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $K = (rqB)^2/2m \propto q^2 m^{-1}$.

(a) $K_\alpha = \left(q_\alpha/q_p\right)^2 \left(m_p/m_\alpha\right) K_p = (2)^2 (1/4) K_p = K_p = 1.0\,\text{MeV}$;

(b) $K_d = \left(q_d/q_p\right)^2 \left(m_p/m_d\right) K_p = (1)^2 (1/2) K_p = 1.0\text{ MeV}/2 = 0.50\,\text{MeV}$.

21. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0\times10^{-6}\,\mathrm{T})(1.60\times10^{-19}\,\mathrm{C})}{2\pi(9.11\times10^{-31}\,\mathrm{kg})} = 9.78\times10^{5}\,\mathrm{Hz}.$$

(b) Using Eq. 28-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2(9.11\times10^{-31}\,\mathrm{kg})(100\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}}{(1.60\times10^{-19}\,\mathrm{C})(35.0\times10^{-6}\,\mathrm{T})} = 0.964\,\mathrm{m}\ .$$

22. Combining Eq. 28-16 with energy conservation ($eV = \frac{1}{2} m_e v^2$ in this particular application) leads to the expression

$$r = \frac{m_e}{e\,B}\sqrt{\frac{2eV}{m_e}}$$

which suggests that the slope of the r versus $\sqrt{V}$ graph should be $\sqrt{2m_e / eB^2}$. From Fig. 28-39, we estimate the slope to be 5×10^{-5} in SI units. Setting this equal to $\sqrt{2m_e / eB^2}$ and solving we find $B = 6.7 \times 10^{-2}$ T.

23. Let ξ stand for the ratio ($m/|q|$) we wish to solve for. Then Eq. 28-17 can be written as $T = 2\pi\xi/B$. Noting that the horizontal axis of the graph (Fig. 28-40) is inverse-field ($1/B$) then we conclude (from our previous expression) that the slope of the line in the graph must be equal to $2\pi\xi$. We estimate that slope as 7.5×10^{-9} T·s, which implies

$$\xi = m/|q| = 1.2\times10^{-9} \text{ kg/C}.$$

24. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent towards the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99 \times 10^{6} \text{ m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find $r = 0.00710$ m.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9}$ s.

25. (a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\mathrm{u}} = \frac{2\left(4.50\times10^{-2}\ \mathrm{m}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(1.20\,\mathrm{T}\right)}{\left(4.00\,\mathrm{u}\right)\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)} = 2.60\times10^{6}\ \mathrm{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2}\ \mathrm{m})/(2.60 \times 10^{6}\ \mathrm{m/s}) = 1.09 \times 10^{-7}\ \mathrm{s}$.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{\left(4.00\,\mathrm{u}\right)\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)\left(2.60\times10^{6}\ \mathrm{m/s}\right)^2}{2\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)} = 1.40\times10^{5}\ \mathrm{eV}\ .$$

(d) $\Delta V = K/q = 1.40 \times 10^{5}\ \mathrm{eV}/2e = 7.00 \times 10^{4}\ \mathrm{V}$.

26. Using $F = mv^2/r$ (for the centripetal force) and $K = mv^2/2$, we can easily derive the relation

$$K = \frac{1}{2} Fr .$$

With the values given in the problem, we thus obtain $K = 2.09 \times 10^{-22}$ J.

27. Reference to Fig. 28-11 is very useful for interpreting this problem. The distance traveled parallel to $\vec{B}$ is $d_{\|} = v_{\|}T = v_{\|}(2\pi m_e/|q|B)$ using Eq. 28-17. Thus,

$$v_{\|} = \frac{d_{\|}\, e\, B}{2\,\pi\, m_e} = 50.3 \text{ km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_{\perp}$, then we find $v_{\perp} = 41.7$ km/s. The speed is therefore $v = \sqrt{v_{\perp}^2 + v_{\|}^2} = 65.3$ km/s.

28. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v} \times \vec{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton).

(a) Eq. 28-17 becomes $T = 2\pi m_{\text{p}} / e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right)=\frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $\left|\vec{B}\right| = 0.252\,\text{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns.

29. (a) If v is the speed of the positron then $v \sin \phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv \sin \phi = m_e(v \sin \phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB) \sin \phi$. The period is given by

$$T = \frac{2\pi r}{v \sin \phi} = \frac{2\pi m_e}{eB} = \frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\,\text{T}\right)} = 3.58\times10^{-10}\,\text{s}.$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT \cos \phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.00\times10^3\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = 2.65\times10^7\ \text{m/s}\,.$$

Thus,

$$p = \left(2.65\times10^7\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89° = 1.66\times10^{-4}\ \text{m}\,.$$

(c) The orbit radius is

$$R = \frac{m_e v \sin \phi}{eB} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.65\times10^7\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{T}\right)} = 1.51\times10^{-3}\ \text{m}\,.$$

30. (a) Eq. 3-20 gives $\phi = \cos^{-1}(2/19) = 84°$.

(b) No, the magnetic field can only change the direction of motion of a free (unconstrained) particle, not its speed or its kinetic energy.

(c) No, as reference to to Fig. 28-11 should make clear.

(d) We find $v_\perp = v \sin \phi = 61.3$ m/s, so $r = mv_\perp / eB = 5.7$ nm.

31. (a) We solve for B from $m = B^2qx^2/8V$ (see Sample Problem 28-3):

$$B = \sqrt{\frac{8Vm}{qx^2}} \; .$$

We evaluate this expression using $x = 2.00$ m:

$$B = \sqrt{\frac{8\left(100\times10^3\ \text{V}\right)\left(3.92\times10^{-25}\ \text{kg}\right)}{\left(3.20\times10^{-19}\ \text{C}\right)\left(2.00\ \text{m}\right)^2}} = 0.495\,\text{T} \; .$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100\times10^{-6}\ \text{kg}}{3600\,\text{s}} = 2.78\times10^{-8}\ \text{kg/s} \; .$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{\left(3.20\times10^{-19}\ \text{C}\right)\left(2.78\times10^{-8}\ \text{kg/s}\right)}{3.92\times10^{-25}\ \text{kg}} = 2.27\times10^{-2}\ \text{A} \; .$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\Delta t = iV\,\Delta t \; .$$

For $\Delta t = 1.0$ h,

$$E = \left(2.27\times10^{-2}\ \text{A}\right)\left(100\times10^3\ \text{V}\right)\left(3600\,\text{s}\right) = 8.17\times10^6\ \text{J} \; .$$

To obtain the second expression, i/q is substituted for N.

32. Eq. 28-17 gives $T = 2\pi m_e/eB$. Thus, the total time is

$$\left(\frac{T}{2}\right)_1 + t_{\text{gap}} + \left(\frac{T}{2}\right)_2 = \frac{\pi m_e}{e}\left(\frac{1}{B_1} + \frac{1}{B_2}\right) + t_{\text{gap}}\,.$$

The time spent in the gap (which is where the electron is accelerating in accordance with Eq. 2-15) requires a few steps to figure out: letting $t = t_{\text{gap}}$ then we want to solve

$$d = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; 0.25\text{ m} = \sqrt{\frac{2K_0}{m_e}}\,t + \frac{1}{2}\left(\frac{e\Delta V}{m_e d}\right)t^2$$

for t. We find in this way that the time spent in the gap is $t \approx 6$ ns. Thus, the total time is 8.7 ns.

33. Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. Therefore, using Eq. 28-17, the time is given by

$$t=\frac{T}{2}=\frac{\pi m}{Bq}=\frac{\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{(3.53\times10^{-3}\,\text{T})\left(1.60\times10^{-19}\,\text{C}\right)}=5.07\times10^{-9}\,\text{s}.$$

34. Let $v_{\parallel} = v\cos\theta$. The electron will proceed with a uniform speed $v_{\parallel}$ in the direction of $\vec{B}$ while undergoing uniform circular motion with frequency f in the direction perpendicular to B: $f = eB/2\pi m_e$. The distance d is then

$$d = v_{\parallel}T = \frac{v_{\parallel}}{f} = \frac{(v\cos\theta)2\pi m_e}{eB} = \frac{2\pi\left(1.5\times10^{7}\ \text{m/s}\right)\left(9.11\times10^{-31}\,\text{kg}\right)\left(\cos10^\circ\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{-3}\,\text{T}\right)} = 0.53\text{m}.$$

35. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6 \text{ eV}}{2\left(80 \times 10^3 \text{eV}\right)} = 104 \ .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} \ .$$

For the average energy

$$r = \frac{\sqrt{2\left(8.3 \times 10^6 \text{ eV}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)\left(3.34 \times 10^{-27} \text{ kg}\right)}}{\left(1.60 \times 10^{-19} \text{ C}\right)\left(1.57 \text{ T}\right)} = 0.375 \text{ m} \ .$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2 \text{ m}.$$

36. (a) Using Eq. 28-23 and Eq. 28-18, we find

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)} = 1.83\times10^{7}\,\text{Hz}.$$

(b) From $r = m_p v/qB = \sqrt{2m_P k}\,/qB$ we have

$$K = \frac{(rqB)^2}{2m_p} = \frac{\left[\left(0.500\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\,\text{T}\right)\right]^2}{2\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.60\times10^{-19}\,\text{ J/eV}\right)} = 1.72\times10^{7}\,\text{eV}.$$

37. (a) By conservation of energy (using qV for the potential energy which is converted into kinetic form) the kinetic energy gained in each pass is 200 eV.

(b) Multiplying the part (a) result by $n = 100$ gives $\Delta K = n(200 \text{ eV}) = 20.0$ keV.

(c) Combining Eq. 28-16 with the kinetic energy relation ($n(200 \text{ eV}) = m_p v^2/2$ in this particular application) leads to the expression

$$r = \frac{m_p}{e\,B}\sqrt{\frac{2n(200 \text{ eV})}{m_p}}\,.$$

which shows that r is proportional to $\sqrt{n}$. Thus, the percent increase defined in the problem in going from $n = 100$ to $n = 101$ is $\sqrt{101/100} - 1 = 0.00499$ or 0.499%.

38. (a) The magnitude of the field required to achieve resonance is

$$B=\frac{2\pi f m_p}{q}=\frac{2\pi(12.0\times10^6\ \text{Hz})\left(1.67\times10^{-27}\,\text{kg}\right)}{1.60\times10^{-19}\,\text{C}}=0.787\,\text{T}.$$

(b) The kinetic energy is given by

$$K=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\text{ m})^2(12.0\times10^6\ \text{Hz})^2$$
$$=1.33\times10^{-12}\,\text{J}=8.34\times10^6\,\text{eV}.$$

(c) The required frequency is

$$f=\frac{qB}{2\pi m_p}=\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.57\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)}=2.39\times10^7\,\text{Hz}.$$

(d) The kinetic energy is given by

$$K=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\text{ m})^2(2.39\times10^7\ \text{Hz})^2$$
$$=5.3069\times10^{-12}\,\text{J}=3.32\times10^7\,\text{eV}.$$

39. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \quad \Rightarrow \quad i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

40. (a) From symmetry, we conclude that any x-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{k}$ direction produces on each part of the bent wire a y-component of force pointing in the $-\hat{j}$ direction; each of these components has magnitude

$$|F_y| = i\ell\,|\vec{B}|\sin 30° = (2.0\text{ A})(2.0\text{ m})(4.0\text{ T})\sin 30° = 8\,\text{N}.$$

Therefore, the force on the wire shown in the figure is $(-16\hat{j})\text{ N}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

41. (a) The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin \phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus,

$$F_B = (5000\,\text{A})(100\,\text{m})(60.0 \times 10^{-6}\,\text{T})\sin 70° = 28.2\,\text{N}\ .$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

42. The magnetic force on the (straight) wire is

$$F_B = iBL \sin\theta = (13.0\,\mathrm{A})\,(1.50\,\mathrm{T})\,(1.80\,\mathrm{m})\,(\sin 35.0°) = 20.1\,\mathrm{N}.$$

43. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\mathrm{i}}\times\left(B_y\hat{\mathrm{j}}+B_z\hat{\mathrm{k}}\right) = iL\left(-B_z\hat{\mathrm{j}}+B_y\hat{\mathrm{k}}\right)\\ &= (0.500\,\mathrm{A})\,(0.500\,\mathrm{m})\left[-(0.0100\,\mathrm{T})\hat{\mathrm{j}}+(0.00300\,\mathrm{T})\hat{\mathrm{k}}\right]\\ &= \left(-2.50\times10^{-3}\hat{\mathrm{j}}+0.750\times10^{-3}\,\hat{\mathrm{k}}\right)\mathrm{N}.\end{aligned}$$

44. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13\times10^{-3}\,\text{A})(2.56\times10^{-2}\,\text{m})(5.63\times10^{-2}\,\text{T})(0.0611\,\text{s})}{2.41\times10^{-5}\,\text{kg}}$$
$$= 3.34\times10^{-2}\,\text{m/s}.$$

(b) The direction is to the left (away from the generator).

45. (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = iLB_d, \quad F_y = iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s\left(mg - iLB_w\right).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \quad \Rightarrow \quad iLB_d = \mu_s\left(mg - iLB_w\right).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\left(mg - iLB\sin\theta\right) \quad \Rightarrow \quad B = \frac{\mu_s mg}{iL\left(\cos\theta + \mu_s\sin\theta\right)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.60\right) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60\left(1.0\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)}{\left(50\,\text{A}\right)\left(1.0\,\text{m}\right)\left(\cos 31° + 0.60\sin 31°\right)} = 0.10\,\text{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.60\right) = 31°$.

46. We use $d\vec{F}_B = id\vec{L} \times \vec{B}$, where $d\vec{L} = dx\hat{\mathrm{i}}$ and $\vec{B} = B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}}$. Thus,

$$\begin{aligned}\vec{F}_B &= \int id\vec{L} \times \vec{B} = \int_{x_i}^{x_f} idx\hat{\mathrm{i}} \times \left(B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}}\right) = i\int_{x_i}^{x_f} B_y\, dx\hat{\mathrm{k}} \\ &= \left(-5.0\,\mathrm{A}\right)\left(\int_{1.0}^{3.0}\left(8.0x^2\, dx\right)\left(\mathrm{m}\cdot\mathrm{mT}\right)\right)\hat{\mathrm{k}} = (-0.35\,\mathrm{N})\hat{\mathrm{k}}.\end{aligned}$$

47. The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight-segment of the rectangular coil, we note that Eq. 28-26 produces a non-zero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by N = 20 due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight-segment of the coil which lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight-segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight-segment located at x = 0.050 m, which has length L = 0.10 m and is shown in Figure 28-47 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight-segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where B = 0.50 T and θ = 30°) produces a force equal to $NiLB_x$ which points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)(x) = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30°$$
$$= 0.0043\text{ N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times 10^{-3}\text{ N}\cdot\text{m})\hat{\text{j}}$

An alternative way to do this problem is through the use of Eq. 28-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 28-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\text{ A})(0.0050\text{ m}^2)$$

and points in the $-z$ direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

48. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the $+y$ axis, while the $\ell_x = 120$ cm side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is

$$\theta = \tan^{-1}(50/120) = 22.6°,$$

relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is 180° – 22.6° = +157°. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750\,\mathrm{T}$,

$$B_x = -B\cos\theta = -0.0692\,\mathrm{T}\ , \quad B_y = B\sin\theta = 0.0288\,\mathrm{T}.$$

(a) Eq. 28-26 produces zero force when $\vec{L}\,||\,\vec{B}$ so there is no force exerted on the hypotenuse of length 130 cm.

(b) On the 50 cm side, the B_x component produces a force $i\ell_y B_x\hat{\mathrm{k}}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00\,\mathrm{A})(0.500\,\mathrm{m})(0.0692\,\mathrm{T}) = 0.138\,\mathrm{N}.$$

(c) On the 120 cm side, the B_y component produces a force $i\ell_x B_y\hat{\mathrm{k}}$, and there is no contribution from the B_x component. The magnitude of the force on the ℓ_x side is also

$$(4.00\,\mathrm{A})(1.20\,\mathrm{m})(0.0288\,\mathrm{T}) = 0.138\,\mathrm{N}.$$

(d) The net force is

$$i\ell_y B_x\hat{\mathrm{k}} + i\ell_x B_y\hat{\mathrm{k}} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$ but $B_y < 0$ and a zero net force would still be the result.

49. Consider an infinitesimal segment of the loop, of length *ds*. The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude

$$dF_h = (iB\cos\theta)ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_y = (iB\sin\theta)ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$\begin{aligned} F_v &= iB\sin\theta\int ds = 2\pi aiB\sin\theta = 2\pi(0.018\text{ m})(4.6\times10^{-3}\text{ A})(3.4\times10^{-3}\text{ T})\sin 20° \\ &= 6.0\times10^{-7}\text{ N}. \end{aligned}$$

We note that *i, B*, and θ have the same value for every segment and so can be factored from the integral.

50. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is $x = 4$ cm (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of x must be ¼ of 8 cm; that is, $x = 2$ cm when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020 \text{ m})^2 = 0.00040 \text{ m}^2$. Since $N = 1$ and the torque in this case is given as 4.8×10^{-4} N·m, then the aforementioned equations lead immediately to $i = 0.0030$ A.

51. (a) The current in the galvanometer should be 1.62 mA when the potential difference across the resistor-galvanometer combination is 1.00 V. The potential difference across the galvanometer alone is

$$iR_g = \left(1.62\times10^{-3}\ \text{A}\right)\left(75.3\,\Omega\right) = 0.122\ \text{V},$$

so the resistor must be in series with the galvanometer and the potential difference across it must be 1.00 V – 0.122 V = 0.878V. The resistance should be

$$R = \left(0.878\ \text{V}\right)\big/\left(1.62\times10^{-3}\ \text{A}\right) = 542\,\Omega.$$

(b) As stated above, the resistor is in series with the galvanometer.

(c) The current in the galvanometer should be 1.62 mA when the total current in the resistor and galvanometer combination is 50.0 mA. The resistor should be in parallel with the galvanometer, and the current through it should be 50.0 mA – 1.62 mA = 48.38 mA. The potential difference across the resistor is the same as that across the galvanometer, 0.122 V, so the resistance should be $R = \left(0.122\ \text{V}\right)\big/\left(48.38\times10^{-3}\ \text{A}\right) = 2.52\,\Omega.$

(d) As stated in (c), the resistor is in parallel with the galvanometer.

52. We use $\tau_{\max} = |\vec{\mu} \times \vec{B}|_{\max} = \mu B = i\pi r^2 B,$ and note that $i = qf = qv/2\pi r$. So

$$\tau_{\max} = \left(\frac{qv}{2\pi r}\right)\pi r^2 B = \frac{1}{2}qvrB = \frac{1}{2}(1.60\times10^{-19}\,\text{C})(2.19\times10^{6}\,\text{m/s})(5.29\times10^{-11}\,\text{m})(7.10\times10^{-3}\,\text{T})$$
$$= 6.58\times10^{-26}\,\text{N}\cdot\text{m}.$$

53. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity *mg*, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction *f*, acting up the incline at the point of contact. We take the *x* axis to be positive down the incline. Then the *x* component of Newton's second law for the center of mass yields

$$mg \sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B \sin\theta$, and the force of friction produces a torque with magnitude *fr*, where *r* is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B \sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate *f* from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length *L* and two of length 2*r*, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$

54. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2$.

(b) The torque is

$$\tau = \left|\vec{\mu}\times\vec{B}\right| = \mu B\sin\theta = \left(0.184\ \text{A}\cdot\text{m}^2\right)\left(12.0\,\text{T}\right)\sin 41.0° = 1.45\,\text{N}\cdot\text{m}.$$

55. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\mathrm{A\cdot m^2}}{(160)(\pi)(0.0190\,\mathrm{m})^2} = 12.7\,\mathrm{A}.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{\max} = \mu B = \left(2.30\ \mathrm{A\cdot m^2}\right)\left(35.0\times10^{-3}\ \mathrm{T}\right) = 8.05\times10^{-2}\ \mathrm{N\cdot m}.$$

56. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\ \text{J/T}}{\pi\left(3500 \times 10^3\ \text{m}\right)^2} = 2.08 \times 10^9\ \text{A}.$$

57. (a) The area of the loop is $A = \frac{1}{2}(30\,\text{cm})(40\,\text{cm}) = 6.0 \times 10^2\,\text{cm}^2$, so

$$\mu = iA = (5.0\,\text{A})\left(6.0 \times 10^{-2}\,\text{m}^2\right) = 0.30\,\text{A} \cdot \text{m}^2 .$$

(b) The torque on the loop is

$$\tau = \mu B \sin\theta = \left(0.30\,\text{A} \cdot \text{m}^2\right)\left(80 \times 10^3\,\text{T}\right)\sin 90° = 2.4 \times 10^{-2}\,\text{N} \cdot \text{m}.$$

58. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - \left(-\mu B\cos 0°\right).$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77°.$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta = 77°$ on the other side of the alignment axis.

59. (a) The magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi (7.00\,\text{A})\left[(0.200\,\text{m})^2 + (0.300\,\text{m})^2\right] = 2.86\,\text{A}\cdot\text{m}^2.$$

(b) Now,

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi (7.00\,\text{A})\left[(0.300\,\text{m})^2 - (0.200\,\text{m})^2\right] = 1.10\,\text{A}\cdot\text{m}^2.$$

60. Eq. 28-39 gives $U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi$, so at $\phi = 0$ (corresponding to the lowest point on the graph in Fig. 28-52) the mechanical energy is

$$K + U = K_o + (-\mu B) = 6.7 \times 10^{-4}\ \text{J} + (-5 \times 10^{-4}\ \text{J}) = 1.7 \times 10^{-4}\ \text{J}.$$

The turning point occurs where $K = 0$, which implies $U_{\text{turn}} = 1.7 \times 10^{-4}$ J. So the angle where this takes place is given by

$$\phi = -\cos^{-1}\left(\frac{1.7 \times 10^{-4}\ \text{J}}{\mu B}\right) = 110°$$

where we have used the fact (see above) that $\mu B = 5 \times 10^{-4}$ J.

61. The magnetic dipole moment is $\vec{\mu} = \mu\left(0.60\,\hat{\mathrm{i}} - 0.80\,\hat{\mathrm{j}}\right)$, where

$$\mu = NiA = Ni\pi r^2 = 1(0.20\text{ A})\pi(0.080\text{ m})^2 = 4.02 \times 10^{-4}\text{ A}\cdot\text{m}^2.$$

Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} = \vec{\mu} \times \vec{B} &= \mu\left(0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}\right) \times \left(0.25\hat{\mathrm{i}} + 0.30\hat{\mathrm{k}}\right) \\ &= \mu\left[(0.60)(0.30)\left(\hat{\mathrm{i}} \times \hat{\mathrm{k}}\right) - (0.80)(0.25)\left(\hat{\mathrm{j}} \times \hat{\mathrm{i}}\right) - (0.80)(0.30)\left(\hat{\mathrm{j}} \times \hat{\mathrm{k}}\right)\right] \\ &= \mu\left[-0.18\hat{\mathrm{j}} + 0.20\hat{\mathrm{k}} - 0.24\hat{\mathrm{i}}\right].\end{aligned}$$

Here $\hat{\mathrm{i}} \times \hat{\mathrm{k}} = -\hat{\mathrm{j}}, \hat{\mathrm{j}} \times \hat{\mathrm{i}} = -\hat{\mathrm{k}},$ and $\hat{\mathrm{j}} \times \hat{\mathrm{k}} = \hat{\mathrm{i}}$ are used. We also use $\hat{\mathrm{i}} \times \hat{\mathrm{i}} = 0$. Now, we substitute the value for μ to obtain

$$\vec{\tau} = \left(-9.7\times10^{-4}\,\hat{\mathrm{i}} - 7.2\times10^{-4}\,\hat{\mathrm{j}} + 8.0\times10^{-4}\,\hat{\mathrm{k}}\right)\mathrm{N\cdot m}.$$

(b) The potential energy of the dipole is given by

$$\begin{aligned}U = -\vec{\mu}\cdot\vec{B} &= -\mu\left(0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}\right)\cdot\left(0.25\hat{\mathrm{i}} + 0.30\hat{\mathrm{k}}\right) \\ &= -\mu(0.60)(0.25) = -0.15\mu = -6.0\times10^{-4}\,\mathrm{J}.\end{aligned}$$

Here $\hat{\mathrm{i}}\cdot\hat{\mathrm{i}} = 1, \hat{\mathrm{i}}\cdot\hat{\mathrm{k}} = 0, \hat{\mathrm{j}}\cdot\hat{\mathrm{i}} = 0,$ and $\hat{\mathrm{j}}\cdot\hat{\mathrm{k}} = 0$ are used.

62. Let a = 30.0 cm, b = 20.0 cm, and c = 10.0 cm. From the given hint, we write

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 = iab\left(-\hat{\mathrm{k}}\right) + iac\left(\hat{\mathrm{j}}\right) = ia\left(c\hat{\mathrm{j}} - b\hat{\mathrm{k}}\right) = (5.00\,\mathrm{A})(0.300\,\mathrm{m})\left[(0.100\,\mathrm{m})\,\hat{\mathrm{j}} - (0.200\,\mathrm{m})\,\hat{\mathrm{k}}\right]$$
$$= \left(0.150\hat{\mathrm{j}} - 0.300\hat{\mathrm{k}}\right)\mathrm{A}\cdot\mathrm{m}^2.$$

63. If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi\left(L/2\pi N\right)^2 = L^2/4\pi N^2$.

(a) For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular (i.e., at a $90°$ angle) to the field.

(b) The magnitude of the torque is then

$$\tau = NiAB = \left(Ni\right)\left(\frac{L^2}{4\pi N^2}\right)B = \frac{iL^2 B}{4\pi N}.$$

To maximize the torque, we take the number of turns N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

(c) The magnitude of the maximum torque is

$$\tau = \frac{iL^2 B}{4\pi} = \frac{(4.51\times10^{-3}\ \text{A})(0.250\ \text{m})^2(5.71\times10^{-3}\,\text{T})}{4\pi} = 1.28\times10^{-7}\ \text{N}\cdot\text{m}$$

64. Looking at the point in the graph (Fig. 28-54(b)) corresponding to $i_2 = 0$ (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0\times10^{-5}\ \text{A}\cdot\text{m}^2$. Looking at the point where the line crosses the axis (at $i_2 = 5.0$ mA) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0\times10^{-5}\ \text{A}\cdot\text{m}^2$ when $i_2 = 0.0050\ \text{A}$ which means (Eq. 28-35)

$$N_2A_2 = \frac{\mu_2}{i_2} = \frac{2.0\times10^{-5}\ \text{A}\cdot\text{m}^2}{0.0050\ \text{A}} = 4.0\times10^{-3}\ \text{m}^2 .$$

Now the problem has us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions – from coil 1 and coil 2 – specifically for the case that $i_2 = 0.007$ A. We find that total moment is

$$\mu = (2.0 \times 10^{-5}\ \text{A·m}^2) + (N_2A_2\ i_2) = 4.8 \times 10^{-5}\ \text{A·m}^2.$$

65. (a) Using Eq. 28-35 and Figure 28-23, we have

$$\vec{\mu} = (NiA)(-\hat{\mathrm{j}}) = -(0.0240\ \mathrm{A \cdot m^2})\hat{\mathrm{j}}.$$

Then, Eq. 28-38 gives

$$U = -\vec{\mu} \cdot \vec{B} = -(-0.0240\ \mathrm{A \cdot m^2})(-3.00 \times 10^{-3}\ \mathrm{T}) = -7.20 \times 10^{-5}\ \mathrm{J}.$$

(b) Using the fact that $\hat{\mathrm{j}} \times \hat{\mathrm{j}} = 0$, Eq. 28-37 leads to

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = (-0.0240\hat{\mathrm{j}}) \times (2.00 \times 10^{-3}\,\hat{\mathrm{i}}) + (-0.0240\hat{\mathrm{j}}) \times (-4.00 \times 10^{-3}\,\hat{\mathrm{k}}) \\ &= (4.80 \times 10^{-5}\,\hat{\mathrm{k}} + 9.60 \times 10^{-5}\,\hat{\mathrm{i}})\ \mathrm{N \cdot m}.\end{aligned}$$

66. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{\mathrm{j}}$. The (infinitesimal) force on this element is

$$d\vec{F} = i\,d\ell\left(-\hat{\mathrm{j}}\right)\times\left(0.3y\hat{\mathrm{i}} + 0.4y\hat{\mathrm{j}}\right)$$

with SI units (and 3 significant figures) understood. Since $\hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}}$ and $\hat{\mathrm{j}}\times\hat{\mathrm{j}} = 0$, we obtain

$$d\vec{F} = 0.3iy\,d\ell\ \hat{\mathrm{k}} = \left(6.00\times10^{-4}\ \mathrm{N/m^2}\right)y\,d\ell\ \hat{\mathrm{k}}.$$

We integrate the force element found above, using the symbol ξ to stand for the coefficient 6.00×10^{-4} N/m^2, and obtain

$$\vec{F} = \int d\vec{F} = \xi\,\hat{\mathrm{k}}\int_0^{0.25} y\,dy = \xi\,\hat{\mathrm{k}}\left(\frac{0.25^2}{2}\right) = (1.88\times10^{-5}\ \mathrm{N})\hat{\mathrm{k}}\ .$$

67. The period of revolution for the iodine ion is $T = 2\pi r/v = 2\pi m/Bq$, which gives

$$m = \frac{BqT}{2\pi} = \frac{\left(45.0\times 10^{-3}\,\mathrm{T}\right)\left(1.60\times 10^{-19}\,\mathrm{C}\right)\left(1.29\times 10^{-3}\,\mathrm{s}\right)}{(7)(2\pi)\left(1.66\times 10^{-27}\,\mathrm{kg/u}\right)} = 127\,\mathrm{u}.$$

68. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 28-3,

$$F_{B,\max} = |q|\, vB \sin(90°) = ev\,B = (1.60 \times 10^{-19}\ \text{C})\,(7.20 \times 10^{6}\ \text{m/s})\,(83.0 \times 10^{-3}\ \text{T})$$
$$= 9.56 \times 10^{-14}\ \text{N}.$$

(b) The smallest value occurs if they are parallel: $F_{B,\min} = |q|\, vB \sin(0) = 0$.

(c) By Newton's second law, $a = F_B/m_e = |q|\, vB \sin\theta / m_e$, so the angle θ between $\vec{v}$ and $\vec{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{|q| vB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\,\text{kg})(4.90\times10^{14}\ \text{m/s}^2)}{(1.60\times10^{-16}\,\text{C})(7.20\times10^{6}\ \text{m/s})(83.0\times10^{-3}\,\text{T})}\right] = 0.267°.$$

69. (a) We use $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since $\vec{B}$ points towards the one-hour (or "5-minute'') mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B \sin 90° = NiAB = \pi N i r^2 B = 6\pi(2.0\,\mathrm{A})(0.15\,\mathrm{m})^2 (70\times10^{-3}\,\mathrm{T})$$
$$= 5.9\times10^{-2}\,\mathrm{N\cdot m}.$$

70. (a) We use Eq. 28-10: $v_d = E/B = (10 \times 10^{-6}\text{V}/1.0 \times 10^{-2}\text{ m})/(1.5\text{ T}) = 6.7 \times 10^{-4}$ m/s.

(b) We rewrite Eq. 28-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

which we use $A = \ell d$. In this experiment, $A = (0.010\text{ m})(10 \times 10^{-6}\text{ m}) = 1.0 \times 10^{-7}\text{ m}^2$. By Eq. 28-10, v_d equals the ratio of the fields (as noted in part (a)), so we are led to

$$n = \frac{Bi}{E\ Ae} = \frac{i}{v_d Ae} = \frac{3.0\,\text{A}}{\left(6.7\times10^{-4}\text{ m/s}\right)\left(1.0\times10^{-7}\text{m}^2\right)\left(1.6\times10^{-19}\text{ C}\right)} = 2.8\times10^{29}/\text{m}^3 .$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north, south, east, west* and vertical *up* and *down* directions. We assume $\vec{B}$ points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage which becomes established).

71. From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x = \sqrt{8Vm/B^2q}$, which we substitute into the expression for Δm to obtain

$$\Delta m = \left(\frac{B^2q}{8V}\right)2\sqrt{\frac{8mV}{B^2q}}\Delta x = B\sqrt{\frac{mq}{2V}}\Delta x .$$

Thus, the distance between the spots made on the photographic plate is

$$\begin{aligned}\Delta x &= \frac{\Delta m}{B}\sqrt{\frac{2V}{mq}} \\ &= \frac{(37\,\mathrm{u}-35\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})}{0.50\,\mathrm{T}}\sqrt{\frac{2(7.3\times10^{3}\,\mathrm{V})}{(36\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})(1.60\times10^{-19}\,\mathrm{C})}} \\ &= 8.2\times10^{-3}\,\mathrm{m}.\end{aligned}$$

72. (a) Equating the magnitude of the electric force ($F_e = eE$) with that of the magnetic force (Eq. 28-3), we obtain $B = E / v \sin \phi$. The field is smallest when the $\sin \phi$ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.5\times 10^3\,\text{eV}\right)\left(1.60\times 10^{-19}\ \text{J/eV}\right)}{9.11\times 10^{-31}\,\text{kg}}} = 2.96\times 10^7\ \text{m/s}.$$

Thus,

$$B = \frac{E}{v} = \frac{10\times 10^3\ \text{V/m}}{2.96\times 10^7\ \text{m/s}} = 3.4\times 10^{-4}\,\text{T}.$$

The direction of the magnetic field must be perpendicular to both the electric field ($-\hat{\mathrm{j}}$) and the velocity of the electron ($+\hat{\mathrm{i}}$). Since the electric force $\vec{F}_e = (-e)\vec{E}$ points in the $+\hat{\mathrm{j}}$ direction, the magnetic force $\vec{F}_B = (-e)\vec{v}\times\vec{B}$ points in the $-\hat{\mathrm{j}}$ direction. Hence, the direction of the magnetic field is $-\hat{\mathrm{k}}$. In unit-vector notation, $\vec{B} = (-3.4\times 10^{-4}\,\text{T})\hat{\mathrm{k}}$.

73. The fact that the fields are uniform, with the feature that the charge moves in a straight line, implies the speed is constant (if it were not, then the magnetic *force* would vary while the electric force could not — causing it to deviate from straight-line motion). This is then the situation leading to Eq. 28-7, and we find

$$|\vec{E}| = v|\vec{B}| = 500\,\text{V/m}.$$

Its direction (so that $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$ vanishes) is downward, or $-\hat{\text{j}}$, in the "page" coordinates. In unit-vector notation, $\vec{E} = (-500\text{ V/m})\hat{\text{j}}$

74. (a) For the magnetic field to have an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 28-16. The kinetic energy of the electron is $K = \frac{1}{2} m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2 B^2}} \ .$$

This must be less than d, so $\sqrt{\frac{2m_e K}{e^2 B^2}} \le d$, or $B \ge \sqrt{\frac{2m_e K}{e^2 d^2}}$.

(b) If the electrons are to travel as shown in Fig. 28-57, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

75. (a) Since $K = qV$ we have $K_p = \frac{1}{2}K_\alpha\left(\text{as } q_\alpha = 2K_p\right)$, or $K_p / K_\alpha = 0.50$.

(b) Similarly, $q_\alpha = 2K_d$, $K_d / K_\alpha = 0.50$.

(c) Since $r = \sqrt{2mK}\big/qB \propto \sqrt{mK}\big/q$, we have

$$r_d = \sqrt{\frac{m_d K_d}{m_p K_p}}\frac{q_p r_p}{q_d} = \sqrt{\frac{(2.00\text{u})K_p}{(1.00\text{u})K_p}}r_p = 10\sqrt{2}\,\text{cm} = 14\text{cm}.$$

(d) Similarly, for the alpha particle, we have

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\frac{q_p r_p}{q_\alpha} = \sqrt{\frac{(4.00\text{u})K_\alpha}{(1.00\text{u})\left(K_\alpha/2\right)}}\frac{er_p}{2e} = 10\sqrt{2}\text{cm} = 14\text{cm}.$$

76. The equation of motion for the proton is

$$\vec{F} = q\vec{v} \times \vec{B} = q\left(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}\right) \times B\hat{\mathrm{i}} = qB\left(v_z\hat{\mathrm{j}} - v_y\hat{\mathrm{k}}\right)$$
$$= m_p\vec{a} = m_p\left[\left(\frac{dv_x}{dt}\right)\hat{\mathrm{i}} + \left(\frac{dv_y}{dt}\right)\hat{\mathrm{j}} + \left(\frac{dv_z}{dt}\right)\hat{\mathrm{k}}\right].$$

Thus,

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = \omega v_z, \quad \frac{dv_z}{dt} = -\omega v_y,$$

where $\omega = eB/m$. The solution is $v_x = v_{0x}$, $v_y = v_{0y}\cos\omega t$ and $v_z = -v_{0y}\sin\omega t$. In summary, we have

$$\vec{v}(t) = v_{0x}\hat{\mathrm{i}} + v_{0y}\cos(\omega t)\hat{\mathrm{j}} - v_{0y}(\sin\omega t)\hat{\mathrm{k}}.$$

77. By the right-hand rule, we see that $\vec{v} \times \vec{B}$ points along $-\vec{\mathrm{k}}$. From Eq. 28-2 $\left(\vec{F} = q\vec{v} \times \vec{B}\right)$, we find that for the force to point along $+\hat{\mathrm{k}}$, we must have $q < 0$. Now, examining the magnitudes in Eq. 28-3, we find $|\vec{F}| = |q|v|\vec{B}|\sin\phi$, or

$$0.48 \text{ N} = |q|(4000 \text{ m/s})\,(0.0050 \text{ T})\sin 35°$$

which yields $|q| = 0.040$ C. In summary, then, $q = -40$ mC.

78. Using Eq. 28-16, the charge-to-mass ratio is $\frac{q}{m}=\frac{v}{B'r}$. With the speed of the ion giving by $v=E/B$ (using Eq. 28-7), the expression becomes

$$\frac{q}{m}=\frac{E/B}{B'r}=\frac{E}{BB'r}.$$

79. (a) We use Eq. 28-2 and Eq. 3-30:

$$
\begin{aligned}
\vec{F} = q\vec{v}\times\vec{B} &= (+e)\left(\left(v_yB_z - v_zB_y\right)\hat{\mathrm{i}} + \left(v_zB_x - v_xB_z\right)\hat{\mathrm{j}} + \left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}\right) \\
&= \left(1.60\times10^{-19}\right)\left(\left((4)(0.008) - (-6)\,(-0.004)\right)\hat{\mathrm{i}} + \right. \\
&\quad \left. \left((-6)(0.002) - (-2)\,(0.008)\right)\hat{\mathrm{j}} + \left((-2)(-0.004) - (4)(0.002)\right)\hat{\mathrm{k}}\right) \\
&= \left(1.28\times10^{-21}\right)\hat{\mathrm{i}} + \left(6.41\times10^{-22}\right)\hat{\mathrm{j}}
\end{aligned}
$$

with SI units understood.

(b) By definition of the cross product, $\vec{v}\perp\vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$
\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{B}}{|\vec{v}\,||\,\vec{B}\,|}\right) = \cos^{-1}\left(\frac{-68}{\sqrt{56}\sqrt{84}}\right) = 173°
$$

80. (a) In Chapter 27, the electric field (called E_C in this problem) which "drives" the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho n e v_d .$$

Now, regarding the Hall effect, we use Eq. 28-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_c = B/ne\rho$.

(b) Using the value of copper's resistivity given in Chapter 26, we obtain

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\ \text{T}}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)} = 2.84\times10^{-3}.$$

81. (a) The textbook uses "geomagnetic north" to refer to Earth's magnetic pole lying in the northern hemisphere. Thus, the electrons are traveling northward. The vertical component of the magnetic field is downward. The right-hand rule indicates that $\vec{v} \times \vec{B}$ is to the west, but since the electron is negatively charged (and $\vec{F} = q\vec{v} \times \vec{B}$), the magnetic force on it is to the east.

We combine $F = m_e a$ with $F = evB \sin \phi$. Here, $B \sin \phi$ represents the downward component of Earth's field (given in the problem). Thus, $a = evB / m_e$. Now, the electron speed can be found from its kinetic energy. Since $K = \frac{1}{2}mv^2$,

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(12.0 \times 10^3 \text{eV}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)}{9.11 \times 10^{-31} \text{kg}}} = 6.49 \times 10^7 \text{ m/s}.$$

Therefore,

$$a = \frac{evB}{m_e} = \frac{\left(1.60\times10^{-19}\text{C}\right)\left(6.49\times10^7 \text{ m/s}\right)\left(55.0\times10^{-6}\text{T}\right)}{9.11\times10^{-31}\text{kg}} = 6.27\times10^{14} \text{ m/s}^2 \approx 6.3\times10^{14} \text{ m/s}^2 .$$

(b) We ignore any vertical deflection of the beam which might arise due to the horizontal component of Earth's field. Technically, then, the electron should follow a circular arc. However, the deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$\Delta x = vt \quad \Rightarrow \quad t = \frac{\Delta x}{v} = \frac{0.200\text{m}}{6.49\times10^7 \text{ m/s}} = 3.08\times10^{-9} \text{ s}.$$

Then, with our y axis oriented eastward,

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}\left(6.27\times10^{14}\right)\left(3.08\times10^{-9}\right)^2 = 0.00298\text{m} \approx 0.0030 \text{ m}.$$

82. (a) We are given $\vec{B} = B_x\hat{\mathrm{i}} = (6\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$, so that $\vec{v}\times\vec{B} = -v_y B_x\hat{\mathrm{k}}$ where $v_y = 4\times10^4$ m/s. We note that the magnetic force on the electron is $(-e)\left(-v_y B_x\hat{\mathrm{k}}\right)$ and therefore points in the $+\hat{\mathrm{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 28-16 becomes

$$r = \frac{m_e v_y}{e\,B_x} = 0.0038\,\mathrm{m}.$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60\ \mu$s, and during that time the "drift" of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_x T = 0.019$ m where $v_x = 32\times10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call "upward'') but "pushed" in the $+z$ direction (to his right). Hence, he describes the electron's spiral as clockwise.

83. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $r \propto \sqrt{mK}/qB$.

(a) $\dfrac{r_d}{r_p} = \sqrt{\dfrac{m_d K_d}{m_p K_p}}\dfrac{q_p}{q_d} = \sqrt{\dfrac{2.0\text{u}}{1.0\text{u}}}\dfrac{e}{e} = \sqrt{2} \approx 1.4$, and

(b) $\dfrac{r_\alpha}{r_p} = \sqrt{\dfrac{m_\alpha K_\alpha}{m_p K_p}}\dfrac{q_p}{q_\alpha} = \sqrt{\dfrac{4.0\text{u}}{1.0\text{u}}}\dfrac{e}{2e} = 1.0.$

84. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 28-2 ($\vec{F} = q\vec{v} \times \vec{B}$) and Eq. 3-30, we obtain (with SI units understood)

$$4\hat{\mathrm{i}} - 20\hat{\mathrm{j}} + 12\hat{\mathrm{k}} = 2\Big((4B_2 - 6B_1)\,\hat{\mathrm{i}} + (6B_1 - 2B_2)\,\hat{\mathrm{j}} + (2B_1 - 4B_1)\,\hat{\mathrm{k}}\Big).$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary,

$$\vec{B} = \left(-3.0\hat{\mathrm{i}} - 3.0\hat{\mathrm{j}} - 4.0\hat{\mathrm{k}}\right)\mathrm{T}.$$

85. The contribution to the force by the magnetic field $\left(\vec{B} = B_x\hat{\mathrm{i}} = (-0.020\ \mathrm{T})\hat{\mathrm{i}}\right)$ is given by Eq. 28-2:

$$\begin{aligned}\vec{F}_B &= q\vec{v} \times \vec{B} = q\left(\left(17000\hat{\mathrm{i}} \times B_x\hat{\mathrm{i}}\right) + \left(-11000\hat{\mathrm{j}} \times B_x\hat{\mathrm{i}}\right) + \left(7000\hat{\mathrm{k}} \times B_x\hat{\mathrm{i}}\right)\right) \\ &= q\left(-220\hat{\mathrm{k}} - 140\hat{\mathrm{j}}\right)\end{aligned}$$

in SI units. And the contribution to the force by the electric field $\left(\vec{E} = E_y\hat{\mathrm{j}} = 300\hat{\mathrm{j}}\ \mathrm{V/m}\right)$ is given by Eq. 23-1: $\vec{F}_E = qE_y\hat{\mathrm{j}}$. Using $q = 5.0 \times 10^{-6}$ C, the net force on the particle is

$$\vec{F} = (0.00080\hat{\mathrm{j}} - 0.0011\hat{\mathrm{k}})\ \mathrm{N}.$$

86. The current is in the $+\hat{\text{i}}$ direction. Thus, the $\hat{\text{i}}$ component of $\vec{B}$ has no effect, and (with x in meters) we evaluate

$$\vec{F} = (3.00\,\text{A})\int_0^1 \left(-0.600\,\text{T/m}^2\right)x^2\,dx\left(\hat{\text{i}}\times\hat{\text{j}}\right) = \left(-1.80\frac{1^3}{3}\,\text{A}\cdot\text{T}\cdot\text{m}\right)\hat{\text{k}} = (-0.600\,\text{N})\hat{\text{k}}.$$

87. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area which was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB\sum_n \Delta A_n = NiAB\sin\theta.$$

Chapter 29

1. (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Since $B = \mu_0 i/2\pi r$,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\,\text{m})(39\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 16\,\text{A}.$$

(b) The current must be from west to east to produce a field which is directed southward at points below it.

2. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem 29-1). Also, the fields from the two semi-circular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

3. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With r = 20 ft = 6.10 m, we have

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(100\,\text{A}\right)}{2\pi\left(6.10\,\text{m}\right)} = 3.3 \times 10^{-6}\ \text{T} = 3.3\,\mu\text{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

4. Eq. 29-1 is maximized (with respect to angle) by setting $\theta = 90°$ ($= \pi/2$ rad). Its value in this case is

$$dB_{\max} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}.$$

From Fig. 29-36(b), we have $B_{\max} = 60\times10^{-12}$ T. We can relate this $B_{\max}$ to our $dB_{\max}$ by setting "ds" equal to 1×10^{-6} m and $R = 0.025$ m. This allows us to solve for the current: $i = 0.375$ A. Plugging this into Eq. 29-4 (for the infinite wire) gives $B_\infty = 3.0$ μT.

5. (a) Recalling the *straight sections* discussion in Sample Problem 29-1, we see that the current in the straight segments collinear with P do not contribute to the field at that point. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$B = \frac{\mu_0 i\theta}{4}\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.411\,\mathrm{A})(74^\circ\cdot\pi/180^\circ)}{4\pi}\left(\frac{1}{0.107\,\mathrm{m}}-\frac{1}{0.135\,\mathrm{m}}\right)$$
$$=1.02\times10^{-7}\,\mathrm{T}.$$

(b) The direction is out of the page.

6. (a) Recalling the *straight sections* discussion in Sample Problem 29-1, we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(0.281\mathrm{A})}{4}\left(\frac{1}{0.0315\mathrm{m}} - \frac{1}{0.0780\mathrm{m}}\right) = 1.67 \times 10^{-6}\ \mathrm{T}.$$

(b) The direction of the field is into the page.

7. (a) The currents must be opposite or antiparallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\text{ m})(300\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 30\text{ A}.$$

8. (a) Recalling the *straight sections* discussion in Sample Problem 29-1, we see that the current in the straight segments collinear with C do not contribute to the field at that point.

Eq. 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. Thus, the magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.0348\mathrm{A})}{4(0.0926\mathrm{m})} = 1.18\times10^{-7}\,\mathrm{T}.$$

(b) The right-hand rule shows that this field is into the page.

9. (a) $B_{P_1} = \mu_0 i_1 / 2\pi r_1$ where i_1 = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2} = \mu_0 i_2 / 2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = (6.5\,\text{A})\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right) = 4.3\,\text{A}.$$

(b) Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page.

10. (a) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current $3.00i$, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi (d-r)} \quad \Rightarrow \quad r = \frac{d}{4} = \frac{16.0 \text{ cm}}{4} = 4.0 \text{ cm}.$$

(b) Doubling the currents does not change the location where the magnetic field is zero.

11. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 29-7) and a semicircular arc (Eq. 29-9 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 29-6(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{2(0.0050\ \mathrm{m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\ \mathrm{T}$$

upon substituting $i = 10$ A and $R = 0.0050$ m.

(b) The direction of this field is out of the page, as Fig. 29-6(c) makes clear.

(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 29-4,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{\pi(0.0050\ \mathrm{m})} = 8.0\times10^{-4}\ \mathrm{T}.$$

(d) This field, too, points out of the page.

12. With the "usual" x and y coordinates used in Fig. 29-43, then the vector $\vec{r}$ pointing from a current element to P is $\vec{r} = -s\,\hat{\text{i}} + R\,\hat{\text{j}}$. Since $d\vec{s} = ds\,\hat{\text{i}}$, then $|d\vec{s} \times \vec{r}| = Rds$. Therefore, with $r = \sqrt{s^2 + R^2}$, Eq. 29-3 gives

$$dB = \frac{\mu_0}{4\pi} \frac{iR\,ds}{(s^2 + R^2)^{3/2}}.$$

(a) Clearly, considered as a function of s (but thinking of "ds" as some finite-sized constant value), the above expression is maximum for $s = 0$. Its value in this case is $dB_{\text{max}} = \mu_0 i\,ds / 4\pi R^2$.

(b) We want to find the s value such that $dB = dB_{\text{max}}/10$. This is a non-trivial algebra exercise, but is nonetheless straightforward. The result is $s = \sqrt{10^{2/3} - 1}\,R$. If we set $R = 2.00$ cm, then we obtain $s = 3.82$ cm.

13. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge q is $\vec{B} = (\mu_0 i / 2\pi d)\,\hat{\mathrm{k}}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 i q}{2\pi d}\left(\vec{v} \times \hat{\mathrm{k}}\right).$$

In this situation, $\vec{v} = v\left(-\hat{\mathrm{j}}\right)$ (where v is the speed and is a positive value), and $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 i q v}{2\pi d}\left(\left(-\hat{\mathrm{j}}\right)\times\hat{\mathrm{k}}\right) = -\frac{\mu_0 i q v}{2\pi d}\,\hat{\mathrm{i}} = -\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.350\mathrm{A})(1.60\times10^{-19}\,\mathrm{C})(200\mathrm{m/s})}{2\pi(0.0289\ \mathrm{m})}\,\hat{\mathrm{i}}$$
$$= (-7.75\times10^{-23}\ \mathrm{N})\hat{\mathrm{i}}.$$

14. The fact that $B_y = 0$ at $x = 10$ cm implies the currents are in opposite directions. Thus

$$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

using Eq. 29-4 and the fact that $i_1 = 4i_2$. To get the maximum, we take the derivative with respect to x and set equal to zero. This leads to $3x^2 - 2Lx - L^2 = 0$ which factors and becomes $(3x + L)(x - L) = 0$, which has the physically acceptable solution: $x = L$. This produces the maximum B_y: $\mu_o\, i_2 / 2\pi L$. To proceed further, we must determine L. Examination of the datum at $x = 10$ cm in Fig. 29-45(b) leads (using our expression above for B_y and setting that to zero) to $L = 30$ cm.

(a) The maximum value of B_y occurs at $x = L = 30$ cm.

(b) With $i_2 = 0.003$ A we find $\mu_o\, i_2 / 2\pi L = 2.0$ nT.

(c) and (d) Fig. 29-45(b) shows that as we get very close to wire 2 (where its field strongly dominates over that of the more distant wire 1) B_y points along the $-y$ direction. The right-hand rule leads us to conclude that wire 2's current is consequently is *into the page*. We previously observed that the currents were in opposite directions, so wire 1's current is *out of the page*.

15. Each of the semi-infinite straight wires contributes $\mu_0 i/4\pi R$ (Eq. 29-7) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 29-9 pointing into the page, and this is able to produce zero total field at that location if $B_{\text{arc}} = 2.00 B_{\text{semiinfinite}}$, or

$$\frac{\mu_0 i \phi}{4\pi R} = 2.00\left(\frac{\mu_0 i}{4\pi R}\right)$$

which yields $\phi = 2.00$ rad.

16. Initially, we have $B_{\text{net},y} = 0$, and $B_{\text{net},x} = B_2 + B_4 = 2(\mu_o\, i\,/2\pi d)$ using Eq. 29-4, where $d = 0.15$ m . To obtain the 30º condition described in the problem, we must have

$$B_{\text{net},y} = B_{\text{net},x}\tan(30°) \quad \Rightarrow\; B_1' - B_3 = 2\left(\frac{\mu_0 i}{2\pi d}\right)\tan(30°)$$

where $B_3 = \mu_o\, i\,/2\pi d$ and $B_1' = \mu_0 i/2\pi d'$. Since $\tan(30º) = 1/\sqrt{3}$, this leads to

$$d' = \frac{\sqrt{3}}{\sqrt{3}+2}d = 0.464d\ .$$

(a) With d = 15.0 cm, this gives d' = 7.0 cm. Being very careful about the geometry of the situation, then we conclude that we must move wire 1 to x = –7.0 cm.

(b) To restore the initial symmetry, we would have to move wire 3 to x = +7.0 cm.

17. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x = -L/2$ to $x = L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(0.0582\ \mathrm{A}\right)}{2\pi\left(0.131\ \mathrm{m}\right)}\frac{0.180\mathrm{m}}{\sqrt{(0.180\mathrm{m})^2+4(0.131\mathrm{m})^2}} = 5.03\times10^{-8}\ \mathrm{T}.$$

18. We consider Eq. 29-6 but with a finite upper limit ($L/2$ instead of ∞). This leads to

$$B = \frac{\mu_0 i}{2\pi R}\frac{L/2}{\sqrt{(L/2)^2 + R^2}} .$$

In terms of this expression, the problem asks us to see how large L must be (compared with R) such that the infinite wire expression B_∞ (Eq. 29-4) can be used with no more than a 1% error. Thus we must solve

$$\frac{B_\infty - B}{B} = 0.01 .$$

This is a non-trivial algebra exercise, but is nonetheless straightforward. The result is

$$L = \frac{200R}{\sqrt{201}} \approx 14.1R \quad \Rightarrow \quad \frac{L}{R} \approx 14.1$$

19. Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

$$B_{\text{total}} = 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a} = \frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20\ \text{A}\right)}{\pi\left(0.20\ \text{m}\right)} = 8.0\times10^{-5}\ \text{T}.$$

In the calculation cos 45° was replaced with $1/\sqrt{2}$. The total field points upward, or in the $+y$ direction. Thus, $\vec{B}_{\text{total}} = (8.0\times10^{-5}\ \text{T})\hat{\text{j}}$.

20. Using the law of cosines and the requirement that B = 100 nT, we have

$$\theta = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1B_2}\right) = 144°,$$

where Eq. 29-10 has been used to determine B_1 (168 nT) and B_2 (151 nT).

21. Our x axis is along the wire with the origin at the right endpoint, and the current is in the positive x direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi} \frac{\sin\theta}{r^2} dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2 + R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2 + R^2}$, we integrate from $x = -L$ to $x = 0$. The total field is

$$B = \frac{\mu_0 i R}{4\pi} \int_{-L}^{0} \frac{dx}{\left(x^2 + R^2\right)^{3/2}} = \frac{\mu_0 i R}{4\pi} \frac{1}{R^2} \frac{x}{\left(x^2 + R^2\right)^{1/2}} \Bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R} \frac{L}{\sqrt{L^2 + R^2}}$$

$$= \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.693\ \text{A}\right)}{4\pi\left(0.251\ \text{m}\right)} \frac{0.136\text{m}}{\sqrt{(0.136\text{m})^2 + (0.251\text{m})^2}} = 1.32 \times 10^{-7}\ \text{T}.$$

22. Using the Pythagorean theorem, we have

$$B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1 \phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i_2}{2\pi R}\right)^2$$

which, when thought of as the equation for a line in a B^2 versus i_2^2 graph, allows us to identify the first term as the "y-intercept" (1×10^{-10}) and the part of the second term which multiplies i_2^2 as the "slope" (5×10^{-10}). The latter observation leads to the conclusion that R = 8.9 mm, and then our observation about the "y-intercept" determines the angle subtended by the arc: ϕ = 1.8 rad.

23. (a) As illustrated in Sample Problem 29-1, the radial segments do not contribute to $\vec{B}_P$ and the arc-segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0\left(0.40\,\mathrm{A}\right)\left(\pi\,\mathrm{rad}\right)}{4\pi\left(0.050\,\mathrm{m}\right)}\hat{\mathrm{k}} - \frac{\mu_0\left(0.80\,\mathrm{A}\right)\left(2\pi/3\,\mathrm{rad}\right)}{4\pi\left(0.040\,\mathrm{m}\right)}\hat{\mathrm{k}} = -(1.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}| = 1.7\times10^{-6}\,\mathrm{T}$.

(b) The direction is $-\hat{\mathrm{k}}$, or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B} = -\frac{\mu_0\left(0.40\,\mathrm{A}\right)\left(\pi\,\mathrm{rad}\right)}{4\pi\left(0.050\,\mathrm{m}\right)}\hat{\mathrm{k}} - \frac{\mu_0\left(0.80\,\mathrm{A}\right)\left(2\pi/3\,\mathrm{rad}\right)}{4\pi\left(0.040\,\mathrm{m}\right)}\hat{\mathrm{k}} = -(6.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}| = 6.7\times10^{-6}\,\mathrm{T}$.

(d) The direction is $-\hat{\mathrm{k}}$, or into the page.

24. In the one case we have $B_{\text{small}} + B_{\text{big}} = 47.25$ μT, and the other case gives $B_{\text{small}} - B_{\text{big}} = 15.75$ μT (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{\text{small}})+(1/r_{\text{big}})}{(1/r_{\text{small}})-(1/r_{\text{big}})}=\frac{1+(r_{\text{small}}/r_{\text{big}})}{1-(r_{\text{small}}/r_{\text{big}})}=3 \ .$$

The solution of this is straightforward: $r_{\text{small}} = r_{\text{big}}/2$. Using the given fact that the $r_{\text{big}} = 4.00$ cm, then we conclude that the small radius is $r_{\text{small}} = 2.00$ cm.

25. We use Eq. 29-4 to relate the magnitudes of the magnetic fields B_1 and B_2 to the currents (i_1 and i_2, respectively) in the two long wires. The angle of their net field is

$$\theta = \tan^{-1}(B_2/B_1) = \tan^{-1}(i_2/i_1) = 53.13^\circ.$$

The accomplish the net field rotation described in the problem, we must achieve a final angle $\theta' = 53.13^\circ - 20^\circ = 33.13^\circ$. Thus, the final value for the current i_1 must be $i_2/\tan\theta'$ = 61.3 mA.

26. Letting “out of the page” in Fig. 29-55(a) be the positive direction, the net field is

$$B = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi(R/2)}$$

from Eqs. 29-9 and 29-4. Referring to Fig. 29-55, we see that $B = 0$ when $i_2 = 0.5$ A, so (solving the above expression with B set equal to zero) we must have

$$\phi = 4(i_2/i_1) = 4(0.5/2) = 1.00 \text{ rad (or } 57.3^\circ).$$

27. The contribution to $\vec{B}_{\text{net}}$ from the first wire is (using Eq. 29-4)

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r_1}\hat{\text{k}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(30\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{k}} = (3.0\times10^{-6}\ \text{T})\hat{\text{k}}.$$

The distance from the second wire to the point where we are evaluating $\vec{B}_{\text{net}}$ is r_2 = 4 m – 2 m = 2 m. Thus,

$$\vec{B}_2 = \frac{\mu_0 i_2}{2\pi r_2}\hat{\text{i}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(40\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{i}} = (4.0\times10^{-6}\ \text{T})\hat{\text{i}}.$$

and consequently is perpendicular to $\vec{B}_1$. The magnitude of $\vec{B}_{\text{net}}$ is therefore

$$|\vec{B}_{\text{net}}| = \sqrt{(3.0\times10^{-6}\ \text{T})^2 + (4.0\times10^{-6}\ \text{T})^2} = 5.0\times10^{-6}\ \text{T}\,.$$

28. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2} = \frac{\mu_0 i}{2R}$. Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right) = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(5.78\times10^{-3}\ \mathrm{A}\right)}{2\left(0.0189\ \mathrm{m}\right)}\left(1+\frac{1}{\pi}\right) = 2.53\times10^{-7}\,\mathrm{T}.$$

$\vec{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\mathrm{T})\hat{\mathrm{k}}$

(b) Now $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(5.78\times10^{-3}\ \mathrm{A}\right)}{2\left(0.0189\ \mathrm{m}\right)}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\mathrm{T}.$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\mathrm{T}(\cos17.66°\hat{\mathrm{i}} + \sin17.66°\hat{\mathrm{k}}) = (1.92\times10^{-7}\,\mathrm{T})\hat{\mathrm{i}} + (6.12\times10^{-8}\,\mathrm{T})\hat{\mathrm{k}}$$

29. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the *y* axis (passing through *P*), consisting of two equal magnetic field *y*-components. Using Eq. 29-17,

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where $i = 4.00$ A, $r = r = \sqrt{d_2^2 + d_1^2/4} = 5.00 \text{ m},$ and

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{4.00 \text{ m}}{6.00 \text{ m}/2}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°.$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(4.00 \text{ A})}{\pi(5.00 \text{ m})}\sin 53.1° = 2.56\times10^{-7} \text{ T}.$$

30. Initially we have

$$B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$$

using Eq. 29-9. In the final situation we use Pythagorean theorem and write

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2.$$

If we square B_i and divide by B_f^2, we obtain

$$\left(\frac{B_i}{B_f}\right)^2 = \frac{[(1/R)+(1/r)]^2}{(1/R)^2+(1/r)^2} .$$

From the graph (see Fig. 29-58(c) – note the maximum and minimum values) we estimate $B_i/B_f = 12/10 = 1.2$, and this allows us to solve for r in terms of R:

$$r = R\frac{1 \pm 1.2\sqrt{2-1.2^2}}{1.2^2-1} = 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}.$$

Since we require $r < R$, then the acceptable answer is $r = 2.3$ cm.

31. Consider a section of the ribbon of thickness *dx* located a distance *x* away from point *P*. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}.$$

Thus,

$$B_P = \int dB_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) = \frac{(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(4.61\times 10^{-6}\ \mathrm{A})}{2\pi\left(0.0491\ \mathrm{m}\right)}\ln\left(1+\frac{0.0491}{0.0216}\right)$$
$$= 2.23\times 10^{-11}\,\mathrm{T}.$$

and $\vec{B}_P$ points upward. In unit-vector notation, $\vec{B}_P = (2.23\times 10^{-11}\,\mathrm{T})\hat{\mathrm{j}}$

32. By the right-hand rule (which is "built-into" Eq. 29-3) the field caused by wire 1's current, evaluated at the coordinate origin, is along the $+y$ axis. Its magnitude B_1 is given by Eq. 29-4. The field caused by wire 2's current will generally have both an x and a y component which are related to its magnitude B_2 (given by Eq. 29-4) and sines and cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle θ_2 (shown in Fig. 29-60) then its components are

$$B_{2x} = B_2 \sin\theta_2, \quad B_{2y} = -B_2 \cos\theta_2 \ .$$

The magnitude-squared of their net field is then (by Pythagoras' theorem) the sum of the square of their net x-component and the square of their net y-component:

$$B^2 = (B_2 \sin\theta_2)^2 + (B_1 - B_2 \cos\theta_2)^2 = B_1^2 + B_2^2 - 2B_1 B_2 \cos\theta_2 \ .$$

(since $\sin^2\theta + \cos^2\theta = 1$), which we could also have gotten directly by using the law of cosines. We have

$$B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}.$$

With the requirement that the net field have magnitude $B = 80$ nT, we find

$$\theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1 B_2}\right) = \cos^{-1}(-1/4) = 104°,$$

where the positive value has been chosen.

33. (a) Recalling the *straight sections* discussion in Sample Problem 29-1, we see that the current in the straight segments collinear with P do not contribute to the field at that point. We use the result of Problem 29-21 to evaluate the contributions to the field at P, noting that the nearest wire-segments (each of length a) produce magnetism into the page at P and the further wire-segments (each of length $2a$) produce magnetism pointing out of the page at P. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} = \frac{\sqrt{2}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(13\ \mathrm{A})}{8\pi\left(0.047\ \mathrm{m}\right)}$$
$$= 1.96\times10^{-5}\,\mathrm{T} \approx 2.0\times10^{-5}\,\mathrm{T}.$$

(b) The direction of the field is into the page.

34. We note that when there is no y-component of magnetic field from wire 1 (which, by the right-hand rule, relates to when wire 1 is at 90º = π/2 rad), the total y-component of magnetic field is zero (see Fig. 29-62(c)). This means wire #2 is either at +π/2 rad or −π/2 rad.

(a) We now make the assumption that wire #2 must be at −π/2 rad (−90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the θ_1 = 90º datum in Fig. 29-62(b)) – where there is a *maximum* in $B_{\text{net}\,x}$ (equal to +6 μT) – we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\ \text{m})(4.0\times10^{-6}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 4.0\ \text{A}\,.$$

(c) The fact that Fig. 29-62(b) increases as θ_1 progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net}\,y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-62(c)).

(d) Referring now to Fig. 29-62(b) we note that there is no x-component of magnetic field from wire 1 when θ_1 = 0, so that plot tells us that B_{2x} = +2.0 μT. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\ \text{m})(2.0\times10^{-6}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 2.0\ \text{A}\,.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

35. Eq. 29-13 gives the magnitude of the force between the wires, and finding the x-component of it amounts to multiplying that magnitude by $\cos\phi = \frac{d_2}{\sqrt{d_1^2 + d_2^2}}$. Therefore, the x-component of the force per unit length is

$$\frac{F_x}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2 + d_2^2)} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(4.00\times10^{-3}\ \mathrm{A})(6.80\times10^{-3}\ \mathrm{A})(0.050\ \mathrm{m})}{2\pi[(0.0240\ \mathrm{m})^2 + (0.050\ \mathrm{m})^2]}$$
$$= 8.84\times10^{-11}\ \mathrm{N/m}.$$

36. Using Eq. 29-13, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing towards wire 3 which is at the lower right). Only the forces (or their components) along the diagonal direction contribute. With $\theta = 45°$, we find the force per unit meter on wire 1 to be

$$F_1 = |\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}| = 2F_{12}\cos\theta + F_{13} = 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} = \frac{3}{2\sqrt{2}\pi}\left(\frac{\mu_0 i^2}{a}\right)$$

$$= \frac{3}{2\sqrt{2}\pi}\frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(15.0\mathrm{A}\right)^2}{\left(8.50\times10^{-2}\,\mathrm{m}\right)} = 1.12\times10^{-3}\,\mathrm{N/m}.$$

The direction of $\vec{F}_1$ is along $\hat{r} = (\hat{\mathrm{i}} - \hat{\mathrm{j}})/\sqrt{2}$. In unit-vector notation, we have

$$\vec{F}_1 = \frac{(1.12\times10^{-3}\,\mathrm{N/m})}{\sqrt{2}}(\hat{\mathrm{i}} - \hat{\mathrm{j}}) = (7.94\times10^{-4}\,\mathrm{N/m})\hat{\mathrm{i}} + (-7.94\times10^{-4}\,\mathrm{N/m})\hat{\mathrm{j}}$$

37. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting $d = 0.500$ m, we find the net force (per meter length) using Eq. 29-13, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.00\times10^{-7}$ N/m .

38. We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\hat{\mathrm{j}} = \frac{25\mu_0 i^2 l}{24\pi d}\hat{\mathrm{j}} = \frac{25\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(3.00\mathrm{A}\right)^2(10.0\mathrm{m})}{24\pi\left(8.00\times10^{-2}\mathrm{m}\right)}\hat{\mathrm{j}}$$

$$= (4.69\times10^{-4}\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(b) Similarly, for wire 2, we have

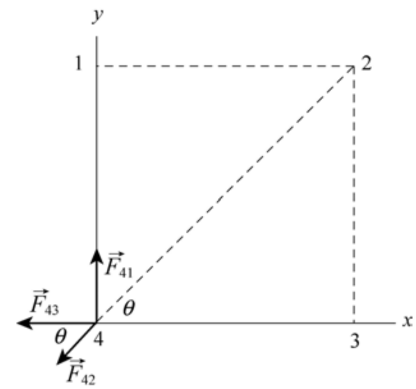
$$\vec{F}_2 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\mathrm{j}} = \frac{5\mu_0 i^2 l}{12\pi d}\hat{\mathrm{j}} = (1.88\times10^{-4}\mathrm{N})\,\hat{\mathrm{j}}.$$

(c) $F_3 = 0$ (because of symmetry).

(d) $\vec{F}_4 = -\vec{F}_2 = (-1.88\times10^{-4}\mathrm{N})\hat{\mathrm{j}}$, and

(e) $\vec{F}_5 = -\vec{F}_1 = -(4.69\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$.

39. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.



The components of $\vec{F}_4$ are given by

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

and

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$

Thus,

$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(7.50\text{A}\right)^2}{4\pi\left(0.135\text{m}\right)}$$

$$= 1.32\times10^{-4}\ \text{N/m}$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°.$$

In unit-vector notation, we have

$$\vec{F}_1 = (1.32\times10^{-4}\ \text{N/m})[\cos 162°\hat{\text{i}} + \sin 162°\hat{\text{j}}] = (-1.25\times10^{-4}\ \text{N/m})\hat{\text{i}} + (4.17\times10^{-5}\ \text{N/m})\hat{\text{j}}$$

40. (a) The fact that the curve in Fig. 29-65(b) passes through zero implies that the currents in wires 1 and 3 exert forces in opposite directions on wire 2. Thus, current i_1 points *out of the page*. When wire 3 is a great distance from wire 2, the only field that affects wire 2 is that caused by the current in wire 1; in this case the force is negative according to Fig. 29-65(b). This means wire 2 is attracted to wire 1, which implies (by the discussion in section 29-2) that wire 2's current is in the same direction as wire 1's current: *out of the page*. With wire 3 infinitely far away, the force per unit length is given (in magnitude) as 6.27×10^{-7} N/m. We set this equal to $F_{12} = \mu_0 i_1 i_2 / 2\pi d$. When wire 3 is at x = 0.04 m the curve passes through the zero point previously mentioned, so the force between 2 and 3 must equal F_{12} there. This allows us to solve for the distance between wire 1 and wire 2:

$$d = (0.04 \text{ m})(0.750 \text{ A})/(0.250 \text{ A}) = 0.12 \text{ m}.$$

Then we solve 6.27×10^{-7} N/m= $\mu_0 i_1 i_2 / 2\pi d$ and obtain i_2 = 0.50 A.

(b) The direction of i_2 is out of the page.

41. The magnitudes of the forces on the sides of the rectangle which are parallel to the long straight wire (with i_1 = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp \text{ sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y} dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(30.0\text{A})(20.0\text{A})(8.00\text{cm})\left(300\times10^{-2}\,\text{m}\right)}{2\pi\left(1.00\text{cm}+8.00\text{cm}\right)} = 3.20\times10^{-3}\,\text{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\text{N})\hat{\text{j}}$ in unit-vector notation.

42. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\,\text{A}+3.0\,\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-2.0\,\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\,\text{A}-5.0\,\text{A}-3.0\,\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-13.0\,\text{A}\right) = -1.6\times10^{-5}\,\text{T}\cdot\text{m}.$$

43. (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\left(2.0\,\text{A}\right) = -2.5\times 10^{-6}\,\text{T}\cdot\text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

44. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(7i - 6i + 3i + i\right) = 5\mu_0 i = 5\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)\left(4.50 \times 10^{-3}\ \mathrm{A}\right) = 2.83 \times 10^{-8}\ \mathrm{T \cdot m}.$$

45. We use Eq. 29-20 $B = \mu_0 ir / 2\pi a^2$ for the B-field inside the wire ($r < a$) and Eq. 29-17 $B = \mu_0 i / 2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

46. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B} \cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(15\ \mathrm{A/m^2}\right)(6)\left(0.20\mathrm{m}\right)^2 = 4.5\times10^{-6}\ \mathrm{T\cdot m}.$$

47. For $r \le a$,

$$B(r)=\frac{\mu_0 i_{\text{enc}}}{2\pi r}=\frac{\mu_0}{2\pi r}\int_0^r J(r)\,2\pi r dr=\frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r dr=\frac{\mu_0 J_0 r^2}{3a}.$$

(a) At $r=0,\ B=0$.

(b) At $r=a/2$, we have

$$B(r)=\frac{\mu_0 J_0 r^2}{3a}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\text{A/m}^2)(3.1\times10^{-3}\,\text{m}/2)^2}{3(3.1\times10^{-3}\,\text{m})}=1.0\times10^{-7}\,\text{T}.$$

(c) At $r=a$,

$$B(r=a)=\frac{\mu_0 J_0 a}{3}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\text{A/m}^2)(3.1\times10^{-3}\,\text{m})}{3}=4.0\times10^{-7}\,\text{T}.$$

48. (a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\text{ wire}} > B_{C,\text{ wire}}$. Thus, for $B_P = B_C = B_{C,\text{ wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\text{A})/8 = 3.00\times10^{-3}\,\text{A}$.

(b) The direction is into the page.

49. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where $i = 0.30$ A, $\ell = 0.25$ m and $N = 200$. This yields $B = 3.0\times10^{-4}$ T.

50. We find N, the number of turns of the solenoid, from the magnetic field $B = \mu_0 in = \mu_o iN/\ell$: $N = B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN = \frac{2\pi rB\ell}{\mu_0 i} = \frac{2\pi\left(2.60\times10^{-2}\ \text{m}\right)\left(23.0\times10^{-3}\ \text{T}\right)\left(1.30\,\text{m}\right)}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(18.0\,\text{A}\right)} = 108\ \text{m}.$$

51. (a) We use Eq. 29-24. The inner radius is r = 15.0 cm, so the field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(0.800\,\mathrm{A})(500)}{2\pi(0.150\,\mathrm{m})} = 5.33\times10^{-4}\ \mathrm{T}.$$

(b) The outer radius is r = 20.0 cm. The field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(0.800\,\mathrm{A})(500)}{2\pi(0.200\mathrm{m})} = 4.00\times10^{-4}\ \mathrm{T}.$$

52. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 i n = \mu_0 i \left(\frac{N}{\ell} \right)$$

where $i = 3.60$ A, $\ell = 0.950$ m and $N = 1200$. This yields $B = 0.00571$ T.

53. (a) We denote the $\vec{B}$-fields at point P on the axis due to the solenoid and the wire as $\vec{B}_s$ and $\vec{B}_w$, respectively. Since $\vec{B}_s$ is along the axis of the solenoid and $\vec{B}_w$ is perpendicular to it, $\vec{B}_s \perp \vec{B}_w$ respectively. For the net field $\vec{B}$ to be at 45° with the axis we then must have $B_s = B_w$. Thus,

$$B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d},$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi\left(20.0\times 10^{-3}\,\text{A}\right)\left(10\,\text{turns/cm}\right)} = 4.77\,\text{cm}\,.$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times 10^{-3}\,\text{A}\right)\left(10\,\text{turns}/0.0100\,\text{m}\right) = 3.55\times 10^{-5}\,\text{T}.$$

54. As the problem states near the end, some idealizations are being made here to keep the calculation straightforward (but are slightly unrealistic). For circular motion (with speed $v_\perp$ which represents the magnitude of the component of the velocity perpendicular to the magnetic field [the field is shown in Fig. 29-19]), the period is (see Eq. 28-17)

$$T = 2\pi r/v_\perp = 2\pi m/eB.$$

Now, the time to travel the length of the solenoid is $t = L/v_\parallel$ where $v_\parallel$ is the component of the velocity in the direction of the field (along the coil axis) and is equal to $v\cos\theta$ where θ = 30º. Using Eq. 29-23 ($B = \mu_0 in$) with $n = N/L$, we find the number of revolutions made is $t/T = 1.6 \times 10^6$.

55. The orbital radius for the electron is

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$i = \frac{mv}{e\mu_0 nr} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)(0.0460)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(100/0.0100\text{m})\left(2.30\times10^{-2}\,\text{m}\right)}$$
$$= 0.272\,\text{A}.$$

56. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0\,.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50\,.$$

57. The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = (200)(0.30\,\text{A})\pi(0.050\,\text{m})^2 = 0.47\,\text{A}\cdot\text{m}^2\ .$$

58. We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 i R^2 N}{2\left[R^2 + (R/2)^2\right]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R} = \frac{8\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)(200)\left(0.0122\,\mathrm{A}\right)}{5\sqrt{5}\,(0.25\,\mathrm{m})} = 8.78\times10^{-6}\,\mathrm{T}.$$

$\vec{B}_P$ is in the positive x direction.

59. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\,\mathrm{A\cdot m^2} \ .$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3} \ .$$

We solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(2.36\,\mathrm{A\cdot m^2})}{2\pi(5.0\times10^{-6}\,\mathrm{T})}\right)^{1/3} = 46\,\mathrm{cm} \ .$$

60. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi\alpha} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})\left(0.0562\,\mathrm{A}\right)}{4}\left(\frac{1}{0.0572\,\mathrm{m}}+\frac{1}{0.0936\,\mathrm{m}}\right)$$
$$= 4.97\times10^{-7}\,\mathrm{T}.$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-6(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$ which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2+b^2) = \frac{\pi(0.0562\mathrm{A})}{2}[(0.0572\mathrm{m})^2 + (0.0936\mathrm{m})^2] = 1.06\times10^{-3}\,\mathrm{A\cdot m^2}.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

61. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\vec{\mu} = \vec{\mu}_{bc\,f\,gb} + \vec{\mu}_{abgha} + \vec{\mu}_{cde\,f\,c} = \left(ia^2\right)\left(\hat{\mathrm{j}} - \hat{\mathrm{i}} + \hat{\mathrm{i}}\right) = ia^2\hat{\mathrm{j}}$$
$$= \left(6.0\,\mathrm{A}\right)\left(0.10\,\mathrm{m}\right)^2\hat{\mathrm{j}} = (6.0\times10^{-2}\ \mathrm{A}\cdot\mathrm{m}^2)\,\hat{\mathrm{j}}\,.$$

(b) Since both points are far from the cube we can use the dipole approximation. For

$(x, y, z) = (0, 5.0\ \mathrm{m}, 0)$

$$\vec{B}(0,\ 5.0\ \mathrm{m},\ 0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\ \mathrm{T}\cdot\mathrm{m/A})(6.0\times10^{-2}\mathrm{m}^2\cdot\mathrm{A})\,\hat{\mathrm{j}}}{2\pi(5.0\ \mathrm{m})^3} = (9.6\times10^{-11}\ \mathrm{T})\,\hat{\mathrm{j}}\,.$$

62. Using Eq. 29-26, we find that the net y-component field is

$$B_y = \frac{\mu_0 i_1 R^2}{2\pi(R^2 + z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2\pi(R^2 + z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ (see Fig. 29-76(a)) and $z_2^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-76(b) corresponding to $B_y = 0$ would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1. We obtain $i_1 = (45/16\pi)$ A ≈ 0.90 A.

(b) With loop 2 at $y = 0.06$ m (see Fig. 29-76(b)) we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2 + L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2 + y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A $\approx$ 2.7 A.

63. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(15\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 7.9 \times 10^{-5}\,\mathrm{T}.$$

(b) The torque has magnitude equal to

$$\begin{aligned}\tau &= |\vec{\mu}_2 \times \vec{B}_1| = \mu_2 B_1 \sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 \\ &= \pi(50)(1.3\,\mathrm{A})(0.82\times10^{-2}\,\mathrm{m})^2(7.9\times10^{-5}\,\mathrm{T}) \\ &= 1.1\times10^{-6}\,\mathrm{N\cdot m}.\end{aligned}$$

64. The radial segments do not contribute to $\vec{B}$ (at the center) and the arc-segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{k}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0 i(\pi \text{ rad})}{4\pi(4.00 \text{ m})}\hat{k} + \frac{\mu_0 i(\pi/2 \text{ rad})}{4\pi(2.00 \text{ m})}\hat{k} - \frac{\mu_0 i(\pi/2 \text{ rad})}{4\pi(4.00 \text{ m})}\hat{k}$$

where $i = 2.00$ A. This yields $\vec{B} = (1.57 \times 10^{-7} \text{ T})\,\hat{k}$, or $|\vec{B}| = 1.57 \times 10^{-7}\,\text{T}$.

65. (a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi\left(a^2 - b^2\right)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})\left(5.25\,\mathrm{A}\right)(0.0200\,\mathrm{m})}{2\pi[(0.0400\,\mathrm{m})^2 - (0.0150\,\mathrm{m})^2]} = 1.53\times10^{-5}\,\mathrm{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2}.$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

Note: One my appy Ampere's law to show that the magnetic field in the hole is uniform. Consider a rectangular path with two long sides (side 1 and 2, each with length L) and two short sides (each of length less than b). If side 1 is directly along the axis of the hole, then side 2 would be also parallel to it and also in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make L *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\oint_{\text{rectangle}} \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$\int_{side\,1} \vec{B} \cdot d\vec{s} + \int_{\text{side}\,2} \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{in hole}}$$

$$\left(B_{\text{side}\,1} - B_{\text{side}\,2}\right) L = 0$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

66. Eq. 29-4 gives

$$i = \frac{2\pi RB}{\mu_0} = \frac{2\pi\left(0.880\,\text{m}\right)\left(7.30\times10^{-6}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 32.1\,\text{A}.$$

67. (a) By the right-hand rule, the magnetic field $\vec{B}_1$ (evaluated at a) produced by wire 1 (the wire at bottom left) is at ϕ = 150° (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at ϕ = 210°. By symmetry $\left(\vec{B}_1 = \vec{B}_2\right)$ we observe that only the x-components survive, yielding

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \left(2\frac{\mu_0 i}{2\pi\ell} \cos 150° \right)\hat{\mathrm{i}} = (-3.46\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$$

where i = 10 A, ℓ = 0.10 m, and Eq. 29-4 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\mathrm{k}}$ direction) of value

$$i_b = B\frac{2\pi r}{\mu_0} = \left(3.46\times10^{-5}\ \mathrm{T}\right)\frac{2\pi(0.087\ \mathrm{m})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 15\,\mathrm{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and Eq. 29-4 has again been used.

(b) As stated above, to cancel this, wire b must carry current into the page (that is, the $-z$ direction)

68. We note that the distance from each wire to P is $r = d/\sqrt{2} = 0.071\,\text{m}$. In both parts, the current is $i = 100$ A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at P) reveals that the vertical components cancel and the horizontal components add, yielding the result:

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos 45.0° = 4.00\times10^{-4}\ \text{T}$$

and directed in the $-x$ direction. In unit-vector notation, we have $\vec{B} = (-4.00\times10^{-4}\ \text{T})\hat{\text{i}}$.

(b) Now, with the currents anti-parallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45.0° = 4.00\times10^{-4}\ \text{T}$$

and directed in the $+y$ direction. In unit-vector notation, we have $\vec{B} = (4.00\times10^{-4}\ \text{T})\hat{\text{j}}$.

69. Since the radius is $R = 0.0013$ m, then the $i = 50$ A produces

$$B = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(50\ \mathrm{A})}{2\pi(0.0013\ \mathrm{m})} = 7.7\times10^{-3}\ \mathrm{T}$$

at the edge of the wire. The three equations, Eq. 29-4, Eq. 29-17 and Eq. 29-20, agree at this point.

70. (a) With cylindrical symmetry, we have, external to the conductors,

$$\left|\vec{B}\right| = \frac{\mu_0 \, i_{\text{enc}}}{2\pi r}$$

which produces i_{enc} = 25 mA from the given information. Therefore, the thin wire must carry 5.0 mA.

(b) The direction is downward, opposite to the 30 mA carried by the thin conducting surface.

71. We use $B(x,y,z) = (\mu_0/4\pi)i\Delta\vec{s}\times\vec{r}/r^3$, where $\Delta\vec{s} = \Delta s\hat{\mathrm{j}}$ and $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\vec{k}$. Thus,

$$\vec{B}(x,y,z) = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,\Delta s\hat{\mathrm{j}}\times\left(x\vec{i} + y\hat{\mathrm{j}} + z\vec{k}\right)}{\left(x^2+y^2+z^2\right)^{3/2}} = \frac{\mu_0 i\,\Delta s\left(z\hat{\mathrm{i}} - x\hat{\mathrm{k}}\right)}{4\pi\left(x^2+y^s+z^2\right)^{3/2}}.$$

(a) The field on the z axis (at $z = 5.0$ m) is

$$\vec{B}(0,\,0,\,5.0\mathrm{m}) = \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(2.0\mathrm{A}\right)\left(3.0\times10^{-2}\,\mathrm{m}\right)\left(5.0\mathrm{m}\right)\hat{\mathrm{i}}}{4\pi\left(0^2+0^2+\left(5.0\mathrm{m}\right)^2\right)^{3/2}} = (2.4\times10^{-10}\,\mathrm{T})\hat{\mathrm{i}}.$$

(b) $\vec{B}$ (0, 6.0 m, 0) = 0, since $x = z = 0$.

(c) The field in the xy plane, at $(x, y) = (7,7)$, is

$$\vec{B}(7.0\mathrm{m},7.0\mathrm{m},0) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(2.0\,\mathrm{A})(3.0\times10^{-2}\,\mathrm{m})(-7.0\,\mathrm{m})\hat{\mathrm{k}}}{4\pi\left(\left(7.0\mathrm{m}\right)^2+\left(7.0\mathrm{m}\right)^2+0^2\right)^{3/2}} = (-4.3\times10^{-11}\,\mathrm{T})\hat{\mathrm{k}}.$$

(d) The field in the xy plane, at $(x, y) = (-3, -4)$, is

$$\vec{B}(-3.0\,\mathrm{m},\,-4.0\mathrm{m},\,0) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(2.0\,\mathrm{A})(3.0\times10^{-2}\,\mathrm{m})(3.0\,\mathrm{m})\hat{\mathrm{k}}}{4\pi\left(\left(-3.0\,\mathrm{m}\right)^2+\left(-4.0\,\mathrm{m}\right)^2+0^2\right)^{3/2}} = (1.4\times10^{-10}\,\mathrm{T})\hat{\mathrm{k}}.$$

72. (a) The radial segments do not contribute to $\vec{B}_P$ and the arc-segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{k}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4 \text{ rad})}{4\pi(4.00 \text{ m})}\hat{k} - \frac{\mu_0 i(7\pi/4 \text{ rad})}{4\pi(2.00 \text{ m})}\hat{k}$$

where $i = 0.200$ A. This yields $\vec{B} = -2.75 \times 10^{-8}\,\hat{k}$ T, or $|\vec{B}| = 2.75 \times 10^{-8}$ T.

(b) The direction is $-\hat{k}$, or into the page.

73. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r,$$

we find that $r = 0.00128$ m gives the desired field value.

74. The points must be along a line parallel to the wire and a distance r from it, where r satisfies $B_{\text{wire}} = \dfrac{\mu_0 i}{2\pi r} = B_{\text{ext}}$, or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{\left(1.26\times10^{-6}\ \text{T}\cdot\text{m/A}\right)\left(100\,\text{A}\right)}{2\pi\left(5.0\times10^{-3}\ \text{T}\right)} = 4.0\times10^{-3}\,\text{m}.$$

75. Let the length of each side of the square be a. The center of a square is a distance $a/2$ from the nearest side. There are four sides contributing to the field at the center. The result is

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2+4(a/2)^2}}\right) = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

On the other hand, the magnetic field at the center of a circular wire of radius R is $\mu_0 i/2R$ (e.g., Eq. 29-10). Thus, the problem is equivalent to showing that

$$\frac{2\sqrt{2}\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad \Rightarrow \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R}\ .$$

To do this we must relate the parameters a and R. If both wires have the same length L then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \quad \Rightarrow \quad a = \frac{\pi R}{2}.$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R},$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

76. We take the current (i = 50 A) to flow in the $+x$ direction, and the electron to be at a point P which is r = 0.050 m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 29-4 with Eq. 28-2, we obtain $\vec{F}_e = (-e\mu_0 i/2\pi r)(\vec{v} \times \hat{\text{k}})$.

(a) The electron is moving down: $\vec{v} = -v\hat{\text{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\left(-\hat{\text{i}}\right) = (3.2\times10^{-16}\text{ N})\,\hat{\text{i}},$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

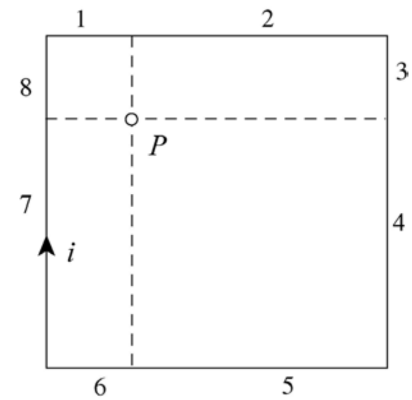
(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{\text{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\left(-\hat{\text{j}}\right) = (3.2\times10^{-16}\text{ N})\,\hat{\text{j}},$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

(c) Now, $\vec{v} = \pm v\hat{\text{k}}$ so $\vec{F}_e \propto \hat{\text{k}} \times \hat{\text{k}} = 0$.

77. The two small wire-segments, each of length $a/4$, shown in Fig. 29-83 nearest to point P, are labeled 1 and 8 in the figure.



Let $-\hat{k}$ be a unit vector pointing into the page. We use the results of Problem 29-21 to calculate B_{P1} through B_{P8}:

$$B_{P1} = B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a},$$

$$B_{P4} = B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a},$$

$$B_{P2} = B_{P7} = \frac{\mu_0 i}{4\pi(a/4)} \cdot \frac{3a/4}{\left[(3a/4)^2 + (a/4)^2\right]^{1/2}} = \frac{3\mu_0 i}{\sqrt{10}\pi a},$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)} \cdot \frac{a/4}{\left[(a/4)^2 + (3a/4)^2\right]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a}.$$

Finally,

$$\begin{aligned}\vec{B}_P &= \sum_{n=1}^{8} B_{Pn}(-\hat{k}) = 2\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6} + \frac{3}{\sqrt{10}} + \frac{1}{3\sqrt{10}}\right)(-\hat{k}) \\ &= \frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(10\text{A})}{\pi\left(8.0\times10^{-2}\,\text{m}\right)}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6} + \frac{3}{\sqrt{10}} + \frac{1}{3\sqrt{10}}\right)(-\hat{k}) \\ &= \left(2.0\times10^{-4}\,\text{T}\right)(-\hat{k}).\end{aligned}$$

78. Eq. 29-17 applies for each wire, with $r=\sqrt{R^2+(d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

$$B=2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right)=\frac{\mu_0 i d}{2\pi\left(R^2+(d/2)^2\right)}=1.25\times10^{-6}\ \text{T},$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement. Using the right-hand rule, we find both horizontal components point in the $+x$ direction. Thus, in unit-vector notation, we have $\vec{B}=(1.25\times10^{-6}\ \text{T})\hat{\text{i}}$.

79. The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it is clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to $\mu_0 JA = \mu_0 J\Delta y\Delta x = \mu_0\lambda\Delta x$.

(a) Figure 29-85 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P' (as reference to Fig. 29-4 will confirm [consider the current elements nearest each of those points]), so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure? Our focus is on point P. Fig. 29-4 suggests that the current element just to the right of the nearest one (the one directly under point P) will contribute a downward component, but by the same reasoning the current element just to the left of the nearest one should contribute an upward component to the field at P. The current elements are all equivalent, as is reflected in the horizontal-translational symmetry built into this problem; therefore, all vertical components should cancel in pairs. The field at P must be purely horizontal, as drawn.

(b) The path used in evaluating $\oint \vec{B} \cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P' respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown next. Ampere's law yields

$$2B\Delta x = \mu_0\lambda\Delta x \;\;\Rightarrow\;\; B = \frac{1}{2}\mu_0\lambda.$$

80. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B = 0.050$ m wire B. Using Eq. 29-4, we have

$$\vec{B}_{\text{net}} = \vec{B}_A + \vec{B}_B = -\frac{\mu_0 i_A}{2\pi r_A}\hat{\text{k}} - \frac{\mu_0 i_B}{2\pi r_B}\hat{\text{k}} = (-52.0\times10^{-6}\ \text{T})\hat{\text{k}}.$$

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0 i_A}{2\pi(r_A - y)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 13/160 \approx 0.0813$ m.

(c) We eliminate the $y < r_B$ possibility due to wire B carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0 i_A}{2\pi(y - r_A)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 7/40 \approx 0.0175$ m.

81. (a) For the circular path L of radius r concentric with the conductor

$$\oint_L \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 i \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}.$$

Thus, $B = \dfrac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\dfrac{r^2 - b^2}{r}\right)$.

(b) At $r = a$, the magnetic field strength is

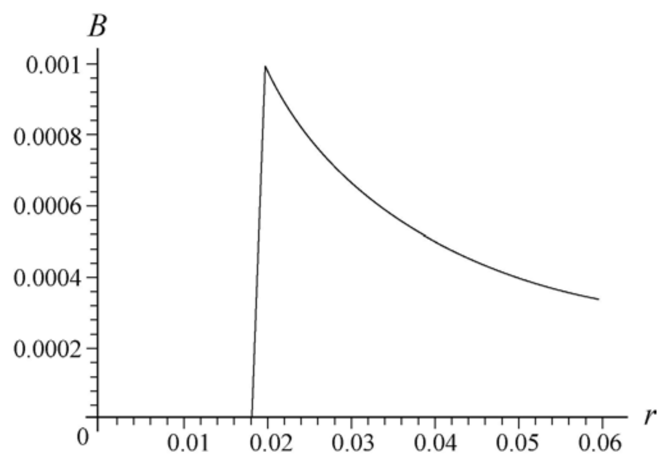
$$\frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{a^2 - b^2}{a}\right) = \frac{\mu_0 i}{2\pi a}.$$

At $r = b, B \propto r^2 - b^2 = 0$. Finally, for $b = 0$

$$B = \frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}$$

which agrees with Eq. 29-20.

(c) The field is zero for $r < b$ and is equal to Eq. 29-17 for $r > a$, so this along with the result of part (a) provides a determination of B over the full range of values. The graph (with SI units understood) is shown below.



82. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$\left|\vec{F}\right| = \frac{\mu_0\, L(5.0\,\mathrm{A})(3.2\,\mathrm{A})}{2\pi(0.10\,\mathrm{m})} + \frac{\mu_0\, L(5.0\,\mathrm{A})(5.0\,\mathrm{A})}{2\pi(0.20\,\mathrm{m})}$$

where L = 3.0 m. Thus, $\left|\vec{F}\right| = 1.7 \times 10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\vec{F}| = \frac{\mu_0 L(5.0\mathrm{A})(3.2\mathrm{A})}{2\pi(0.10\mathrm{m})} - \frac{\mu_0 L(5.0\mathrm{A})(5.0\mathrm{A})}{2\pi(0.20\mathrm{m})} = 2.1\times10^{-5}\,\mathrm{N}\,.$$

83. We refer to the center of the circle (where we are evaluating $\vec{B}$) as C. Recalling the *straight sections* discussion in Sample Problem 29-1, we see that the current in the straight segments which are collinear with C do not contribute to the field there. Eq. 29-9 (with ϕ = π/2 rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at C. Thus, the non-zero contributions come from those straight-segments which are not collinear with C. There are two of these "semi-infinite" segments, one a vertical distance R above C and the other a horizontal distance R to the left of C. Both contribute fields pointing out of the page (see Fig. 29-6(c)). Since the magnitudes of the two contributions (governed by Eq. 29-7) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 29-4). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 29-6(c)).

84. Using Eq. 29-20 and Eq. 29-17, we have

$$|\vec{B}_1| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_1 \qquad |\vec{B}_2| = \frac{\mu_0 i}{2\pi r_2}$$

where $r_1 = 0.0040\,\text{m}$, $\left|\vec{B}_1\right| = 2.8\times10^{-4}\,\text{T}$, $r_2 = 0.010\,\text{m}$ and $|\vec{B}_2| = 2.0\times10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find i = 10 A. Plugging this into the first equation yields $R = 5.3 \times 10^{-3}$ m.

85. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 29-17,

$$\left|\vec{B}\right| = \frac{\mu_0 \, i_w}{2\pi r} = 4.8 \times 10^{-3} \text{ T}$$

where i_w = 24 A and r = 0.0010 m.

(b) Now the field consists of two contributions (which are anti-parallel) — from the wire (Eq. 29-17) and from a portion of the conductor (Eq. 29-20 modified for annular area):

$$|\vec{B}| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_0^2 - \pi R_i^2}\right)$$

where r = 0.0030 m, R_i = 0.0020 m, R_o = 0.0040 m and i_c = 24 A. Thus, we find $|\vec{B}| = 9.3 \times 10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

86. (a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 29-26:

$$B = \frac{N\mu_0 iR^2}{2(R^2+z^2)^{3/2}},$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense, and the fields they produce are in the same direction in the region between them. We place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop, we set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces, we put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 iR^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right].$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 iR^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right].$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 iR^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0$$

independent of the value of s.

(b) The second derivative is

$$\frac{d^2B}{dx^2} = \frac{N\mu_0 iR^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}} - \frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right].$$

At $x = s/2$,

$$\left.\frac{d^2B}{dx^2}\right|_{s/2} = \frac{N\mu_0 iR^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right]$$
$$= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 iR^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}}.$$

Clearly, this is zero if $s = R$.

87. The center of a square is a distance $R = a/2$ from the nearest side (each side being of length $L = a$). There are four sides contributing to the field at the center. The result is

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2 + 4(a/2)^2}}\right) = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

88. We refer to the side of length L as the long side and that of length W as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of Problem 29-17 leads to

$$B = 2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2 + 4(W/2)^2}} + 2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2 + 4(L/2)^2}}.$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2 + W^2}{\sqrt{W^2 + L^2}} = \sqrt{W^2 + L^2}.$$

89. We imagine the square loop in the yz plane (with its center at the origin) and the evaluation point for the field being along the x axis (as suggested by the notation in the problem). The origin is a distance $a/2$ from each side of the square loop, so the distance from the evaluation point to each side of the square is, by the Pythagorean theorem,

$$R = \sqrt{(a/2)^2 + x^2} = \frac{1}{2}\sqrt{a^2 + 4x^2}.$$

Only the x components of the fields (contributed by each side) will contribute to the final result (other components cancel in pairs), so a trigonometric factor of

$$\frac{a/2}{R} = \frac{a}{\sqrt{a^2 + 4x^2}}$$

multiplies the expression of the field given by the result of Problem 29-17 (for each side of length $L = a$). Since there are four sides, we find

$$B(x) = 4\left(\frac{\mu_0 i}{2\pi R}\right)\left(\frac{a}{\sqrt{a^2 + 4R^2}}\right)\left(\frac{a}{\sqrt{a^2 + 4x^2}}\right) = \frac{4\mu_0\, i\, a^2}{2\pi\left(\frac{1}{2}\right)\left(\sqrt{a^2 + 4x^2}\right)^2 \sqrt{a^2 + 4(a/2)^2 + 4x^2}}$$

which simplifies to the desired result. It is straightforward to set $x = 0$ and see that this reduces to the expression found in Problem 29-87 (noting that $4/\sqrt{2} = 2\sqrt{2}$).

90. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 29-12 to find the magnetic force on the segment, and Eq. 29-7 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\mathrm{i}}$ direction, and the current in rail 2 is in the $-\hat{\mathrm{i}}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{\mathrm{k}}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\mathrm{j}}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = i dy\left(-\hat{\mathrm{j}}\right)\times \vec{B}_1 + dy\left(-\hat{\mathrm{j}}\right)\times \vec{B}_2 = i\left[B_1 + B_2\right]\hat{\mathrm{i}}\ dy$$
$$= i\left[\frac{\mu_0 i}{4\pi\left(2R+w-y\right)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\mathrm{i}}\ dy.$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R+w-y} + \frac{1}{y}\right)dy\hat{\mathrm{i}} = \frac{\mu_0 i^2}{2\pi}\ \ln\left(1+\frac{w}{R}\right)\hat{\mathrm{i}}.$$

(b) Using the work-energy theorem, we have

$$\Delta K = \tfrac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL.$$

Thus, the final speed of the projectile is

$$v_f = \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)L\right]^{1/2}$$
$$= \left[\frac{2\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(450\times10^3\ \mathrm{A}\right)^2\ \ln\left(1+1.2\ \mathrm{cm}/6.7\ \mathrm{cm}\right)\left(4.0\ \mathrm{m}\right)}{2\pi\left(10\times10^{-3}\ \mathrm{kg}\right)}\right]^{1/2}$$
$$= 2.3\times10^3\ \mathrm{m/s}.$$

91. We use Ampere's law. For the dotted loop shown on the diagram $i = 0$. The integral $\int \vec{B} \cdot d\vec{s}$ is zero along the bottom, right, and top sides of the loop. Along the right side the field is zero, along the top and bottom sides the field is perpendicular to $d\vec{s}$. If ℓ is the length of the left edge, then direct integration yields $\oint \vec{B} \cdot d\vec{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not discontinuously as suggested by the figure.

92. In this case $L = 2\pi r$ is roughly the length of the toroid so

$$B = \mu_0 i_0 \left(\frac{N}{2\pi r} \right) = \mu_0 n i_0$$

This result is expected, since from the perspective of a point inside the toroid the portion of the toroid in the vicinity of the point resembles part of a long solenoid.

93. (a) Eq. 29-20 applies for $r < c$. Our sign choice is such that i is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2}, \; r \le c.$$

(b) Eq. 29-17 applies in the region between the conductors.

$$B = \frac{\mu_0 i}{2\pi r}, \; c \le r \le b.$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 29-17) plus the field due to the (negative) current in that part of the outer conductor at radius less than r. The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right), \;\; b < r \le a.$$

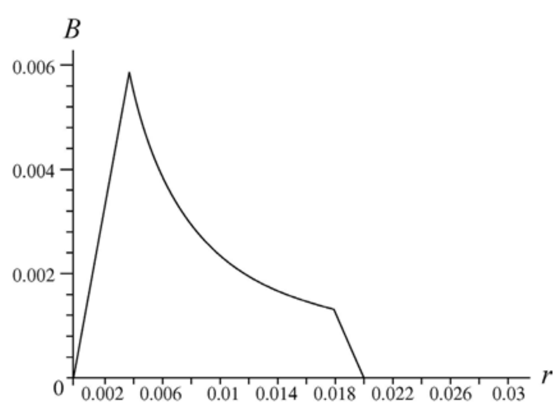
If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right).$$

(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \ge a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown below:



Chapter 30

1. (a) The magnitude of the emf is

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12(2.0) + 7.0 = 31\,\text{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to left through R.

2. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi\left(0.12\text{m}\right)^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times10^{-2}\ \text{V}.$$

(b) For 2.0 s $< t <$ 4.0 s: $\varepsilon \propto dB/dt = 0$.

(c) For 4.0 s $< t <$ 6.0 s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi\left(0.12\text{m}\right)^2\left(\frac{-0.5\text{T}}{6.0\text{s}-4.0\text{s}}\right) = 1.1\times10^{-2}\ \text{V}.$$

3. The amplitude of the induced emf in the loop is

$$\begin{aligned}\varepsilon_m &= A\mu_0 n i_0 \omega = (6.8\times10^{-6}\,\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\text{ A})(212\text{ rad/s})\\ &= 1.98\times10^{-4}\,\text{V}.\end{aligned}$$

4. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi r B\frac{dr}{dt} \\ &= -2\pi(0.12\,\mathrm{m})(0.800\,\mathrm{T})(-0.750\,\mathrm{m/s}) \\ &= 0.452\,\mathrm{V}.\end{aligned}$$

5. The total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

$$= -(120)(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(22000/\mathrm{m})\,\pi\left(0.016\mathrm{m}\right)^2\left(\frac{1.5\ \mathrm{A}}{0.025\ \mathrm{s}}\right)$$

$$= 0.16\mathrm{V}.$$

Ohm's law then yields $i = |\varepsilon|/R = 0.016\ \mathrm{V}/5.3\Omega = 0.030\ \mathrm{A}$.

6. The resistance of the loop is

$$R = \rho\frac{L}{A} = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{\pi\left(0.10\,\text{m}\right)}{\pi\left(2.5\times10^{-3}\ \text{m}\right)^2/4} = 1.1\times10^{-3}\,\Omega.$$

We use $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{\left(10\,\text{A}\right)\left(1.1\times10^{-3}\,\Omega\right)}{\pi\left(0.05\,\text{m}\right)^2} = 1.4\ \text{T/s}.$$

7. The field (due to the current in the straight wire) is out-of-the-page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

8. From the datum at $t = 0$ in Fig. 30-41(b) we see 0.0015 A = V_{battery}/R, which implies that the resistance is

$$R = (6.00\ \mu\text{V})/(0.0015\ \text{A}) = 0.0040\ \Omega.$$

Now, the value of the current during 10 s < t < 20 s leads us to equate

$$(V_{\text{battery}} + \varepsilon_{\text{induced}})/R = 0.00050\ \text{A}.$$

This shows that the induced emf is $\varepsilon_{\text{induced}} = -4.0\ \mu\text{V}$. Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -A\,a\ .$$

Plugging in $\varepsilon = -4.0 \times 10^{-6}$ V and $A = 5.0 \times 10^{-4}\ \text{m}^2$, we obtain $a = 0.0080$ T/s.

9. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

10. Fig. 30-43(b) demonstrates that dB/dt (the slope of that line) is 0.003 T/s. Thus, in absolute value, Faraday's law becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}$$

where $A = 8 \times 10^{-4}$ m^2. We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-43(c) to be $i = dq/dt =$ 0.002 A (the slope of that line). Therefore, the resistance of the loop is

$$R = \frac{|\varepsilon|}{i} = \frac{A\,|dB/dt|}{i} = \frac{(8.0\times10^{-4}\ \text{m}^2)(0.0030\ \text{T/s})}{0.0020\ \text{A}} = 0.0012\,\Omega\,.$$

11. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\text{ m})^2}{2}(0.870\text{ T/s}) = 1.74\text{ V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\text{ V} + 1.74\text{ V} = 21.7\text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

12. (a) Since the flux arises from a dot product of vectors, the result of one sign for B_1 and B_2 and of the opposite sign for B_3 (we choose the minus sign for the flux from B_1 and B_2, and therefore a plus sign for the flux from B_3). The induced emf is

$$\varepsilon = -\Sigma \frac{d\Phi_B}{dt} = A\left(\frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt}\right)$$
$$= (0.10\text{ m})(0.20\text{ m})\left(2.0\times 10^{-6}\text{ T/s} + 1.0\times 10^{-6}\text{ T/s} - 5.0\times 10^{-6}\text{ T/s}\right)$$
$$= -4.0\times 10^{-8}\text{ V}.$$

The minus sign meaning that the effect is dominated by the changes in B_3. Its magnitude (using Ohm's law) is $|\varepsilon|/R = 8.0$ μA.

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

13. (a) It should be emphasized that the result, given in terms of sin(2π *ft*), could as easily be given in terms of cos(2π *ft*) or even cos(2π *ft* + ϕ) where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta+\phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi fN\,abB$.

(b) We solve

$$\varepsilon_0 = 150\text{ V} = 2\pi fN\,abB$$

when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $N\,ab = 0.796\text{ m}^2$.

14. (a) The magnetic flux Φ_B through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)\left(\cos 45°\right) = \pi r^2 B/\sqrt{2}\,.$$

Thus,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) = -\frac{\pi\left(3.7\times10^{-2}\,\text{m}\right)^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\text{T}}{4.5\times10^{-3}\,\text{s}}\right)$$

$$= 5.1\times10^{-2}\,\text{V}.$$

(a) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

15. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40 \text{ rev/s})(2\pi \text{ rad/rev})}{2\pi} = 40 \text{ Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-48, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + (\pi a^2/2)\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\frac{d\cos\left(2\pi ft\right)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f \sin(2\pi ft)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020 \text{ T})\pi^2 (0.020 \text{ m})^2 (40/\text{s}) = 3.2\times10^{-3} \text{V}.$$

16. We note that 1 gauss = 10^{-4} T. The amount of charge is

$$
\begin{aligned}
q(t) &= \frac{N}{R}[BA\cos 20° - (-BA\cos 20°)] = \frac{2NBA\cos 20°}{R} \\
&= \frac{2(1000)(0.590\times10^{-4}\ \text{T})\pi(0.100\ \text{m})^2(\cos 20°)}{85.0\ \Omega + 140\ \Omega} = 1.55\times10^{-5}\ \text{C}\ .
\end{aligned}
$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

17. First we write $\Phi_B = BA \cos \theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA \cos \theta$ (as opposed to, say, $BA \sin \theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and f = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is A = (0.500 m) × (0.300 m) = 0.150 m^2, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBA2\pi f\sin(2\pi ft)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi fNAB = 2\pi(16.7\text{ rev/s})(100\text{ turns})(0.15\text{ m}^2)(3.5\text{ T}) = 5.50\times10^3\text{ V} .$$

18. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For $\vec{B} = (4.00\times10^{-2}\,\mathrm{T/m})y\hat{\mathrm{k}}$, $dB/dt = 0$ and hence $\varepsilon = 0$.

(b) None.

(c) For $\vec{B} = (6.00\times10^{-2}\,\mathrm{T/s})t\hat{\mathrm{k}}$,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(0.400\text{ m} \times 0.250\text{ m})(0.0600\text{ T/s}) = -6.00\text{ mV},$$

or $|\varepsilon| = 6.00$ mV.

(d) Clockwise.

(e) For $\vec{B} = (8.00\times10^{-2}\,\mathrm{T/m\cdot s})yt\,\hat{\mathrm{k}}$,

$$\Phi_B = (0.400)(0.0800t)\int y\,dy = 1.00\times10^{-3}t,$$

in SI units. The induced emf is $\varepsilon = -d\Phi B/dt = -1.00$ mV, or $|\varepsilon| = 1.00$ mV.

(f) Clockwise.

(g) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$.

(h) None.

(i) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$

(j) None.

19. The amount of charge is

$$q(t)=\frac{1}{R}[\Phi_B(0)-\Phi_B(t)]=\frac{A}{R}[B(0)-B(t)]=\frac{1.20\times10^{-3}\,\mathrm{m}^2}{13.0\,\Omega}[1.60\,\mathrm{T}-(-1.60\,\mathrm{T})]$$

$$=2.95\times10^{-2}\mathrm{C}\ .$$

20. Since $\frac{d\cos\phi}{dt} = -\sin\phi\frac{d\phi}{dt}$, Faraday's law (with $N = 1$) becomes

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(BA\cos\phi)}{dt} = BA\sin\phi\frac{d\phi}{dt}.$$

Substituting the values given yields $|\varepsilon| = 0.018$ V.

21. (a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 29-27, with $z = x$ (taken to be much greater than R), gives

$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\,\hat{\mathrm{i}}$$

where the $+x$ direction is upward in Fig. 30-50. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases, and we have a situation like that shown in Fig. 30-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

22. (a) Since $\vec{B} = B\,\hat{\mathrm{i}}$ uniformly, then only the area "projected" onto the *yz* plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux Φ_B through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \frac{1}{4}\pi r^2 B\ .$$

Thus,

$$|\,\varepsilon\,| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| = \frac{1}{4}\pi(0.10\,\mathrm{m})^2(3.0\times 10^{-3}\,\mathrm{T/s}) = 2.4\times 10^{-5}\,\mathrm{V}\ .$$

(b) We have a situation analogous to that shown in Fig. 30-5(a). Thus, the current in segment *bc* flows from *c* to *b* (following Lenz's law).

23. (a) Eq. 29-10 gives the field at the center of the large loop with R = 1.00 m and current $i(t)$. This is approximately the field throughout the area (A = 2.00 × 10^{-4} m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where i_0 = 200 A and

$$k = (-200\text{ A} - 200\text{ A})/1.00\text{ s} = -400\text{ A/s},$$

we find

(a) $B(t=0) = \dfrac{\mu_0 i_0}{2R} = \dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left(200\,\text{A}\right)}{2\left(1.00\text{m}\right)} = 1.26\times10^{-4}\,\text{T},$

(b) $B(t=0.500\text{s}) = \dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left[200\,\text{A} - \left(400\,\text{A/s}\right)\left(0.500\,\text{s}\right)\right]}{2\left(1.00\text{m}\right)} = 0,$ and

(c) $B(t=1.00\text{s}) = \dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left[200\,\text{A} - \left(400\,\text{A/s}\right)\left(1.00\,\text{s}\right)\right]}{2\left(1.00\text{m}\right)} = -1.26\times10^{-4}\,\text{T},$

or $|B(t=1.00\text{s})| = 1.26\times10^{-4}\,\text{T}.$

(d) Yes, as indicated by the flip of sign of B(t) in (c).

(e) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right)\\ &= -(2.00\times10^{-4}\,\text{m}^2)\left(\frac{-1.26\times10^{-4}\,\text{T} - 1.26\times10^{-4}\,\text{T}}{1.00\text{ s}}\right)\\ &= 5.04\times10^{-8}\ \text{V}.\end{aligned}$$

24. (a) First, we observe that a large portion of the figure contributes flux which "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal-but-opposite flux which cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B dA = \int_{b-a}^{a} \left(\frac{\mu_0 i}{2\pi r} \right) (b\, dr) = \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a} \right).$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}
\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a} \right) \right] = -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a} \right) \frac{di}{dt} \\
&= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a} \right) \frac{d}{dt}\left(\frac{9}{2} t^2 - 10t \right) \\
&= \frac{-\mu_0 b (9t - 10)}{2\pi} \ln\left(\frac{a}{b-a} \right).
\end{aligned}$$

With $a = 0.120$ m and $b = 0.160$ m, then, at $t = 3.00$ s, the magnitude of the emf induced in the rectangular loop is

$$|\varepsilon| = \frac{(4\pi \times 10^{-7})(0.16)(9(3) - 10)}{2\pi} \ln\left(\frac{0.12}{0.16 - 0.12} \right) = 5.98 \times 10^{-7} \text{ V}.$$

(b) We note that $di/dt > 0$ at $t = 3$ s. The situation is roughly analogous to that shown in Fig. 30-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

25. (a) Consider a (thin) strip of area of height *dy* and width $\ell = 0.020\,\text{m}$. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = BdA = \left(4t^2 y\right)\left(\ell dy\right)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^{\ell} \left(4t^2 y \ell\right) dy = 2t^2 \ell^3 \ .$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 \ .$$

At $t = 2.5$ s, the magnitude of the induced emf is 8.0×10^{-5} V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

26. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi}\ \ln\left(\frac{r+b/2}{r-b/2}\right).$$

When $r = 1.5b$, we have

$$|\Phi_B| = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(4.7\mathrm{A})(0.022\mathrm{m})}{2\pi}\ln(2.0) = 1.4\times10^{-8}\,\mathrm{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$\begin{aligned} i_{\mathrm{loop}} &= \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 i a}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+b/2}{r-b/2}\right)\right| = \frac{\mu_0 i a b v}{2\pi R[r^2-(b/2)^2]} \\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(4.7\mathrm{A})(0.022\mathrm{m})(0.0080\mathrm{m})(3.2\times10^{-3}\,\mathrm{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\mathrm{m})^2]} \\ &= 1.0\times10^{-5}\,\mathrm{A}. \end{aligned}$$

27. (a) We refer to the (very large) wire length as L and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of anti-parallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\left(L\,dx\right) + 2\int_{d/2}^{\ell/2} B\left(L\,dx\right)$$

where d = 0.0025 m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R\left(\frac{\mu_0 i}{2\pi R^2}r + \frac{\mu_0 i}{2\pi(\ell - r)}\right)dr + 2\int_R^{\ell/2}\left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)}\right)dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{\ell - R}{\ell}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{\ell - R}{R}\right) \\ &= 0.23\times10^{-5}\,\text{T}\cdot\text{m} + 1.08\times10^{-5}\,\text{T}\cdot\text{m}\end{aligned}$$

which yields $\Phi_B/L = 1.3 \times 10^{-5}$ T·m or 1.3×10^{-5} Wb/m.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wires was computed above to be 0.23×10^{-5} T·m. Thus,

$$\frac{0.23\times10^{-5}\,\text{T}\cdot\text{m}}{1.3\times10^{-5}\,\text{T}\cdot\text{m}} = 0.17 = 17\%\,.$$

(c) What was described in part (a) as a symmetry plane at $x = \ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0 < x < \ell/2$ region is now of opposite sign of the flux in the $\ell/2 < x < \ell$ region which causes the total flux (or, in this case, flux per meter) to be zero.

28. Eq. 27-23 gives ε^2/R as the rate of energy transfer into thermal forms (dE_{th}/dt, which, from Fig. 30-55(c), is roughly 40 nJ/s). Interpreting ε as the induced emf (in absolute value) in the single-turn loop ($N = 1$) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Eq. 29-23 gives $B = \mu_0 ni$ for the solenoid (and note that the field is zero outside of the solenoid – which implies that $A = A_{coil}$), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = A\frac{dB}{dt} = A_{coil}\frac{d}{dt}\left(\mu_0 n i_{coil}\right) = \mu_0 n A_{coil}\frac{di_{coil}}{dt}.$$

where Fig. 30-55(b) suggests that $di_{coil}/dt = 0.5$ A/s. With $n = 8000$ (in SI units) and $A_{coil} = \pi(0.02)^2$ (note that the loop radius does not come into the computations of this problem, just the coil's), we find V = 6.3 μV. Returning to our earlier observations, we can now solve for the resistance: $R = \varepsilon^2/(dE_{th}/dt) = 1.0$ mΩ.

29. Thermal energy is generated at the rate $P = \varepsilon^2/R$ (see Eq. 27-23). Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is 1.69×10^{-8} Ω·m (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire (d = 0.00100 m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire (L = 0.500 m) is the circumference of the loop. This enclosed area is used in Faraday's law (where we ignore minus signs in the interest of finding the magnitudes of the quantities):

$$\varepsilon = \frac{d\Phi_B}{dt} = A_{\text{loop}}\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

where the rate of change of the field is dB/dt = 0.0100 T/s. Consequently, we obtain

$$P = \frac{\varepsilon^2}{R} = \frac{(L^2/4\pi)^2(dB/dt)^2}{\rho L/(\pi d^2/4)} = \frac{d^2L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(1.00\times10^{-3}\text{ m})^2(0.500\text{ m})^3}{64\pi(1.69\times10^{-8}\,\Omega\cdot\text{m})}\left(0.0100\text{ T/s}\right)^2$$
$$= 3.68\times10^{-6}\text{ W}.$$

30. Noting that $|\Delta B| = B$, we find the thermal energy is

$$
\begin{aligned}
P_{\text{thermal}}\Delta t &= \frac{\varepsilon^2 \Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2 B^2}{R\Delta t} \\
&= \frac{(2.00\times 10^{-4}\,\text{m}^2)^2 (17.0\times 10^{-6}\,\text{T})^2}{(5.21\times 10^{-6}\,\Omega)(2.96\times 10^{-3}\,\text{s})} \\
&= 7.50\times 10^{-10}\,\text{J}.
\end{aligned}
$$

31. (a) Eq. 30-8 leads to

$$\varepsilon = BLv = (0.350 \text{ T})(0.250 \text{ m})(0.55 \text{ m/s}) = 0.0481 \text{ V} .$$

(b) By Ohm's law, the induced current is i = 0.0481 V/18.0 Ω = 0.00267 A. By Lenz's law, the current is clockwise in Fig. 30-56.

(c) Eq. 26-22 leads to $P = i^2R = 0.000129$ W.

32. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time v: $d = vt$, where $v = 5.20$ m/s. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2 \ .$$

Since the field is a uniform $B = 0.350$ T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2.$$

At $t = 3.00$ s, we obtain $\Phi_B = 85.2$ Wb.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 9.46\frac{dt^2}{dt} = 18.9t$$

in SI units. At $t = 3.00$ s, this yields $\varepsilon = 56.8$ V.

(c) Our calculation in part (b) shows that $n = 1$.

33. (a) Eq. 30-8 leads to

$$\varepsilon = BLv = (1.2\,\text{T})(0.10\text{ m})(5.0\text{ m/s}) = 0.60\text{ V}.$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is $i = 0.60\text{ V}/0.40\ \Omega = 1.5\text{ A}$.

(d) The direction is clockwise.

(e) Eq. 27-22 leads to $P = i^2R = 0.90\text{ W}$.

(f) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\text{ A})(0.10\text{ m})(1.2\text{ T}) = 0.18\text{ N}.$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(g) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent:

$$P = Fv = (0.18\text{ N})(5.0\text{ m/s}) = 0.90\text{ W},$$

which is the same as our result from part (e).

34. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R} \ ,$$

which yields $v_t = mgR/B^2L^2$.

35. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux is

$$d\Phi_B = B dA = \frac{\mu_0 i}{2\pi r} x dr .$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 i x}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i x}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{a+L}{a}\right) \\ &= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)(100\,\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) = 2.40\times10^{-4}\,\mathrm{V}.\end{aligned}$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon / R = \left(2.40\times10^{-4}\,\mathrm{V}\right)/\left(0.400\,\Omega\right) = 6.00\times10^{-4}\,\mathrm{A}.$$

Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = \left(6.00\times10^{-4}\,\mathrm{A}\right)^2\left(0.400\,\Omega\right) = 1.44\times10^{-7}\,\mathrm{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is

$$dF_B = i_\ell B\,dr = \left(\mu_0 i_\ell i / 2\pi r\right) dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$
\begin{aligned}
F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right) \\
&= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(6.00\times10^{-4}\,\mathrm{A}\right)\left(100\,\mathrm{A}\right)}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\
&= 2.87\times10^{-8}\ \mathrm{N}.
\end{aligned}
$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of 2.87×10^{-8} N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\ \mathrm{N})(5.00\ \mathrm{m/s}) = 1.44 \times 10^{-7}\ \mathrm{W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

36. (a) For path 1, we have

$$\oint_1 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B1}}{dt} = \frac{d}{dt}\left(B_1 A_1\right) = A_1\frac{dB_1}{dt} = \pi r_1^2 \frac{dB_1}{dt} = \pi\left(0.200\text{m}\right)^2\left(-8.50\times 10^{-3}\ \text{T/s}\right)$$
$$= -1.07\times 10^{-3}\ \text{V}$$

(b) For path 2, the result is

$$\oint_2 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B2}}{dt} = \pi r_2^2 \frac{dB_2}{dt} = \pi\left(0.300\text{m}\right)^2\left(-8.50\times 10^{-3}\ \text{T/s}\right) = -2.40\times 10^{-3}\ \text{V}$$

(c) For path 3, we have

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times 10^{-3}\ \text{V} - \left(-2.4\times 10^{-3}\ \text{V}\right) = 1.33\times 10^{-3}\ \text{V}$$

37. (a) The point at which we are evaluating the field is inside the solenoid, so Eq. 30-25 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}\left(6.5\times10^{-3}\ \mathrm{T/s}\right)\left(0.0220\ \mathrm{m}\right) = 7.15\times10^{-5}\ \mathrm{V/m}.$$

(b) Now the point at which we are evaluating the field is outside the solenoid and Eq. 30-27 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}\left(6.5\times10^{-3}\ \mathrm{T/s}\right)\frac{\left(0.0600\ \mathrm{m}\right)^2}{\left(0.0820\ \mathrm{m}\right)} = 1.43\times10^{-4}\ \mathrm{V/m}.$$

38. From the "kink" in the graph of Fig. 30-61, we conclude that the radius of the circular region is 2.0 cm. For values of r less than that, we have (from the absolute value of Eq. 30-20)

$$E(2\pi r) = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi r^2 a$$

which means that $E/r = a/2$. This corresponds to the slope of that graph (the linear portion for small values of r) which we estimate to be 0.015 (in SI units). Thus, $a = 0.030$ T/s.

39. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where B_0 = (30.0 T + 29.6 T)/2 = 29.8 T and B_1 = (30.0 T – 29.6 T)/2 = 0.200 T. Then from Eq. 30-25

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}\left[B_0 + B_1 \sin(\omega t + \phi_0)\right] = \frac{1}{2}B_1 \omega r \cos(\omega t + \phi_0).$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\text{max}} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\text{T})(2\pi)(15\text{ Hz})(1.6\times 10^{-2}\text{ m}) = 0.15\text{ V/m}.$$

40. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{\left(8.0\times10^{-3}\ \text{H}\right)\left(5.0\times10^{-3}\ \text{A}\right)}{400} = 1.0\times10^{-7}\ \text{Wb}.$$

41. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB\left(\pi r^2\right) = (30.0)\left(2.60\times10^{-3}\ \text{T}\right)(\pi)(0.100\ \text{m})^2 = 2.45\times10^{-3}\ \text{Wb}.$$

(b) Eq. 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\ \text{Wb}}{3.80\ \text{A}} = 6.45\times10^{-4}\ \text{H}.$$

42. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B=\mu_0 n\Delta i=\mu_0\left(\frac{N}{W}\right)\left(\frac{i}{N}\right)=\frac{\mu_0 i}{W}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.035\text{A})}{0.16\text{m}}=2.7\times10^{-7}\,\text{T}.$$

(b) Eq. 30-33 leads to

$$L=\frac{\Phi_B}{i}=\frac{\pi R^2 B}{i}=\frac{\pi R^2\left(\mu_0 i/W\right)}{i}=\frac{\pi\mu_0 R^2}{W}=\frac{\pi(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.018\text{m})^2}{0.16\text{m}}=8.0\times10^{-9}\,\text{H}.$$

43. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B / ℓ. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B(\ell\,dx) + 2\int_a^{d/2} B(\ell\,dx)$$

where d = 0.0025 m is the diameter of each wire. We will use r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2} r + \frac{\mu_0 i}{2\pi(d-r)}\right) dr + 2\int_a^{d/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi \ln((d-a)/a)$. Now, we use Eq. 30-33 (with $N = 1$) to obtain the inductance per unit length:

$$\frac{L}{\ell} = \frac{\Phi_B}{\ell i} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = 1.81\times10^{-6}\,\text{H/m}.$$

44. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\varepsilon}{L} = -\frac{60\,\text{V}}{12\,\text{H}} = -5.0\,\text{A/s},$$

or $|di/dt| = 5.0\,\text{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then *i* must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8 \times 10^{-4}\ \text{H}.$$

46. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes $|\varepsilon| = L|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})(7.0\,\mathrm{A}-0)}{2.0\times 10^{-3}\,\mathrm{s}} = 1.6\times 10^{4}\ \mathrm{V}.$$

(b) For $2\ \mathrm{ms} < t < 5$ ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})(5.0\,\mathrm{A}-7.0\,\mathrm{A})}{(5.0-2.0)10^{-3}\,\mathrm{s}} = 3.1\times 10^{3}\ \mathrm{V}.$$

(c) For $5\ \mathrm{ms} < t < 6$ ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})(0-5.0\,\mathrm{A})}{(6.0-5.0)10^{-3}\,\mathrm{s}} = 2.3\times 10^{4}\ \mathrm{V}.$$

47. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add (V_1 + V_2), then inductances in series must *add*, $L_{eq} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{eq} = \sum_{n=1}^{N} L_n$.

48. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also applies to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that the field of one inductor not to have significant influence (or "coupling") in the next.

(b) Just as with resistors, $\frac{1}{L_{\text{eq}}} = \sum_{n=1}^{N} \frac{1}{L_n}$.

49. Using the results from Problems 30-47 and 30-48, the equivalent resistance is

$$L_{eq} = L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3} = 30.0\text{mH} + 15.0\text{mH} + \frac{(50.0\text{mH})(20.0\text{mH})}{50.0\text{mH} + 20.0\text{mH}}$$

$$= 59.3 \text{ mH}.$$

50. (a) Immediately after the switch is closed $\varepsilon - \varepsilon_L = iR$. But $i = 0$ at this instant, so $\varepsilon_L = \varepsilon$, or $\varepsilon_L/\varepsilon = 1.00$

(b) $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon$, or $\varepsilon_L/\varepsilon = 0.135$.

(c) From $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \quad \Rightarrow \quad t = \tau_L \ln 2 = 0.693\tau_L \quad \Rightarrow \quad t/\tau_L = 0.693.$$

51. Starting with zero current at $t = 0$ (the moment the switch is closed) the current in the circuit increases according to

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$ is the inductive time constant and ε is the battery emf. To calculate the time at which $i = 0.9990\varepsilon/R$, we solve for t:

$$0.990\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \;\Rightarrow\; \ln\left(0.0010\right) = -\left(t/\tau\right) \;\Rightarrow\; t/\tau_L = 6.91.$$

52. The steady state value of the current is also its maximum value, ε/R, which we denote as i_m. We are told that $i = i_m/3$ at t_0 = 5.00 s. Eq. 30-41 becomes $i = i_m\left(1 - e^{-t_0/\tau_L}\right)$ which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00\,\text{s}}{\ln(1 - 1/3)} = 12.3\ \text{s}.$$

53. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,\text{s}}{\ln\left((10\times10^{-3}\,\text{A})/(1.0\,\text{A})\right)} = 0.217\,\text{s}.$$

Therefore, $R = L/\tau_L = 10$ H/0.217 s = 46 Ω.

54. From the graph we get $\Phi/i = 2 \times 10^{-4}$ in SI units. Therefore, with $N = 25$, we find the self-inductance is $L = N\Phi/i = 5 \times 10^{-3}$ H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol V to stand for the battery emf)

$$\frac{di}{dt} = \frac{V}{R}\frac{R}{L}\, e^{-t/\tau_L} = \frac{V}{L} e^{-t/\tau_L} = 7.1 \times 10^{2} \text{ A/s} .$$

55. (a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \Rightarrow e^{-t/\tau_L} = 0.200 .$$

Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s} .$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A} .$$

56. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\,\text{A}.$$

(b) $i_2 = i_1 = 3.33\,\text{A}$.

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0$$
$$\varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1 = \frac{\varepsilon (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 4.55\,\text{A},$$

(d) and

$$i_2 = \frac{\varepsilon R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 2.73\,\text{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, i.e., $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

57. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\, di/dt = 0$. So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0$ A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\,\mathrm{A})(5.0\,\mathrm{H})}{10\,\mathrm{V}} = 1.5\ \mathrm{s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

58. Applying the loop theorem

$$\varepsilon - L\left(\frac{di}{dt}\right) = iR \ ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\begin{aligned}\varepsilon &= L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0+5.0t) + (3.0+5.0t)R = (6.0)(5.0) + (3.0+5.0t)(4.0) \\ &= (42+20t).\end{aligned}$$

59. (a) We assume i is from left to right through the closed switch. We let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1R = 0.$$

This equation is similar to Eq. 30-46, and its solution is the function given as Eq. 30-47:

$$i_1 = i_0e^{-Rt/L},$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L}, \quad i_2 = i - i_1 = i\left(1 - e^{-Rt/L}\right).$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} \;\Rightarrow\; e^{-Rt/L} = \frac{1}{2}.$$

Taking the natural logarithm of both sides (and using $\ln(1/2) = -\ln 2$) we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \;\Rightarrow\; t = \frac{L}{R}\ln 2.$$

60. (a) Our notation is as follows: h is the height of the toroid, a its inner radius, and b its outer radius. Since it has a square cross section, $h = b - a = 0.12 \text{ m} - 0.10 \text{ m} = 0.02 \text{ m}$. We derive the flux using Eq. 29-24 and the self-inductance using Eq. 30-33:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right)$$

and

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right).$$

Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10 \text{ cm}) \approx 62.8$ cm, the number of turns of the toroid is roughly $N \approx 62.8 \text{ cm}/1.0 \text{ mm} = 628$. Thus

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \approx \frac{\left(4\pi\times 10^{-7}\text{ H/m}\right)(628)^2(0.02\text{m})}{2\pi}\ln\left(\frac{12}{10}\right) = 2.9\times 10^{-4}\text{H}.$$

(b) Noting that the perimeter of a square is four times its sides, the total length ℓ of the wire is $\ell = (628)4(2.0\,\text{cm}) = 50\,\text{m}$, the resistance of the wire is

$$R = (50 \text{ m})(0.02\ \Omega/\text{m}) = 1.0\ \Omega.$$

Thus,

$$\tau_L = \frac{L}{R} = \frac{2.9\times 10^{-4}\,\text{H}}{1.0\Omega} = 2.9\times 10^{-4}\,\text{s}.$$

61. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt}=\frac{d\left(Li^2/2\right)}{dt}=Li\frac{di}{dt}=L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}}=i^2R=\frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2R=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}\quad\Rightarrow\quad t=\tau_L\ln 2=\left(37.0\,\text{ms}\right)\ln 2=25.6\,\text{ms}.$$

62. Let $U_B(t) = \frac{1}{2} Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2} U_B(t \to \infty) = \frac{1}{4} Li_f^2$. This gives $i(t) = i_f \big/ \sqrt{2}$. But $i(t) = i_f (1 - e^{-t/\tau_L})$, so

$$1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{t}{\tau_L} = -\ln\left(1 - \frac{1}{\sqrt{2}}\right) = 1.23.$$

63. (a) If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where ε is the emf of the battery, R is the resistance, and τ_L is the inductive time constant (L/R). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\varepsilon} \Rightarrow -\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\varepsilon}\right).$$

Since

$$\ln\left(1 - \frac{iR}{\varepsilon}\right) = \ln\left[1 - \frac{\left(2.00 \times 10^{-3}\,\text{A}\right)\left(10.0 \times 10^{3}\,\Omega\right)}{50.0\,\text{V}}\right] = -0.5108,$$

the inductive time constant is

$$\tau_L = t/0.5108 = (5.00 \times 10^{-3}\text{ s})/0.5108 = 9.79 \times 10^{-3}\text{ s}$$

and the inductance is

$$L = \tau_L R = \left(9.79 \times 10^{-3}\,\text{s}\right)\left(10.0 \times 10^{3}\,\Omega\right) = 97.9\,\text{H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\text{H})\left(2.00 \times 10^{-3}\,\text{A}\right)^2 = 1.96 \times 10^{-4}\,\text{J}.$$

64. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt}=\frac{d\left(\frac{1}{2}Li^2\right)}{dt}=Li\frac{di}{dt}=L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0 \text{ H}/10\ \Omega = 0.20 \text{ s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

65. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\int_0^t P_{\text{battery}} dt = \int_0^t \frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right) dt = \frac{\varepsilon^2}{R}\left[t + \frac{L}{R}\left(e^{-Rt/L}-1\right)\right]$$

$$= \frac{(10.0\,\text{V})^2}{6.70\,\Omega}\left[2.00\text{ s} + \frac{(5.50\,\text{H})\left(e^{-(6.70\,\Omega)(2.00\text{ s})/5.50\text{ H}}-1\right)}{6.70\,\Omega}\right]$$

$$= 18.7\text{ J}.$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B = \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2 = \frac{1}{2}(5.50\,\text{H})\left(\frac{10.0\,\text{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\,\Omega)(2.00\text{ s})/5.50\text{ H}}\right]^2$$

$$= 5.10\text{ J}.$$

(c) The difference of the previous two results gives the amount "lost" in the resistor: 18.7 J – 5.10 J = 13.6 J.

66. It is important to note that the x that is used in the graph of Fig. 30-71(b) is not the x at which the energy density is being evaluated. The x in Fig. 30-71(b) is the location of wire 2. The energy density (Eq. 30-54) is being evaluated at the coordinate origin throughout this problem. We note the curve in Fig. 30-71(b) has a zero; this implies that the magnetic fields (caused by the individual currents) are in opposite directions (at the origin), which further implies that the currents have the same direction. Since the magnitudes of the fields must be equal (for them to cancel) when the x of Fig. 30-71(b) is equal to 0.20 m, then we have (using Eq. 29-4) $B_1 = B_2$, or

$$\frac{\mu_0 i_1}{2\pi d} = \frac{\mu_0 i_2}{2\pi(0.20\ \text{m})}$$

which leads to $d = (0.20\ \text{m})/3$ once we substitute $i_1 = i_2/3$ and simplify. We can also use the given fact that when the energy density is completely caused by B_1 (this occurs when x becomes infinitely large because then $B_2 = 0$) its value is $u_B = 1.96 \times 10^{-9}$ (in SI units) in order to solve for B_1:

$$B_1 = \sqrt{2\mu_0 \mu_B}\ .$$

(a) This combined with $B_1 = \mu_0 i_1 / 2\pi d$ allows us to find wire 1's current: $i_1 \approx 23$ mA.

(b) Since $i_2 = 3i_1$ then $i_2 = 70$ mA (approximately).

67. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2} B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{0.50\,\mathrm{T}}{\sqrt{\left(8.85\times10^{-12}\ \mathrm{F/m}\right)\left(4\pi\times10^{-7}\ \mathrm{H/m}\right)}} = 1.5\times10^{8}\ \mathrm{V/m}\ .$$

68. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2} L i^2$, where L is its inductance and i is the current. By Eq. 30-54, the energy is also given by $U_B = u_B \mathcal{V}$, where u_B is the average energy density and $\mathcal{V}$ is the volume. Thus

$$i = \sqrt{\frac{2u_B \mathcal{V}}{L}} = \sqrt{\frac{2\left(70.0\,\mathrm{J/m^3}\right)\left(0.0200\,\mathrm{m^3}\right)}{90.0\times10^{-3}\,\mathrm{H}}} = 5.58\,\mathrm{A}\ .$$

69. (a) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 ni$, where n, for the solenoid of this problem, is

$$n = (950 \text{ turns})/(0.850 \text{ m}) = 1.118 \times 10^3 \text{ m}^{-1}.$$

The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(1.118\times 10^3 \text{ m}^{-1}\right)^2 \left(6.60\text{ A}\right)^2 = 34.2 \text{ J/m}^3 .$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid. $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = \left(34.2\text{ J/m}^3\right)\left(17.0\times 10^{-4}\text{ m}^2\right)\left(0.850\text{ m}\right) = 4.94\times 10^{-2}\text{ J} .$$

70. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{\left(4\pi \times 10^{-7}\ \text{H/m}\right)\left(100\,\text{A}\right)}{2\left(50 \times 10^{-3}\ \text{m}\right)} = 1.3 \times 10^{-3}\ \text{T}\ .$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(1.3 \times 10^{-3}\ \text{T}\right)^2}{2\left(4\pi \times 10^{-7}\ \text{H/m}\right)} = 0.63\ \text{J/m}^3\ .$$

71. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2R}\right)^2 = \frac{\mu_0 i^2}{8R^2} = \frac{\left(4\pi\times10^{-7}\ \text{H/m}\right)\left(10\,\text{A}\right)^2}{8\left(2.5\times10^{-3}\ \text{m}/2\right)^2} = 1.0\ \text{J/m}^3\ .$$

(b) The electric energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\ \text{F/m}\right)\left[\left(10\text{A}\right)\left(3.3\Omega/10^3\,\text{m}\right)\right]^2$$
$$= 4.8\times10^{-15}\ \text{J/m}^3\ .$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$.

72. We use $\varepsilon_2 = -M\, di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times 10^3\ \text{V}}{6.0\,\text{A}\big/\left(2.5\times 10^{-3}\ \text{s}\right)} = 13\,\text{H}\ .$$

73. (a) Eq. 30-65 yields

$$M = \frac{\varepsilon_1}{|di_2/dt|} = \frac{25.0\,\text{mV}}{15.0\,\text{A/s}} = 1.67\,\text{mH} \ .$$

(b) Eq. 30-60 leads to

$$N_2\Phi_{21} = Mi_1 = (1.67\,\text{mH})(3.60\,\text{A}) = 6.00\,\text{mWb} \ .$$

74. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\,\text{mH})(6.0\,\text{mA})}{100} = 1.5\,\mu\text{Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1 \frac{di_1}{dt} = (25\,\text{mH})(4.0\text{ A/s}) = 1.0\times 10^2\text{ mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\,\text{mH})(6.0\,\text{mA})}{200} = 90\,\text{nWb}\,.$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\frac{di_1}{dt} = (3.0\,\text{mH})(4.0\text{ A/s}) = 12\,\text{mV}.$$

75. (a) We assume the current is changing at (nonzero) rate di/dt and calculate the total emf across both coils. First consider the coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus, the induced emf's are

$$\varepsilon_1 = -(L_1 + M)\frac{di}{dt} \text{ and } \varepsilon_2 = -(L_2 + M)\frac{di}{dt}.$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -(L_1 + L_2 + 2M)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{eq} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of coil rather than the front (as pictured in the diagram). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -(L_1 - M)\frac{di}{dt}.$$

Similarly, the emf across coil 2 is

$$\varepsilon_2 = -(L_2 - M)\frac{di}{dt}.$$

The total emf across both coils is

$$\varepsilon = -(L_1 + L_2 - 2M)\frac{di}{dt}.$$

This the same as the emf that would be produced by a single coil with inductance

$$L_{eq} = L_1 + L_2 - 2M.$$

76. (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n\pi R^2\right)}{i_s} = \mu_0 \pi R^2 nN \ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

77. The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} \ln\left(1 + \frac{b}{a}\right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1 + \frac{b}{a}\right).$$

From the formula for M obtained above, we have

$$M = \frac{(100)(4\pi \times 10^{-7}\ \text{H/m})(0.30\ \text{m})}{2\pi} \ln\left(1 + \frac{8.0}{1.0}\right) = 1.3 \times 10^{-5}\ \text{H}.$$

78. In absolute value, Faraday's law (for a single turn, with B changing in time) gives

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

for the magnitude of the induced emf. Dividing it by R^2 then allows us to relate this to the slope of the graph in Fig. 30-75(b) [particularly the first part of the graph], which we estimate to be 80 μV/m^2.

(a) Thus, $\frac{dB_1}{dt}$ = (80 μV/m^2)/π ≈ 25 μT/s .

(b) Similar reasoning for region 2 (corresponding to the slope of the second part of the graph in Fig. 30-75(b)) leads to an emf equal to

$$\pi r_1^2\left(\frac{dB_1}{dt} - \frac{dB_2}{dt}\right) + \pi R^2 \frac{dB_2}{dt} \; .$$

which means the second slope (which we estimate to be 40 μV/m^2) is equal to $\pi\frac{dB_2}{dt}$.

Therefore, $\frac{dB_2}{dt}$ = (40 μV/m^2)/π ≈ 13 μT/s.

(c) Considerations of Lenz's law leads to the conclusion that B_2 is increasing.

79. The induced electric field E as a function of r is given by $E(r) = (r/2)(dB/dt)$.

(a) The acceleration of the electron released at point a is

$$\vec{a}_a = \frac{eE}{m}\hat{\mathrm{i}} = \frac{er}{2m}\left(\frac{dB}{dt}\right)\hat{\mathrm{i}} = \frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(5.0\times10^{-2}\,\mathrm{m}\right)\left(10\times10^{-3}\,\mathrm{T/s}\right)}{2\left(9.11\times10^{-27}\,\mathrm{kg}\right)}\hat{\mathrm{i}} = (4.4\times10^{7}\,\mathrm{m/s^2})\hat{\mathrm{i}}.$$

(b) At point b we have $a_b \propto r_b = 0$.

(c) The acceleration of the electron released at point c is

$$\vec{a}_c = -\vec{a}_a = -(4.4\times10^{7}\,\mathrm{m/s^2})\hat{\mathrm{i}}.$$

80. (a) From Eq. 30-35, we find $L = (3.00 \text{ mV})/(5.00 \text{ A/s}) = 0.600 \text{ mH}$.

(b) Since $N\Phi = iL$ (where $\Phi = 40.0\ \mu\text{Wb}$ and $i = 8.00$ A), we obtain $N = 120$.

81. (a) The magnitude of the average induced emf is

$$\varepsilon_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BA_i}{t} = \frac{(2.0\,\text{T})(0.20\,\text{m})^2}{0.20\,\text{s}} = 0.40\,\text{V}.$$

(b) The average induced current is

$$i_{\text{avg}} = \frac{\varepsilon_{\text{avg}}}{R} = \frac{0.40\,\text{V}}{20\times 10^{-3}\,\Omega} = 20\,\text{A}.$$

82. Since $A = \ell^2$, we have $dA/dt = 2\ell d\ell/dt$. Thus, Faraday's law, with $N = 1$, becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -2\ell B\frac{d\ell}{dt}$$

which yields $\varepsilon = 0.0029$ V.

83. The energy stored when the current is i is

$$U_B = \frac{1}{2}Li^2$$

where L is the self-inductance. The rate at which this is developed is

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

where i is given by Eq. 30-41 and di/dt is obtained by taking the derivative of that equation (or by using Eq. 30-37). Thus, using the symbol V to stand for the battery voltage (12.0 volts) and R for the resistance (20.0 Ω), we have, at $t = 1.61\tau_L$,

$$\frac{dU_B}{dt} = \frac{V^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} = \frac{(12.0\ \text{V})^2}{20.0\Omega}\left(1-e^{-1.61}\right)e^{-1.61} = 1.15\ \text{W}\ .$$

84. We write $i = i_0 e^{-t/\tau_L}$ and note that $i = 10\%\ i_0$. We solve for t:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R}\ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\mathrm{H}}{3.00\,\Omega}\ln\left(\frac{i_0}{0.100\,i_0}\right) = 1.54\,\mathrm{s}\,.$$

85. (a) When switch S is just closed, $V_1 = \varepsilon$ and $i_1 = \varepsilon/R_1 = 10 \text{ V}/5.0 \ \Omega = 2.0$ A.

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) $i_s = i_1 + i_2 = 2.0 \text{ A} + 0 = 2.0$ A.

(d) Since $V_L = \varepsilon$, $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) $V_L = \varepsilon = 10$ V.

(f) $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = \dfrac{\varepsilon}{L} = \dfrac{10 \text{ V}}{5.0 \text{ H}} = 2.0 \text{ A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1 = 2.0$ A.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2 = 10 \text{ V}/10 \ \Omega = 1.0$ A.

(i) $i_s = i_1 + i_2 = 2.0 \text{ A} + 1.0 \text{ A} = 3.0$ A.

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon = 10$ V.

(k) $V_L = 0$.

(l) $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = 0$.

86. Because of the decay of current (Eq. 30-45) that occurs after the switches are closed on *B*, the flux will decay according to

$$\Phi_1 = \Phi_{10} e^{-t/\tau_{L_1}}, \quad \Phi_2 = \Phi_{20} e^{-t/\tau_{L_2}}$$

where each time-constant is given by Eq. 30-42. Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{20}/\Phi_{10})}{(R_2/L_2) - (R_1/L_1)} = \frac{\ln(1.50)}{(30.0\ \Omega/0.0030\ \text{H}) - (25\ \Omega/0.0050\ \text{H})} = 81.1 \mu\text{s} \ .$$

87. Eq. 30-41 applies, and the problem requires

$$iR = L\frac{di}{dt} = \varepsilon - iR$$

at some time t (where Eq. 30-39 has been used in that last step). Thus, we have $2iR = \varepsilon$, or

$$\varepsilon = 2iR = 2\left[\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right]R = 2\varepsilon\left(1 - e^{-t/\tau_L}\right)$$

where Eq. 30-42 gives the inductive time constant as $\tau_L = L/R$. We note that the emf ε cancels out of that final equation, and we are able to rearrange (and take natural log) and solve. We obtain $t = 0.520$ ms.

88. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt}=\frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right]=\frac{\varepsilon}{R\tau_L}e^{-t/\tau_L}=\frac{\varepsilon}{L}e^{-t/\tau_L}.$$

With $\tau_L = L/R$ (Eq. 30-42), L = 0.023 H and ε = 12 V, t = 0.00015 s, and di/dt = 280 A/s, we obtain e^{-t/τ_L} = 0.537. Taking the natural log and rearranging leads to R = 95.4 Ω.

89. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt}=\frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right]=\frac{\varepsilon}{R\tau_L}e^{-t/\tau_L}=\frac{\varepsilon}{L}e^{-t/\tau_L}$$

With τ_L = 0.28 ms (by Eq. 30-42), L = 0.050 H and ε = 45 V, we obtain di/dt = 12 A/s when t = 1.2 ms.

90. (a) From Eq. 30-28, we have

$$L = \frac{N\Phi}{i} = \frac{(150)(50\times10^{-9}\ \mathrm{T\cdot m^2})}{2.00\times10^{-3}\ \mathrm{A}} = 3.75\ \mathrm{mH}\,.$$

(b) The answer for L (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is 2(50) = 100 nWb.

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L\left.\frac{di}{dt}\right|_{\max} = (0.00375\ \mathrm{H})(0.0030\ \mathrm{A})(377\ \mathrm{rad/s}) = 0.00424\ \mathrm{V}\,.$$

91. (a) $i_0 = \varepsilon/R = 100$ V/10 $\Omega = 10$ A.

(b) $U_B = \frac{1}{2}Li_0^2 = \frac{1}{2}(2.0\,\mathrm{H})(10\,\mathrm{A})^2 = 1.0\times10^2\,\mathrm{J}$.

92. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = \left(4\pi \times 10^{-7}\ \mathrm{H/m}\right)\left(100\ \mathrm{turns/cm}\right)^2 (\pi)\left(1.6\,\mathrm{cm}\right)^2 = 0.10\,\mathrm{H/m}.$$

(b) The induced emf per meter is

$$\frac{\varepsilon}{\ell} = \frac{L}{\ell}\frac{di}{dt} = \left(0.10\,\mathrm{H/m}\right)\left(13\,\mathrm{A/s}\right) = 1.3\,\mathrm{V/m}.$$

93. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf (ε_{L1}) of the $L_1 = 0.30$ H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt} = \frac{|\varepsilon_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\,\text{A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emf's of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\varepsilon - iR_1 - |\varepsilon_{L1}| - |\varepsilon_{L2}| = 0 \;\Rightarrow\; i = \frac{6.0\,\text{V}}{R_1} = 0.75\,\text{A}.$$

94. Using Eq. 30-41

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$$

where τ_L = 2.0 ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0\,\text{ns}.$$

95. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t = 0$, the loop rule requires the emf of the inductor ε_L to cancel that of the battery ($\varepsilon = 40$ V). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{40\text{ V}}{0.050\text{ H}} = 8.0\times10^2\text{ A/s}.$$

(c) This circuit becomes equivalent to that analyzed in §30-9 when we replace the parallel set of 20000 Ω resistors with $R = 10000$ Ω. Now, with $\tau_L = L/R = 5\times10^{-6}$ s, we have $t/\tau_L = 3/5$, and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\varepsilon}{R}\left(1-e^{-3/5}\right) \approx 1.8\times10^{-3}\text{ A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\varepsilon - i_{\text{bat}}R - |\varepsilon_L| = 0.$$

Using the values from part (c), we obtain $|\varepsilon_L| \approx 22$ V. Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{22\text{ V}}{0.050\text{ H}} \approx 4.4\times10^2\text{ A/s}.$$

(e) As $t\to\infty$, the circuit reaches a steady state condition, so that $di_{\text{bat}}/dt = 0$ and $\varepsilon_L = 0$. The loop rule then leads to

$$\varepsilon - i_{\text{bat}}R - |\varepsilon_L| = 0 \;\Rightarrow\; i_{\text{bat}} = \frac{40\text{ V}}{10000\,\Omega} = 4.0\times10^{-3}\text{ A}.$$

(f) As $t\to\infty$, the circuit reaches a steady state condition, $di_{\text{bat}}/dt = 0$.

96. (a) $L = \Phi/i = 26 \times 10^{-3}$ Wb/5.5 A $= 4.7 \times 10^{-3}$ H.

(b) We use Eq. 30-41 to solve for t:

$$t = -\tau_L \ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{4.7\times10^{-3}\,\text{H}}{0.75\,\Omega}\ln\left[1 - \frac{(2.5\,\text{A})(0.75\,\Omega)}{6.0\,\text{V}}\right]$$

$$= 2.4\times10^{-3}\,\text{s}.$$

97. Using Ohm's law, we relate the induced current to the emf and (the absolute value of) Faraday's law:

$$i = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right|.$$

As the loop is crossing the boundary between regions 1 and 2 (so that "x" amount of its length is in region 2 while "$D - x$" amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}H(B_2 - B_1) = vH(B_2 - B_1) \implies i = vH(B_2 - B_1)/R.$$

Similar considerations hold (replacing "B_1" with 0 and "B_2" with B_1) for the loop crossing initially from the zero-field region (to the left of Fig. 30-81(a)) into region 1.

(a) In this latter case, appeal to Fig. 30-81(b) leads to

$$3.0 \times 10^{-6}\,\text{A} = (0.40\text{ m/s})(0.015\text{ m})\,B_1/(0.020\,\Omega)$$

which yields $B_1 = 10\ \mu\text{T}$.

(b) Lenz's law considerations lead us to conclude that the direction of the region 1 field is *out of the page*.

(c) Similarly, $i = vH(B_2 - B_1)/R$ leads to $B_2 = 3.3\mu\text{T}$.

(d) The direction of $\vec{B}_2$ is out of the page.

98. (a) We use $U_B = \frac{1}{2}Li^2$ to solve for the self-inductance:

$$L = \frac{2U_B}{i^2} = \frac{2\left(25.0\times10^{-3}\text{ J}\right)}{\left(60.0\times10^{-3}\text{ A}\right)^2} = 13.9\text{ H}.$$

(b) Since $U_B \propto i^2$, for U_B to increase by a factor of 4, i must increase by a factor of 2. Therefore, i should be increased to 2(60.0 mA) = 120 mA.

99. (a) The current is given by Eq. 30-41

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) = 2.00 \text{ A} ,$$

where $L = 0.018$ H and $\varepsilon = 12$ V. If $R = 1.00\ \Omega$ (so $\tau_L = L/R = 0.018$ s), we obtain $t = 0.00328$ s when we solve this equation.

(b) For $R = 5.00\ \Omega$ we find $t = 0.00645$ s.

(c) If we set $R = 6.00\ \Omega$ then $\varepsilon/R = 2.00$ A so $e^{-t/\tau_L} = 0$, which means $t = \infty$.

(d) The trend in our answers to parts (a), (b) and (c) lead us to expect the smaller the resistance then the smaller to value of t. If we consider what happens to Eq. 30-39 in the extreme case where $R \to 0$, we find that the time-derivative of the current becomes equal to the emf divided by the self-inductance, which leads to a linear dependence of current on time: $i = (\varepsilon/L)t$. In fact, this is what one have obtained starting from Eq. 30-41 and considering its $R \to 0$ limit. Thus, this case seems self-consistent, so we conclude that it is meaningful and that $R = 0$ is actually a valid answer here.

(e) Thus $t = Li/\varepsilon = 0.00300$ s in this "least-time" scenario.

100. Faraday's law (for a single turn, with B changing in time) gives

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}.$$

In this problem, we find $\frac{dB}{dt} = -\frac{B_0}{\tau} e^{-t/\tau}$. Thus, $\varepsilon = \pi r^2 \frac{B_0}{\tau} e^{-t/\tau}$.

101. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ any current through the battery is also that through the 20 Ω and 10 Ω resistors. Hence,

$$i = \frac{\varepsilon}{30.0\,\Omega} = 0.400\,\text{A}$$

which results in a voltage drop across the 10 Ω resistor equal to (0.400 A)(10 Ω) = 4.0 V. The inductor must have this same voltage across it $|\varepsilon_L|$, and we use (the absolute value of) Eq. 30-35:

$$\frac{di}{dt} = \frac{|\varepsilon_L|}{L} = \frac{4.00\text{ V}}{0.0100\text{ H}} = 400\,\text{A/s}.$$

(b) Applying the loop rule to the outer loop, we have

$$\varepsilon - (0.50\,\text{A})(20\,\Omega) - |\varepsilon_L| = 0.$$

Therefore, $|\varepsilon_L| = 2.0$ V, and Eq. 30-35 leads to

$$\frac{di}{dt} = \frac{|\varepsilon_L|}{L} = \frac{2.00\text{ V}}{0.0100\text{ H}} = 200\,\text{A/s}.$$

(c) As $t \to \infty$, the inductor has $\varepsilon_L = 0$ (since the current is no longer changing). Thus, the loop rule (for the outer loop) leads to

$$\varepsilon - i(20\,\Omega) - |\varepsilon_L| = 0 \Rightarrow i = 0.60\,\text{A}.$$

102. The flux Φ_B over the toroid cross-section is (see, for example Problem 30-60)

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right).$$

Thus, the coil-toroid mutual inductance is

$$M_{ct} = \frac{N_c \Phi_{ct}}{i_t} = \frac{N_c}{i_t}\frac{\mu_0 i_t N_t h}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu_0 N_1 N_2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

where $N_t = N_1$ and $N_c = N_2$.

103. From the given information, we find

$$\frac{dB}{dt} = \frac{0.030\,\mathrm{T}}{0.015\,\mathrm{s}} = 2.0\,\mathrm{T/s}.$$

Thus, with $N = 1$ and $\cos 30° = \sqrt{3}/2$, and using Faraday's law with Ohm's law, we have

$$i = \frac{|\varepsilon|}{R} = \frac{N\pi r^2}{R}\frac{\sqrt{3}}{2}\frac{dB}{dt} = \frac{\pi(0.14\text{ m})^2}{5.0\,\Omega}\frac{\sqrt{3}}{2}\left(2.0\text{ T/s}\right) = 0.021\,\mathrm{A}.$$

104. The area enclosed by any turn of the coil is πr^2 where $r = 0.15$ m, and the coil has $N = 50$ turns. Thus, the magnitude of the induced emf, using Eq. 30-5, is

$$|\varepsilon| = N\pi r^2 \left|\frac{dB}{dt}\right| = \left(3.53\,\mathrm{m}^2\right)\left|\frac{dB}{dt}\right|$$

where $\left|\frac{dB}{dt}\right| = \left(0.0126\,\mathrm{T/s}\right)\left|\cos\omega t\right|$. Thus, using Ohm's law, we have

$$i = \frac{|\varepsilon|}{R} = \frac{\left(3.53\ \mathrm{m}^2\right)\left(0.0126\ \mathrm{T/s}\right)}{4.0\,\Omega}\left|\cos\omega t\right| .$$

When $t = 0.020$ s, this yields $i = 0.011$ A.

Chapter 31

1. (a) The period is $T = 4(1.50\ \mu\text{s}) = 6.00\ \mu\text{s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{6.00\,\mu\text{s}} = 1.67 \times 10^{5}\ \text{Hz}.$$

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or $3.00\ \mu\text{s}$.

2. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{\left(1.60\times10^{-6}\,\text{C}\right)^2}{2\left(140\times10^{-6}\,\text{J}\right)} = 9.14\times10^{-9}\ \text{F}.$$

3. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\,\text{C}}{\sqrt{(1.10 \times 10^{-3}\,\text{H})(4.00 \times 10^{-6}\,\text{F})}} = 4.52 \times 10^{-2}\,\text{A}.$$

4. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00 \times 10^3 \text{ Hz}} = n(5.00\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t_A = 5.00\,\mu$s.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 31-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}(2n-1)T = \frac{(2n-1)}{2f} = \frac{(2n-1)}{2(2\times10^3\,\text{Hz})} = (2n-1)(2.50\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t = 2.50\,\mu$s.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 31-1). Later this will repeat every half-period (compare steps c and g in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t = 1.25\,\mu$s.

5. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{\left(2.90\times 10^{-6}\,\mathrm{C}\right)^2}{2\left(3.60\times 10^{-6}\,\mathrm{F}\right)} = 1.17\times 10^{-6}\,\mathrm{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.168\times 10^{-6}\,\mathrm{J}\right)}{75\times 10^{-3}\,\mathrm{H}}} = 5.58\times 10^{-3}\,\mathrm{A}.$$

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\,\mathrm{N}}{\left(2.0\times10^{-13}\,\mathrm{m}\right)\left(0.50\,\mathrm{kg}\right)}} = 89\,\mathrm{rad/s}.$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{89\,\mathrm{rad/s}} = 7.0\times10^{-2}\,\mathrm{s}.$$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{\left(89\,\mathrm{rad/s}\right)^2\left(5.0\,\mathrm{H}\right)} = 2.5\times10^{-5}\,\mathrm{F}.$$

7. (a) The mass m corresponds to the inductance, so m = 1.25 kg.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\ \text{C}\right)^2}{2\left(5.70 \times 10^{-6}\ \text{J}\right)} = 2.69 \times 10^{-3}\ \text{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\ \text{m/N}} = 372\ \text{N/m}.$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\max} = 1.75 \times 10^{-4}$ m.

(d) The maximum speed $v_{\max}$ corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\ \text{C}}{\sqrt{(1.25\ \text{H})\left(2.69 \times 10^{-3}\ \text{F}\right)}} = 3.02 \times 10^{-3}\ \text{A}.$$

Consequently, $v_{\max} = 3.02 \times 10^{-3}$ m/s.

8. We find the inductance from $f = \omega / 2\pi = \left(2\pi\sqrt{LC}\right)^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10\times 10^3\ \text{Hz}\right)^2 \left(6.7\times 10^{-6}\ \text{F}\right)} = 3.8\times 10^{-5}\ \text{H}.$$

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}.$ Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\mathrm{H})(4.0\times10^{-6}\,\mathrm{F})}}{4} = 7.0\times10^{-4}\ \mathrm{s}.$$

10. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j} \right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j \right) = L \frac{di}{dt} + \frac{q}{C} + iR$$

with

$$L = \sum_j L_j, \quad \frac{1}{C} = \sum_j \frac{1}{C_j}, \quad R = \sum_j R_j$$

and we require $\varepsilon_{\text{total}} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-27(b).

11. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0\times10^{-3}\text{ H})(6.20\times10^{-6}\text{ F})}} = 275\text{ Hz}.$$

(b) When the switch is thrown, the capacitor is charged to $V = 34.0$ V and the current is zero. Thus, the maximum charge on the capacitor is $Q = VC = (34.0\text{ V})(6.20 \times 10^{-6}\text{ F}) = 2.11 \times 10^{-4}$ C. The current amplitude is

$$I = \omega Q = 2\pi f Q = 2\pi(275\text{ Hz})(2.11\times10^{-4}\text{ C}) = 0.365\text{ A}.$$

12. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\ \mathrm{H})(2.0\times10^{-6}\ \mathrm{F}+5.0\times10^{-6}\ \mathrm{F})}} = 6.0\times10^2\ \mathrm{Hz}.$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\ \mathrm{H})(5.0\times10^{-6}\ \mathrm{F})}} = 7.1\times10^2\ \mathrm{Hz}.$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\ \mathrm{H})(2.0\times10^{-6}\ \mathrm{F})}} = 1.1\times10^3\ \mathrm{Hz}.$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\ \mathrm{F}+5.0\times10^{-6}\ \mathrm{F}}{(1.0\times10^{-2}\ \mathrm{H})(2.0\times10^{-6}\ \mathrm{F})(5.0\times10^{-6}\ \mathrm{F})}} = 1.3\times10^3\ \mathrm{Hz}.$$

13. (a) The maximum charge is $Q = CV_{\max} = (1.0 \times 10^{-9}\text{ F})(3.0\text{ V}) = 3.0 \times 10^{-9}\text{ C}$.

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times 10^{-9}\text{ C}}{\sqrt{(3.0\times 10^{-3}\text{ H})(1.0\times 10^{-9}\text{ F})}} = 1.7\times 10^{-3}\text{ A}.$$

(c) When the current is at a maximum, the magnetic energy is at a maximum also:

$$U_{B,\max} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times 10^{-3}\text{ H})(1.7\times 10^{-3}\text{ A})^2 = 4.5\times 10^{-9}\text{ J}.$$

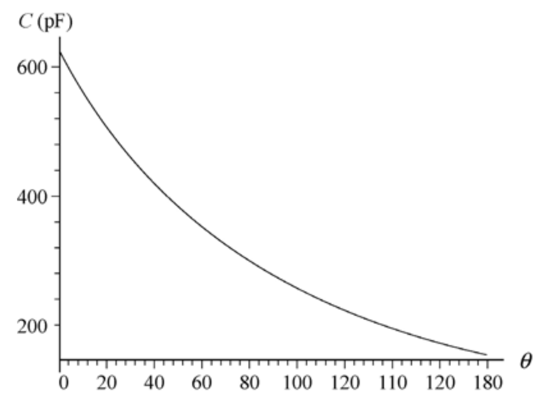
14. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1+\frac{\theta}{180°}\right) \Rightarrow \theta = 180°\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2 \times 10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2 \left(1+\theta/180°\right)^2} = \frac{81}{400000\pi^2\left(180°+\theta\right)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted below:



15. (a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0.$$

(b) An additional capacitance C is chosen so the ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads (pF), then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}.$$

(c) We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency C = 365 pF + 36 pF = 401 pF and f = 0.54 MHz. Thus

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2 (401 \times 10^{-12}\ \text{F})(0.54 \times 10^6\ \text{Hz})^2} = 2.2 \times 10^{-4}\ \text{H}.$$

16. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\mathrm{eq}}C_{\mathrm{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$

$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

17. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2 (64.0\times 10^{-6}\,\text{F})} = 2.50\times 10^{-3}\,\text{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times 10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20\times 10^{-3}\,\text{J}.$$

18. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1-75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q/Q = \sqrt{0.250} = 0.500.$

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\%,$$

we find $i/I = \sqrt{0.750} = 0.866.$

19. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} = \frac{\left(3.80\times10^{-6}\,\text{C}\right)^2}{2\left(7.80\times10^{-6}\,\text{F}\right)} + \frac{\left(9.20\times10^{-3}\,\text{A}\right)^2\left(25.0\times10^{-3}\,\text{H}\right)}{2} = 1.98\times10^{-6}\,\text{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2\left(7.80\times10^{-6}\,\text{F}\right)\left(1.98\times10^{-6}\,\text{J}\right)} = 5.56\times10^{-6}\,\text{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.98\times10^{-6}\,\text{J}\right)}{25.0\times10^{-3}\,\text{H}}} = 1.26\times10^{-2}\,\text{A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\,\text{C}}{5.56\times10^{-6}\,\text{C}}\right) = \pm46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$. We obtain $-\omega Q\sin\phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9°$.

20. (a) From $V = IX_C$ we find $\omega = I/CV$. The period is then $T = 2\pi/\omega = 2\pi CV/I = 46.1\ \mu s$.

(b) The maximum energy stored in the capacitor is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}(2.20\times10^{-7}\text{ F})(0.250\text{ V})^2 = 6.88\times10^{-9}\text{ J}.$$

(c) The maximum energy stored in the inductor is also $U_B = LI^2/2 = 6.88\text{ nJ}$.

(d) We apply Eq. 30-35 as $V = L(di/dt)_{max}$. We can substitute $L = CV^2/I^2$ (combining what we found in part (a) with Eq. 31-4) into Eq. 30-35 (as written above) and solve for $(di/dt)_{max}$. Our result is

$$\left(\frac{di}{dt}\right)_{max} = \frac{V}{L} = \frac{V}{CV^2/I^2} = \frac{I^2}{CV} = \frac{(7.50\times10^{-3}\text{ A})^2}{(2.20\times10^{-7}\text{ F})(0.250\text{ V})} = 1.02\times10^3\text{ A/s}.$$

(e) The derivative of $U_B = \frac{1}{2}Li^2$ leads to

$$\frac{dU_B}{dt} = LI^2\omega\sin\omega t\cos\omega t = \frac{1}{2}LI^2\omega\sin 2\omega t.$$

Therefore, $\left(\frac{dU_B}{dt}\right)_{max} = \frac{1}{2}LI^2\omega = \frac{1}{2}IV = \frac{1}{2}(7.50\times10^{-3}\text{ A})(0.250\text{ V}) = 0.938\text{ mW}.$

21. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC},$

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 1.80\times10^{-4}\,\text{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2\sin^2\omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega\,\sin\omega t\cos\omega t}{C}$$

We use the trigonometric identity $\cos\omega t\sin\omega t = \tfrac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 7.07\times10^{-5}\,\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 5.655\times10^{-4}\,\text{s},$ so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi(1.80\times10^{-4}\,\text{C})^2}{(5.655\times10^{-4}\,\text{s})(2.70\times10^{-6}\,\text{F})} = 66.7\,\text{W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

22. (a) We use $U = \frac{1}{2} LI^2 = \frac{1}{2} Q^2 / C$ to solve for L:

$$L = \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\max}}{I}\right)^2 = C\left(\frac{V_{\max}}{I}\right)^2 = \left(4.00\times10^{-6}\,\mathrm{F}\right)\left(\frac{1.50\,\mathrm{V}}{50.0\times10^{-3}\,\mathrm{A}}\right)^2 = 3.60\times10^{-3}\,\mathrm{H}.$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\,\mathrm{H}\right)\left(4.00\times10^{-6}\,\mathrm{F}\right)}} = 1.33\times10^{3}\,\mathrm{Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^{3}\ \mathrm{Hz}\right)} = 1.88\times10^{-4}\ \mathrm{s}.$$

23. The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Eq. 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\, d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

24. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{max}^2/2C$, where q_{max} is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \Rightarrow \quad q_{max} = \frac{Q}{\sqrt{2}}.$$

Now q_{max} (referred to as the *exponentially decaying amplitude* in §31-5) is related to Q (and the other parameters of the circuit) by

$$q_{max} = Qe^{-Rt/2L} \Rightarrow \ln\left(\frac{q_{max}}{Q}\right) = -\frac{Rt}{2L}.$$

Setting $q_{max} = Q/\sqrt{2}$, we solve for t:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2 .$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$t = 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{\left(220\times10^{-3}\,\mathrm{H}\right)\left(12.0\times10^{-6}\,\mathrm{F}\right)}\right)$$
$$= 0.5104\,\mathrm{s}.$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in §31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2\left(220\times10^{-3}\,\mathrm{H}\right)}{0.5104\,\mathrm{s}}\ln\left(0.99\right) = 8.66\times10^{-3}\ \Omega.$$

26. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$
\begin{aligned}
q &= Qe^{-Rt/2L}\cos(\omega' t+\phi) = Qe^{-RNT/2L}\cos\left[\omega'(2\pi N/\omega')+\phi\right] \\
&= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos(2\pi N+\phi) \\
&= Qe^{-N\pi R\sqrt{C/L}}\cos\phi.
\end{aligned}
$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\ \mu$C is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 \exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = (6.2\,\mu\text{C})\exp\left(-5\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\,\mu\text{C}.$

(b) For $N = 10$, $q_{10} = (6.2\,\mu\text{C})\exp\left(-10\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\,\mu\text{C}.$

(c) For $N = 100$, $q_{100} = (6.2\,\mu\text{C})\exp\left(-100\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\,\mu\text{C}.$

27. Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{\max 1}^2}{2C} = \frac{Q^2}{2C} e^{-Rt/L}$$

where

$$q_{\max 1} = Qe^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in §31-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2} = Qe^{-R(t+T)2/L} \qquad \text{where } T = \frac{2\pi}{\omega'},$$

and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C} e^{-R(t+T)/L} .$$

The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L} .$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots .$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \approx 1 - \left(1 - \frac{RT}{L} + \cdots\right) \approx \frac{RT}{L} = \frac{2\pi R}{\omega L}.$$

28. (a) The current through the resistor is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\text{V}}{50.0\,\Omega} = 0.600\ \text{A}\ .$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600\ \text{A}$.

29. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\,\text{H})(10\times10^{-6}\text{F})}} = 6.5\times10^{2}\ \text{Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650\ \text{Hz})(6.0\times10^{-3}\ \text{H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

30. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I = \omega_d C\varepsilon_m = 2\pi f_d C\varepsilon_m = 2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\text{ V}) = 0.283\text{ A}.$$

(b) $I = 2\pi(8.00 \times 10^3\text{ Hz})(1.50 \times 10^{-6}\text{ F})(30.0\text{ V}) = 2.26\text{ A}$.

31. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\varepsilon_m}{2\pi f_d L} = \frac{30.0\,\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\,\text{H})} = 0.0955\,\text{A} = 95.5\text{ mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955\text{ A})/8 = 0.0119\text{ A} = 11.9\text{ mA}.$$

32. (a) The circuit consists of one generator across one capacitor; therefore, $\varepsilon_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\varepsilon_m}{X_C} = \omega C\varepsilon_m = (377 \text{ rad/s})(4.15\times10^{-6}\text{F})(25.0 \text{ V}) = 3.91\times10^{-2}\text{ A} .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\max}$), but rather as it passes through the (momentary) states of being uncharged ($q = 0$). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\varepsilon(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\varepsilon(t) = 0$ when $i(t) = I$. The fact that $\phi = -90°$ $= -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\varepsilon = -\frac{1}{2}\varepsilon_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = (3.91\times10^{-3}\text{ A})\left(-\frac{\sqrt{3}}{2}\right) = -3.38\times10^{-2}\text{ A},$$

or $|i| = 3.38\times10^{-2}$ A.

33. (a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or

$$\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350 \text{ rad/s})} = 6.73 \times 10^{-3} \text{ s}.$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or

$$\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

The first time this occurs after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$ (as in part (a), $n = 0$). Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350 \text{ rad/s})} = 1.12 \times 10^{-2} \text{ s}.$$

(c) The current lags the emf by $+\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \varepsilon_m$. Thus, $\varepsilon_m = I\omega_d L$ and

$$L = \frac{\varepsilon_m}{I\omega_d} = \frac{30.0\text{V}}{(620 \times 10^{-3}\text{A})(350 \text{ rad/s})} = 0.138 \text{ H}.$$

34. (a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0 \text{ V}}{(377 \text{ rad/s})(12.7 \text{ H})} = 5.22 \times 10^{-3} \text{ A} .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m / 2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I \sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22 \times 10^{-3} \text{A})\left(\frac{\sqrt{3}}{2}\right) = 4.51 \times 10^{-3} \text{ A} .$$

35. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

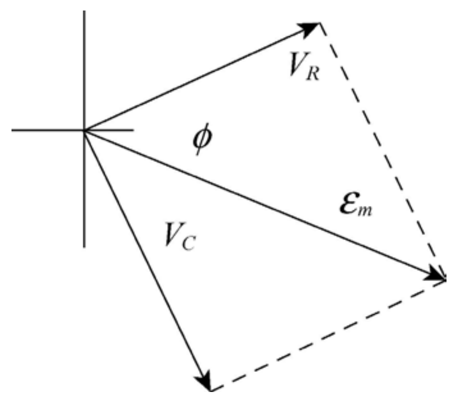
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{267\,\Omega} = 0.135\ \text{A}\ .$$



(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so I leads ε_m. The phasor diagram is drawn to scale on the right.

36. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance Z becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find R: 8.0/4.0 = 2.0 Ω.

37. (a) Now $X_C = 0$, while $R = 200\ \Omega$ and $X_L = \omega L = 2\pi f_d L = 86.7\ \Omega$ remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200\ \Omega)^2 + (86.7\ \Omega)^2} = 218\ \Omega\ .$$

(b) The phase angle is, from Eq. 31-65,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{200\ \Omega}\right) = 23.4°\ .$$

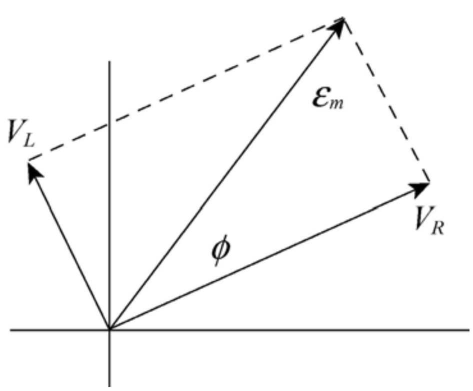
(c) The current amplitude is now found to be

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{218\ \Omega} = 0.165\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.165\ \text{A})(200\Omega) \approx 33\,\text{V}$$
$$V_L = IX_L = (0.165\,\text{A})(86.7\Omega) \approx 14.3\,\text{V}$$

This is an inductive circuit, so ε_m leads I. The phasor diagram is drawn to scale below.



38. (a) Since $Z = \sqrt{R^2 + X_L^2}$ and $X_L = \omega_d L$, then as $\omega_d \to 0$ we find $Z \to R = 40\ \Omega$.

(b) $L = X_L/\omega_d = slope = 60$ mH.

39. (a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{F})} = 37.9\,\Omega\ .$$

The inductive reactance 86.7 Ω is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\Omega)^2 + (37.9\Omega - 86.7\Omega)^2} = 206\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\Omega - 37.9\Omega}{200\Omega}\right) = 13.7°.$$

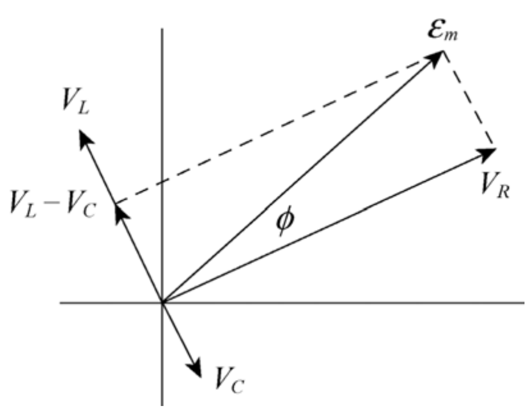
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{206\Omega} = 0.175\,\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.175\text{ A})(200\,\Omega) = 35.0\text{ V}$$
$$V_L = IX_L = (0.175\text{ A})(86.7\,\Omega) = 15.2\text{ V}$$
$$V_C = IX_C = (0.175\text{ A})(37.9\,\Omega) = 6.62\,\text{V}$$

Note that $X_L > X_C$, so that ε_m leads *I*. The phasor diagram is drawn to scale below.



40. (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: $R = 500\ \Omega$.

(b) We describe three methods here (each using information from different points on the graph):

method 1: At $\omega_d = 50$ rad/s, we have $Z \approx 700\ \Omega$ which gives $C = (\omega_d\sqrt{Z^2 - R^2}\,)^{-1} = 41\ \mu\text{F}$.

method 2: At $\omega_d = 50$ rad/s, we have $X_C \approx 500\ \Omega$ which gives $C = (\omega_d X_C)^{-1} = 40\ \mu\text{F}$.

method 3: At $\omega_d = 250$ rad/s, we have $X_C \approx 100\ \Omega$ which gives $C = (\omega_d X_C)^{-1} = 40\ \mu\text{F}$.

41. The rms current in the motor is

$$I_{\mathrm{rms}} = \frac{\mathcal{E}_{\mathrm{rms}}}{Z} = \frac{\mathcal{E}_{\mathrm{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\mathrm{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\mathrm{A}.$$

42. A phasor diagram very much like Fig. 31-11(d) leads to the condition:

$$V_L - V_C = (6.00 \text{ V})\sin(30^\circ) = 3.00 \text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is -8.00 V.

43. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi ,$$

which we solve for *R*:

$$R = \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) = \frac{1}{\tan 75°}\left[(2\pi)(930\,\text{Hz}(8.8\times10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times10^{-6}\text{F}}\right]$$

$$= 89\,\Omega.$$

44. (a) A sketch of the phasors would be very much like Fig. 31-9(c) but with the label "I_C" on the green arrow replaced with "V_R."

(b) We have $I\,R = I\,X_C$, or

$$I\,R = I\,X_C \;\rightarrow\; R = \frac{1}{\omega_d C}$$

which yields $f = \dfrac{\omega_d}{2\pi} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(50.0\ \Omega)(2.00\times10^{-5}\ \text{F})} = 159\ \text{Hz}$.

(c) $\phi = \tan^{-1}(-V_C/V_R) = -45°$.

(d) $\omega_d = 1/RC = 1.00\times10^3$ rad/s.

(e) $I = (12\ \text{V})/\sqrt{R^2 + X_C^2} = 6/(25\sqrt{2}) \approx 170$ mA.

45. (a) For a given amplitude ε_m of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}.$$

We find the maximum by setting the derivative with respect to ω_d equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m [R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right]\left[L + \frac{1}{\omega_d^2 C}\right].$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$; it does so for $\omega_d = 1/\sqrt{LC} = \omega$. For this

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\ \text{H})(20.0\times 10^{-6}\ \text{F})}} = 224\ \text{rad/s}.$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\ \text{V}}{5.00\ \Omega} = 6.00\ \text{A}.$$

(c) We want to find the (positive) values of ω_d for which $I = \varepsilon_m / 2R$:

$$\frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\varepsilon_m}{2R}.$$

This may be rearranged to yield

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2.$$

Taking the square root of both sides (acknowledging the two ± roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0.$$

Using the quadratic formula, we find the smallest positive solution

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{-\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\ \Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$+\frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$= 219\ \text{rad/s}\ .$$

(d) The largest positive solution

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{+\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\,\Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$+\frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$= 228\ \text{rad/s}\ .$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\text{rad/s} - 219\,\text{rad/s}}{224\ \text{rad/s}} = 0.040\,.$$

46. (a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to

$$X_{\text{net}} = (12\ \text{V})/(0.447\ \text{A}) = 26.85\ \Omega.$$

With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = \frac{X_{\text{net}}}{\tan\phi} = \frac{26.85\ \Omega}{\tan 15°} = 100\ \Omega\ .$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find

$$X_{\text{net first}} = R\tan\phi' = (100\ \Omega)\tan(-30.9°) = -59.96\ \Omega.$$

We observe that the effect of switch 1 implies

$$X_C = X_{\text{net}} - X_{\text{net firsrt}} = 26.85\ \Omega - (-59.96\ \Omega) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{\text{net}} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH .

47. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\text{ H}}{\sqrt{(1.0\text{ H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega\ .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\text{ V}}{10\ \Omega} = 1.0\ \text{A}\ .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\ \Omega) = 1.0\times10^3\ \text{V}$$

which is much larger than the amplitude of the generator emf.

48. (a) A sketch of the phasors would be very much like Fig. 31-10(c) but with the label "I_L" on the green arrow replaced with "V_R."

(b) We have $V_R = V_L$, which implies

$$I R = I X_L \;\rightarrow\; R = \omega_d L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times 10^3$ rad/s.

(e) $I = (6 \text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

49. We use the expressions found in Problem 31-45:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}, \quad \omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \; .$$

We also use Eq. 31-4. Thus,

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

For the data of Problem 31-45,

$$\frac{\Delta\omega_d}{\omega} = (5.00\,\Omega)\sqrt{\frac{3(20.0\times10^{-6}\text{ F})}{1.00\,\text{H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 31-45. The method of Problem 31-45, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

50. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(400\text{ Hz})(24.0\times10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi f L - X_C)^2}$$
$$= \sqrt{(220\,\Omega)^2 + [2\pi(400\text{ Hz})(150\times10^{-3}\text{ H}) - 16.6\ \Omega]^2} = 422\ \Omega\ .$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\text{ A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\text{ Hz})(150\times10^{-3}\text{ H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

51. (a) Since $L_{eq} = L_1 + L_2$ and $C_{eq} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\omega = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_1 + C_2 + C_3)}}$$

$$= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\text{ H} + 2.30\times10^{-3}\text{ H})(4.00\times10^{-6}\text{ F} + 2.50\times10^{-6}\text{ F} + 3.50\times10^{-6}\text{ F})}}$$

$$= 796\,\text{Hz}.$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{eq}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.

52. The amplitude (peak) value is

$$V_{\text{max}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}\left(100\,\text{V}\right) = 141\,\text{V}.$$

53. The average power dissipated in resistance R when the current is alternating is given by $P_{avg} = I_{rms}^2 R,$ where I_{rms} is the root-mean-square current. Since $I_{rms} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{avg} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R.$ Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\,\text{A}}{\sqrt{2}} = 1.84\,\text{A}.$$

54. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

55. (a) Using Eq. 31-61, the impedance is

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\,\Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\,\text{V})^2 (12.0\,\Omega)}{(12.07\,\Omega)^2} = 1.186\times10^3\ \text{W} \approx 1.19\times10^3\ \text{W}.$$

56. This circuit contains no reactances, so $\varepsilon_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_{\text{m}}^2(r-R)}{(r+R)^3} = 0 \quad \Rightarrow \quad R = r$$

57. (a) The power factor is cos ϕ, where ϕ is the phase constant defined by the expression $i = I \sin(\omega t - \phi)$. Thus, $\phi = -42.0°$ and $\cos \phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan \phi = (X_L - X_C)/R$. We have

$$\tan \phi = \tan(-42.0°) = -0.900,$$

a negative number. Therefore, $X_L - X_C$ is negative, which leads to $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance X_L would be the same as X_C, tan ϕ would be zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since tan ϕ is negative and finite, neither the capacitive reactance nor the resistance are zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I \cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}.$$

(i) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L and C then the value of the frequency would also be needed to compute the power factor.

58. The current in the circuit satisfies $i(t) = I \sin(\omega_d t - \phi)$, where

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$
$$= \frac{45.0\ \text{V}}{\sqrt{(16.0\ \Omega)^2 + \{(3000\ \text{rad/s})(9.20\ \text{mH}) - 1/[(3000\ \text{rad/s})(31.2\ \mu\text{F})]\}^2}}$$
$$= 1.93\ \text{A}$$

and

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right)$$
$$= \tan^{-1}\left[\frac{(3000\ \text{rad/s})(9.20\ \text{mH})}{16.0\ \Omega} - \frac{1}{(3000\ \text{rad/s})(16.0\ \Omega)(31.2\ \mu\text{F})}\right]$$
$$= 46.5°.$$

(a) The power supplied by the generator is

$$P_g = i(t)\varepsilon(t) = I \sin(\omega_d t - \phi)\varepsilon_m \sin \omega_d t$$
$$= (1.93\ \text{A})(45.0\ \text{V})\sin[(3000\ \text{rad/s})(0.442\ \text{ms})]\sin[(3000\ \text{rad/s})(0.442\ \text{ms}) - 46.5°]$$
$$= 41.4\ \text{W}.$$

(b) With

$$v_c(t) = V_c \sin(\omega_d t - \phi - \pi/2) = -V_c \cos(\omega_d t - \phi)$$

where $V_c = I/\omega_d C$, the rate at which the energy in the capacitor changes is

$$P_c = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = i\frac{q}{C} = iv_c$$
$$= -I \sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin[2(\omega_d t - \phi)]$$
$$= -\frac{(1.93\ \text{A})^2}{2(3000\ \text{rad/s})(31.2\times10^{-6}\ \text{F})}\sin[2(3000\ \text{rad/s})(0.442\ \text{ms}) - 2(46.5°)]$$
$$= -17.0\ \text{W}.$$

(c) The rate at which the energy in the inductor changes is

$$P_L = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}\left[I\sin(\omega_d t - \phi)\right] = \frac{1}{2}\omega_d LI^2 \sin\left[2(\omega_d t - \phi)\right]$$
$$= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right]$$
$$= 44.1\ \text{W}.$$

(d) The rate at which energy is being dissipated by the resistor is

$$P_R = i^2 R = I^2 R \sin^2(\omega_d t - \phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right]$$
$$= 14.4\ \text{W}.$$

(e) Equal. $P_L + P_R + P_c = 44.1\,\text{W} - 17.0\ \text{W} + 14.4\,\text{W} = 41.5\ \text{W} = P_g$.

59. We shall use

$$P_{\text{avg}} = \frac{\varepsilon_m^2 R}{2Z^2} = \frac{\varepsilon_m^2 R}{2\left[R^2 + \left(\omega_d L - 1/\omega_d C\right)^2\right]}.$$

where $Z = \sqrt{R^2 + \left(\omega_d L - 1/\omega_d C\right)^2}$ is the impedance.

(a) Considered as a function of *C*, P_{avg} has its largest value when the factor $R^2 + \left(\omega_d L - 1/\omega_d C\right)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 \left(60.0\times10^{-3}\,\text{H}\right)} = 1.17\times10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as *C* becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is $\cos\phi = \cos 0° = 1$.

(f) The minimum average power is $P_{\text{avg}} = 0$ (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(h) At minimum power, the power factor is $\cos\phi = \cos(-90°) = 0$.

60. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{max}/P_{min} = (I/I_{min})^2 = 5$, or

$$\left(\frac{I}{I_{min}}\right)^2 = \left(\frac{\varepsilon_m / Z_{min}}{\varepsilon_m / Z_{max}}\right)^2 = \left(\frac{Z_{max}}{Z_{min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{max})^2}}{R}\right)^2 = 5.$$

We solve for L_{max}:

$$L_{max} = \frac{2R}{\omega} = \frac{2(120\,\text{V})^2 / 1000\,\text{W}}{2\pi(60.0\,\text{Hz})} = 7.64 \times 10^{-2}\,\text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{max} + R_{bulb}}{R_{bulb}}\right)^2 = 5,$$

or

$$R_{max} = \left(\sqrt{5} - 1\right)R_{bulb} = \left(\sqrt{5} - 1\right)\frac{(120\,\text{V})^2}{1000\,\text{W}} = 17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

61. (a) The rms current is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + \left(2\pi fL - 1/2\pi fC\right)^2}}$$
$$= \frac{75.0\,\text{V}}{\sqrt{(15.0\,\Omega)^2 + \left\{2\pi(550\,\text{Hz})(25.0\,\text{mH}) - 1/\left[2\pi(550\,\text{Hz})(4.70\,\mu\text{F})\right]\right\}^2}}$$
$$= 2.59\,\text{A}.$$

(b) The rms voltage across R is

$$V_{ab} = I_{\text{rms}}R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\,\text{V}\,.$$

(c) The rms voltage across C is

$$V_{bc} = I_{\text{rms}}X_C = \frac{I_{\text{rms}}}{2\pi fC} = \frac{2.59\,\text{A}}{2\pi(550\,\text{Hz})(4.70\,\mu\text{F})} = 159\,\text{V}\,.$$

(d) The rms voltage across L is

$$V_{cd} = I_{\text{rms}}X_L = 2\pi I_{\text{rms}} fL = 2\pi(2.59\,\text{A})(550\,\text{Hz})(25.0\,\text{mH}) = 224\,\text{V}\,.$$

(e) The rms voltage across C and L together is

$$V_{bd} = \left|V_{bc} - V_{cd}\right| = \left|159.5\,\text{V} - 223.7\,\text{V}\right| = 64.2\text{V}$$

(f) The rms voltage across R, C and L together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,\text{V})^2 + (64.2\,\text{V})^2} = 75.0\,\text{V}$$

(g) For R,

$$P_R = \frac{V_{ab}^2}{R} = \frac{(38.8\,\text{V})^2}{15.0\,\Omega} = 100\,\text{W}.$$

(h) No energy dissipation in C.

(i) No energy dissipation in L.

62. For step-up trasnformer:

(a) The smallest value of the ratio V_s / V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: V_{13}/V_{23} = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: V_{23}/V_{13} = 800/200 = 4.00.

(c) The largest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: V_{13}/V_{12} = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

63. (a) The stepped-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) By Ohm's law, the current in the secondary is $I_s = \dfrac{V_s}{R_s} = \dfrac{2.4\,\text{V}}{15\,\Omega} = 0.16\,\text{A}.$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times 10^{-3}\,\text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}$.

64. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00\times 10^3\ \text{V}.$$

65. (a) The rms current in the cable is $I_{\text{rms}} = P/V_t = 250\times10^3\,\text{W}/\left(80\times10^3\ \text{V}\right) = 3.125\,\text{A}$. Therefore, the rms voltage drop is $\Delta V = I_{\text{rms}}R = (3.125\,\text{A})(2)(0.30\,\Omega) = 1.9\ \text{V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2 R = (3.125\,\text{A})(2)(0.60\,\Omega) = 5.9\,\text{W}$.

(c) Now $I_{\text{rms}} = 250\times10^3\ \text{W}/\left(8.0\times10^3\ \text{V}\right) = 31.25\,\text{A}$, so $\Delta V = (31.25\,\text{A})(0.60\Omega) = 19\text{V}$.

(d) $P_d = (3.125\,\text{A})^2(0.60\,\Omega) = 5.9\times10^2\ \text{W}$.

(e) $I_{\text{rms}} = 250\times10^3\ \text{W}/\left(0.80\times10^3\ \text{V}\right) = 312.5\ \text{A}$, so $\Delta V = (312.5\,\text{A})(0.60\,\Omega) = 1.9\times10^2\ \text{V}$.

(f) $P_d = (312.5\,\text{A})^2\,(0.60\ \Omega) = 5.9\times10^4\ \text{W}$.

66. (a) The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\,\text{hp})(746\,\text{W/hp})}{(0.650\,\text{A})^2} = 177\,\Omega.$$

(b) This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\text{rms}}^2 R$ would not give $P_{\text{mechanical}}$ but rather the rate of energy loss due to thermal dissipation.

67. (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}A\cos(\omega_d t - 60°)$$

where we use

$$\sin\alpha - \sin\beta = 2\sin\left[(\alpha-\beta)/2\right]\cos\left[(\alpha+\beta)/2\right]$$

and $\sin 60° = \sqrt{3}/2$. Similarly,

$$\Delta V_{13} = A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right) = \sqrt{3}A\cos(\omega_d t - 120°)$$

and

$$\Delta V_{23} = A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right)$$
$$= \sqrt{3}A\cos(\omega_d t - 180°)$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

68. (a) Eq. 31-39 gives $f = \omega/2\pi = (2\pi C X_C)^{-1} = 8.84$ kHz.

(b) Because of its inverse relationship with frequency, then the reactance will go down by a factor of 2 when f increases by a factor of 2. The answer is $X_C = 6.00\ \Omega$.

69. (a) The impedance is $Z = \dfrac{\varepsilon_m}{I} = \dfrac{125\,\text{V}}{3.20\,\text{A}} = 39.1\ \Omega.$

(b) From $V_R = IR = \varepsilon_m \cos\phi,$ we get

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(125\,\text{V})\cos(0.982\,\text{rad})}{3.20\,\text{A}} = 21.7\,\Omega.$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\,\text{rad}),$ we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

70. (a) Eq. 31-4 directly gives $1/\sqrt{LC} \approx 5.77\times10^3$ rad/s.

(b) Eq. 16-5 then yields $T = 2\pi/\omega = 1.09$ ms.

(c) Although we do not show the graph here, we describe it: it is a cosine curve with amplitude 200 μC and period given in part (b).

71. (a) The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{R}\right) = \tan^{-1}\left(\frac{V_L - V_L / 2.00}{V_L / 2.00}\right) = \tan^{-1}(1.00) = 45.0°.$$

(b) We solve R from $\varepsilon_m \cos\phi = IR$:

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(30.0\,\text{V})(\cos 45°)}{300 \times 10^{-3}\,\text{A}} = 70.7\ \Omega.$$

72. From Eq. 31-4, we have $C = (\omega^2 L)^{-1} = ((2\pi f)^2 L)^{-1} = 1.59\ \mu\text{F}$.

73. (a) We solve L from Eq. 31-4, using the fact that $\omega = 2\pi f$:

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10.4 \times 10^3 \text{ Hz}\right)^2 \left(340 \times 10^{-6} \text{ F}\right)} = 6.89 \times 10^{-7} \text{ H}.$$

(b) The total energy may be calculated from the inductor (when the current is at maximum):

$$U = \frac{1}{2} L I^2 = \frac{1}{2}\left(6.89 \times 10^{-7} \text{ H}\right)\left(7.20 \times 10^{-3} \text{ A}\right)^2 = 1.79 \times 10^{-11} \text{ J}.$$

(c) We solve for Q from $U = \frac{1}{2} Q^2 / C$:

$$Q = \sqrt{2CU} = \sqrt{2\left(340 \times 10^{-6} \text{ F}\right)\left(1.79 \times 10^{-11} \text{ J}\right)} = 1.10 \times 10^{-7} \text{ C}.$$

74. (a) With a phase constant of 45º the (net) reactance must equal the resistance in the circuit, which means the circuit impedance becomes

$$Z = R\sqrt{2} \Rightarrow R = Z/\sqrt{2} = 707\ \Omega.$$

(b) Since $f = 8000$ Hz then $\omega_d = 2\pi(8000)$ rad/s. The net reactance (which, as observed, must equal the resistance) is therefore $X_L - X_C = \omega_d L - (\omega_d C)^{-1} = 707\ \Omega$. We are also told that the resonance frequency is 6000 Hz, which (by Eq. 31-4) means

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (6000\text{ Hz})^2 L}.$$

Substituting this in for C in our previous expression (for the net reactance) we obtain an equation that can be solved for the self-inductance. Our result is $L = 32.2$ mH.

(c) $C = ((2\pi(6000))^2 L)^{-1} = 21.9$ nF.

75. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu\text{H}$.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{\text{max}} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{\text{max}} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{\text{max}}} = 82.2$ nC.

76. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}.$$

77. (a) The impedance is $Z = (80.0\ \text{V})/(1.25\ \text{A}) = 64.0\ \Omega$.

(b) We can write $\cos\phi = R/Z \Rightarrow R = (64.0\ \Omega)\cos(0.650\ \text{rad}) = 50.9\ \Omega$.

(c) Since the “current leads the emf” the circuit is capacitive.

78. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^3\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^3\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\left(4.60\times10^3\,\text{Hz}\right)\left(1.30\times10^3\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus, $X_L = 2(1.30 \times 10^3\ \Omega) = 2.60 \times 10^3\ \Omega$.

(d) $X_C = 1.30 \times 10^3\ \Omega/2 = 6.50 \times 10^2\ \Omega$.

79. (a) Using $\omega = 2\pi f$, $X_L = \omega L$, $X_C = 1/\omega C$ and $\tan(\phi) = (X_L - X_C)/R$, we find

$$\phi = \tan^{-1}[(16.022 - 33.157)/40.0] = -0.40473 \approx -0.405 \text{ rad}.$$

(b) Eq. 31-63 gives $I = 120/\sqrt{40^2 + (16\text{-}33)^2} = 2.7576 \approx 2.76$ A.

(c) $X_C > X_L \Rightarrow$ capacitive.

80. From $U_{\max} = \frac{1}{2}LI^2$ we get $I = 0.115$ A.

81. From Eq. 31-4 we get $f = 1/2\pi\sqrt{LC} = 1.84$ kHz

82. (a) The reactances are as follows:

$$X_L = 2\pi f_d L = 2\pi(400\text{ Hz})(0.0242\text{ H}) = 60.82\ \Omega$$

$$X_C = (2\pi f_d C)^{-1} = [2\pi(400\text{ Hz})(1.21\times10^{-5}\text{ F})]^{-1} = 32.88\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0\ \Omega)^2 + (60.82\ \Omega - 32.88\ \Omega)^2} = 34.36\ \Omega$$

With $\varepsilon = 90.0\text{ V}$, we have

$$I = \frac{\varepsilon}{Z} = \frac{90.0\text{ V}}{34.36\ \Omega} = 2.62\text{ A} \ \Rightarrow\ I_{\text{rms}} = \frac{I}{\sqrt{2}} = \frac{2.62\text{ A}}{\sqrt{2}} = 1.85\text{ A}.$$

Therefore, the rms potential difference across the resistor is $V_{R\,\text{rms}} = I_{\text{rms}} R = 37.0$ V.

(b) Across the capacitor, the rms potential difference is $V_{C\,\text{rms}} = I_{\text{rms}} X_C = 60.9$ V.

(c) Similarly, across the inductor, the rms potential difference is $V_{L\,\text{rms}} = I_{\text{rms}} X_L = 113$ V.

(d) The average rate of energy dissipation is $P_{\text{avg}} = (I_{\text{rms}})^2 R = 68.6$ W.

83. (a) At any time, the total energy U in the circuit is the sum of the energy U_E in the capacitor and the energy U_B in the inductor. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor. Thus,

$$\frac{Q^2}{2C} = \frac{3.00q^2}{2C} \quad \Rightarrow \quad q = \frac{Q}{\sqrt{3.00}} = 0.577Q .$$

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t$ =0.557, or ωt = 0.955 rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$, or $t/T = 0.152$.

84. From Eq. 31-60, we have $(220\text{ V}/3.00\text{ A})^2 = R^2 + X_L^2 \Rightarrow X_L = 69.3\,\Omega$.

85. (a) The energy stored in the capacitor is given by $U_E = q^2/2C$. Since q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}i^2 L$. Since i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the generator is

$$P_{\text{avg}}T = \left(I_{\text{rms}}\varepsilon_{\text{rms}}\cos\phi\right)T = \left(\frac{1}{2}T\right)\varepsilon_m I\cos\phi$$

where we substitute $I_{\text{rms}} = I/\sqrt{2}$ and $\varepsilon_{\text{rms}} = \varepsilon_m/\sqrt{2}$.

(d) The energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\,T = \left(I_{\text{rms}}V_R\right)T = I_{\text{rms}}\left(I_{\text{rms}}R\right)T = \left(\frac{1}{2}T\right)I^2R.$$

(e) Since $\varepsilon_m I\cos\phi = \varepsilon_m I\left(V_R/\varepsilon_m\right) = \varepsilon_m I\left(IR/\varepsilon_m\right) = I^2R$, the two quantities are indeed the same.

86. (a) We note that we obtain the maximum value in Eq. 31-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417 \text{ s}$$

or 4.17 ms. The result is $\varepsilon_m \sin(\pi/2) = \varepsilon_m \sin(90°) = 36.0 \text{ V}$.

(b) At t = 4.17 ms, the current is

$$i = I \sin(\omega_d t - \phi) = I \sin(90° - (-24.3°)) = (0.164\,\text{A}) \cos(24.3°) = 0.1495\,\text{A} \approx 0.150 \text{ A}.$$

using Eq. 31-29 and the results of the Sample Problem. Ohm's law directly gives

$$v_R = iR = (0.1495\,\text{A})(200\Omega) = 29.9\,\text{V}.$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at t = 4.17 ms, we obtain

$$v_C = I \sin(90° - (-24.3°) - 90°) X_C = I X_C \sin(24.3°) = (0.164\,\text{A})(177\Omega)\sin(24.3°) = 11.9\,\text{V}.$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at t = 4.17 ms, we find

$$\begin{aligned} v_L &= I \sin(90° - (-24.3°) + 90°) X_L = -I X_L \sin(24.3°) = -(0.164\,\text{A})(86.7\Omega)\sin(24.3°) \\ &= -5.85\,\text{V}. \end{aligned}$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

87. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/\left[4(350\,\text{rad/s})\right] = 6.73\times10^{-3}$ s.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/\left[4(350\,\text{rad/s})\right] = 2.24\times10^{-3}$ s.

(c) Since i leads ε in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \varepsilon_m / I$:

$$C = \frac{I}{\varepsilon_m \omega} = \frac{6.20\times10^{-3}\text{ A}}{(30.0\text{ V})(350\,\text{rad/s})} = 5.90\times10^{-5}\text{ F}.$$

88. (a) The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings.

(b) If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = \left(N_p / N_s\right) I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R.$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\varepsilon_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\varepsilon_{\text{rms}}}{r + \left(N_p / N_s\right)^2 R}$$

where Eq. 31-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\varepsilon^2 (N_p / N_s)^2 R}{\left[r + (N_p / N_s)^2 R\right]^2}.$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\varepsilon^2 R x}{(r + xR)^2},$$

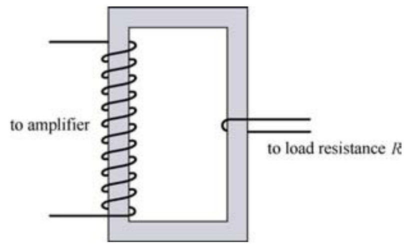
and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\varepsilon^2 R (r - xR)}{(r + xR)^3}.$$

This is zero for $x = r / R = (1000\,\Omega)/(10\,\Omega) = 100$. We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for

$$N_p / N_s = \sqrt{x} = 10.$$

The diagram that follows is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



89. Resonance occurs when the inductive reactance equals the capacitive reactance. Reactances of a certain type add (in series) just like resistances did in Chapter 28. Thus, since the resonance ω values are the same for both circuits, we have for each circuit:

$$\omega L_1 = \frac{1}{\omega C_1}, \quad \omega L_2 = \frac{1}{\omega C_2}$$

and adding these equations we find

$$\omega(L_1 + L_2) = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

Since $L_{eq} = L_1 + L_2$ and $C_{eq}^{-1} = (C_1^{-1} + C_2^{-1})$,

$$\omega L_{eq} = \frac{1}{\omega C_{eq}} \Rightarrow \text{ resonance in the combined circuit.}$$

90. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C \tau_L}$.

91. When the switch is open, we have a series *LRC* circuit involving just the one capacitor near the upper right corner. Eq. 31-65 leads to

$$\frac{\omega_d L-\frac{1}{\omega_d C}}{R}=\tan\phi_o=\tan(-20°)=-\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is 2*C*. In this case, we have

$$\frac{\omega_d L-\frac{1}{2\omega_d C}}{R}=\tan\phi_1=\tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an *LC* circuit with current amplitude

$$I_2=\frac{\varepsilon_m}{Z_{LC}}=\frac{\varepsilon_m}{\sqrt{\left(\omega_d L-\frac{1}{\omega_d C}\right)^2}}=\frac{\varepsilon_m}{\frac{1}{\omega_d C}-\omega_d L}$$

where we use the fact that $(\omega_d C)^{-1}>\omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for *L, R* and *C* from the three equations above, and the results are

(a) $R=\dfrac{-\varepsilon_m}{I_2\tan\phi_o}=\dfrac{-120\text{V}}{(2.00\,\text{A})\tan(-20.0°)}=165\,\Omega.$

(b) $L=\dfrac{\varepsilon_m}{\omega_d I_2}\left(1-2\dfrac{\tan\phi_1}{\tan\phi_o}\right)=\dfrac{120\,\text{V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1-2\dfrac{\tan 10.0°}{\tan(-20.0°)}\right)=0.313\text{ H}.$

(c)

$$C=\frac{I_2}{2\omega_d\varepsilon_m\left(1-\tan\phi_1/\tan\phi_0\right)}=\frac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1-\tan 10.0°/\tan(-20.0°)\right)}$$
$$=1.49\times10^{-5}\text{ F}$$

92. (a) Eqs. 31-4 and 31-14 lead to $Q = \frac{1}{\omega} = I\sqrt{LC} = 1.27\times10^{-6}\,\text{C}$.

(b) We choose the phase constant in Eq. 31-12 to be $\phi = -\pi/2$, so that i_0 = I in Eq. 31-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2 \ .$$

Differentiating and using the fact that 2 sin θ cos θ = sin 2θ, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\omega\sin 2\omega t \ .$$

We find the maximum value occurs whenever sin $2\omega t = 1$, which leads (with n = *odd* integer) to

$$t = \frac{1}{2\omega}\frac{n\pi}{2} = \frac{n\pi}{4\omega} = \frac{n\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\,\text{s}, \ 2.49\times10^{-4}\,\text{s}, \ldots \ .$$

The earliest time is $t = 8.31\times10^{-5}$ s.

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\frac{I}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\,\text{J/s} \ .$$

93. (a) We observe that $\omega = 6597$ rad/s, and, consequently, $X_L = 594\ \Omega$ and $X_C = 303\ \Omega$. Since $X_L > X_C$, the phase angle is positive: $\phi = +60.0°$.

From Eq. 31-65, we obtain $R = \dfrac{X_L - X_C}{\tan\phi} = 168\,\Omega$.

(b) Since we are already on the "high side" of resonance, increasing f will only decrease the current further, but *decreasing* f brings us closer to resonance and, consequently, large values of I.

(c) Increasing L increases X_L, but we already have $X_L > X_C$. Thus, if we wish to move closer to resonance (where X_L must equal X_C), we need to *decrease* the value of L.

(d) To change the present condition of $X_C < X_L$ to something closer to $X_C = X_L$ (resonance, large current), we can increase X_C. Since X_C depends inversely on C, this means *decreasing* C.

94. (a) We observe that ω_d = 12566 rad/s. Consequently, X_L = 754 Ω and X_C = 199 Ω. Hence, Eq. 31-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22 \text{ rad} .$$

(b) We find the current amplitude from Eq. 31-60: $I = \dfrac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288\,\text{A}$.

95. From Eq. 31-4, with $\omega = 2\pi f = 4.49 \times 10^3$ rad / s, we obtain

$$L = \frac{1}{\omega^2 C} = 7.08 \times 10^{-3} \text{ H}.$$

96. (a) From Eq. 31-4, with $\omega = 2\pi f$, C = 2.00 nF and L = 2.00 mH, we have

$$f = \frac{1}{2\pi\sqrt{LC}} = 7.96\times10^{4}\,\text{Hz}.$$

(b) The maximum current in the oscillator is $i_{\text{max}} = I_C = \dfrac{V_C}{X_C} = \omega C v_{\text{max}} = 4.00\times10^{-3}$ A.

(c) Using Eq. 30-49, we find the maximum magnetic energy:

$$U_{B,\text{max}} = \frac{1}{2}Li_{\text{max}}^2 = 1.60\times10^{-8}\,\text{J}.$$

(d) Adapting Eq. 30-35 to the notation of this chapter, $v_{\text{max}} = L\,|\,di/dt\,|_{\text{max}}$, which yields a (maximum) time rate of change (for i) equal to 2.00×10^{3} A/s.

97. Reading carefully, we note that the driving frequency of the source is permanently set at the resonance frequency of the *initial* circuit (with switches open); it is set at $\omega_d = 1/\sqrt{LC} = 1.58 \times 10^4$ rad/s and does not correspond to the resonance frequency once the switches are closed. In our table, below, C_{eq} is in μF, f is in kHz, and R_{eq} and Z are in Ω. Steady state conditions are assumed in calculating the current amplitude (which is in amperes); this I is the current through the source (or through the inductor), as opposed to the (generally smaller) current in one of the resistors. Resonant frequencies f are computed with $\omega = 2\pi f$. Reducing capacitor and resistor combinations is explained in chapters 26 and 28, respectively.

switch	(a) C_{eq}(μF)	(b) f(kHz)	(c) R_{eq}(Ω)	(d) Z(Ω)	(e) I (A)
S_1	4.00	1.78	12.0	19.8	0.605
S_2	5.00	1.59	12.0	22.4	0.535
S_3	5.00	1.59	6.0	19.9	0.603
S_4	5.00	1.59	4.0	19.4	0.619

Chapter 32

1. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -\left(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}\right) = +3\,\text{Wb}\ .$$

2. (a) The flux through the top is $+(0.30\text{ T})\pi r^2$ where $r = 0.020$ m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. (a) We use Gauss' law for magnetism: $\oint \vec{B}\cdot d\vec{A}=0$. Now,

$$\oint \vec{B}\cdot d\vec{A}=\Phi_1+\Phi_2+\Phi_C,$$

where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is Φ_1 = –25.0 μWb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. It value is

$$\Phi_2=\pi(0.120\,\text{m})^2\left(1.60\times10^{-3}\,\text{T}\right)=+7.24\times10^{-5}\,\text{Wb}=+72.4\,\mu\text{Wb}\ .$$

Since the three fluxes must sum to zero,

$$\Phi_C=-\Phi_1-\Phi_2=25.0\,\mu\text{Wb}-72.4\,\mu\text{Wb}=-47.4\,\mu\text{Wb}\ .$$

Thus, the magnitude is $|\Phi_C|=47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_c indicates that the flux is inward through the curved surface.

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1) = \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx = 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx = 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx = \frac{\mu_0 iL}{\pi}\ln 3\,.$$

5. We use the result of part (b) in Sample Problem 32-1:

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}, \qquad (r \geq R)$$

to solve for dE/dt:

$$\frac{dE}{dt} = \frac{2Br}{\mu_0 \varepsilon_0 R^2} = \frac{2\left(2.0\times10^{-7}\ \mathrm{T}\right)\left(6.0\times10^{-3}\ \mathrm{m}\right)}{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(3.0\times10^{-3}\,\mathrm{m}\right)^2} = 2.4\times10^{13}\ \frac{\mathrm{V}}{\mathrm{m\cdot s}}.$$

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d \left(\frac{\text{enclosed area}}{\text{total area}} \right) = \mu_0 (0.75 \text{ A}) \frac{(4.0 \text{ cm})(2.0 \text{ cm})}{12 \text{ cm}^2} = 52 \text{ nT} \cdot \text{m} .$$

7. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where $d = 5.0$ mm), we use the result of part (a) in Sample Problem 32-1

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d}\frac{dV}{dt} \qquad (r \le R).$$

We also use the fact that the time derivative of sin (ωt) (where $\omega = 2\pi f = 2\pi(60) \approx 377/\text{s}$ in this problem) is $\omega \cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \le R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max} \omega \cos(\omega t) \quad \Rightarrow \quad B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max} \omega}{2d}$$

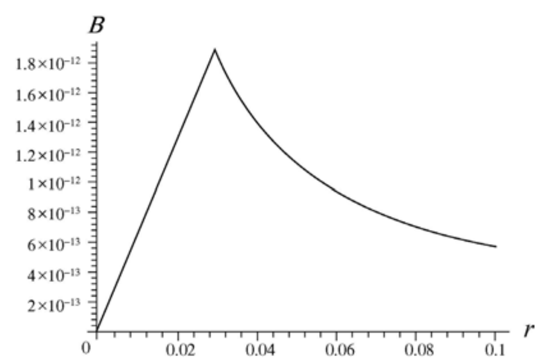
where $V_{\max} = 150$ V. This grows with r until reaching its highest value at $r = R = 30$ mm:

$$B_{\max}\Big|_{r=R} = \frac{\mu_0 \varepsilon_0 R V_{\max} \omega}{2d} = \frac{(4\pi \times 10^{-7}\ \text{H/m})(8.85 \times 10^{-12}\ \text{F/m})(30 \times 10^{-3}\ \text{m})(150\ \text{V})(377/\text{s})}{2(5.0 \times 10^{-3}\ \text{m})}$$
$$= 1.9 \times 10^{-12}\ \text{T}.$$

(b) For $r \le 0.03$ m, we use the expression $B_{\max} = \mu_0 \varepsilon_0 r V_{\max} \omega / 2d$ found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03$ m we perform a similar calculation starting with the result of part (b) in Sample Problem 32-1:

$$B_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd} V_{\max} \omega \cos(\omega t)\right)_{\max}$$
$$= \frac{\mu_0 \varepsilon_0 R^2 V_{\max} \omega}{2rd} \qquad (\text{for}\, r \ge R)$$

(note the $B \propto r^{-1}$ dependence — See also Eqs. 32-16 and 32-17). The plot (with SI units understood) is shown below.



8. From Sample Problem 32-1 we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of B_{max}. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\ B_{max}/B_{max} = r_1/R$, or $r_1 = 0.75\ R = 0.75\ (40\text{ mm}) = 30\text{ mm}$.

(b) Outside the capacitor, $0.75\ B_{max}/B_{max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40\text{ mm}) = 53\text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(6.0\,\text{A}\right)}{2\pi\left(0.040\,\text{m}\right)} = 3.0 \times 10^{-5}\ \text{T}.$$

9. (a) Inside we have (by Eq. 32-16) $B = \mu_0 i_d r_1 / 2\pi R^2$, where $r_1 = 0.0200$ m, $R = 0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s}) = 2.66\times10^{-14}\ \text{A}\ .$$

Thus we find

$$B = \frac{\mu_0 i_d r_1}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2} = 1.18\times10^{-19}\ \text{T}\ .$$

(b) Outside we have (by Eq. 32-17) $B = \mu_0 i_d / 2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B = \frac{\mu_0 i_d}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})} = 1.06\times10^{-19}\ \text{T}$$

10. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}.$$

Using $r = 0.0200$ m (which, in any case, cancels out) and $R = 0.0300$ m, we obtain

$$\begin{aligned} B &= \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})} \\ &= 3.54\times10^{-17}\ \text{T}. \end{aligned}$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set $r = 0.0500$ m and solve. We now find

$$\begin{aligned} B &= \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})} \\ &= 2.13\times10^{-17}\ \text{T}. \end{aligned}$$

11. (a) Application of Eq. 32-7 with $A = \pi r^2$ (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r) = \varepsilon_0 \mu_0 \pi r^2 \left(0.00450 \text{ V/m} \cdot \text{s}\right).$$

For $r = 0.0200$ m, this gives

$$\begin{aligned} B &= \frac{1}{2}\varepsilon_0 \mu_0 r (0.00450 \text{ V/m} \cdot \text{s}) \\ &= \frac{1}{2}(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.0200 \text{ m})(0.00450 \text{ V/m}\cdot\text{s}) \\ &= 5.01\times10^{-22} \text{ T}. \end{aligned}$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0 \mu_0 \pi R^2 \left(0.00450 \text{ V/m} \cdot \text{s}\right).$$

Substituting $r = 0.050$ m and $R = 0.030$ m, we obtain $B = 4.51 \times 10^{-22}$ T.

12. (a) Here, the enclosed electric flux is found by integrating

$$\Phi_E = \int_0^r E\, 2\pi r dr = t(0.500\ \text{V/m}\cdot\text{s})(2\pi)\int_0^r \left(1-\frac{r}{R}\right) r dr = t\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Then (after taking the derivative with respect to time) Eq. 32-3 leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right).$$

For r = 0.0200 m and R = 0.0300 m, this gives $B = 3.09 \times 10^{-20}$ T.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$\Phi_E = t\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{1}{6}t\pi R^2 .$$

Consequently, Eq. 32-3 becomes

$$B(2\pi r) = \frac{1}{6}\varepsilon_0\mu_0\pi R^2$$

which for r = 0.0500 m, yields

$$B = \frac{\varepsilon_0\mu_0 R^2}{12r} = \frac{(8.85\times10^{-12})(4\pi\times10^{-7})(0.030)^2}{12(0.0500)} = 1.67\times10^{-20}\ \text{T} .$$

13. The displacement current is given by $i_d = \varepsilon_0 A(dE/dt)$, where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus,

$$i_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt}.$$

14. We use Eq. 32-14: $i_d = \varepsilon_0 A(dE/dt)$. Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\text{A}}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.10\,\text{m}\right)^2} = 7.2\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

15. Let the area plate be *A* and the plate separation be *d*. We use Eq. 32-10:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(AE) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\,\text{A}}{2.0\times 10^{-6}\,\text{F}} = 7.5\times 10^5\ \text{V/s}.$$

Therefore, we need to change the voltage difference across the capacitor at the rate of $7.5\times 10^5\ \text{V/s}$.

16. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation , $i_d = \varepsilon_0 A(dE/dt)$, so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt}.$$

17. (a) We use $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ to find

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0 \left(J_d \pi r^2\right)}{2\pi r} = \frac{1}{2}\mu_0 J_d r = \frac{1}{2}\left(1.26\times10^{-6}\ \text{H/m}\right)\left(20\ \text{A/m}^2\right)\left(50\times10^{-3}\ \text{m}\right)$$
$$= 6.3\times10^{-7}\ \text{T}.$$

(b) From $i_d = J_d \pi r^2 = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \dfrac{dE}{dt}$, we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\ \text{A/m}^2}{8.85\times10^{-12}\ \text{F/m}} = 2.3\times10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

18. (a) From Eq. 32-10,

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} \varepsilon_0 A \frac{d}{dt}\left[\left(4.0\times10^5\right)-\left(6.0\times10^4 t\right)\right] = -\varepsilon_0 A\left(6.0\times10^4\ \mathrm{V/m\cdot s}\right) \\ &= -\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.0\times10^{-2}\,\mathrm{m^2}\right)\left(6.0\times10^4\ \mathrm{V/m\cdot s}\right) \\ &= -2.1\times10^{-8}\,\mathrm{A}. \end{aligned}$$

Thus, the magnitude of the displacement current is $|\,i_d\,| = 2.1\times10^{-8}\,\mathrm{A}$.

(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop *s* around the plates, then according to Eq. 32-18

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

19. (a) In region *a* of the graph,

$$|i_d| = \varepsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\ \text{F/m}\right)\left(1.6\,\text{m}^2\right)\left|\frac{4.5\times10^5\ \text{N/C} - 6.0\times10^5\ \text{N/C}}{4.0\times10^{-6}\text{s}}\right| = 0.71\,\text{A}.$$

(b) $i_d \propto dE/dt = 0$.

(c) In region *c* of the graph,

$$|i_d| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\ \text{F/m}\right)\left(1.6\text{m}^2\right)\left|\frac{-4.0\times10^5\ \text{N/C}}{2.0\times10^{-6}\text{s}}\right| = 2.8\text{A}.$$

20. (a) Since $i = i_d$ (Eq. 32-15) then the portion of displacement current enclosed is

$$i_{d,\text{enc}} = i\frac{\pi\left(R/3\right)^2}{\pi R^2} = \frac{i}{9} = 1.33\,\text{A}.$$

(b) We see from Sample Problems 32-1 and 32-2 that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to r. Therefore,

$$\frac{B}{B_{\text{max}}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{r}{R}$$

which yields $r = R/4 = (1.20\text{ cm})/4 = 0.300\text{ cm}$.

(c) We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-17):

$$\frac{B}{B_{\text{max}}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{R}{r}$$

which yields $r = 4R = 4(1.20\text{ cm}) = 4.80\text{ cm}$.

21. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0$ A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\text{A}}{\left(8.85\times 10^{-12}\ \text{F/m}\right)\left(1.0\,\text{m}\right)^2} = 2.3\times 10^{11}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i'_d = i_d\left(\frac{d^2}{L^2}\right) = (2.0\text{A})\left(\frac{0.50\text{m}}{1.0\text{m}}\right)^2 = 0.50\text{A}.$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i'_d = \left(1.26\times 10^{-16}\ \text{H/m}\right)\left(0.50\,\text{A}\right) = 6.3\times 10^{-7}\ \text{T}\cdot\text{m}.$$

22. From Eq. 28-11, we have $i = (\varepsilon / R)\, e^{-t/\tau}$ since we are ignoring the self-inductance of the capacitor. Eq. 32-16 gives

$$B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

Furthermore, Eq. 25-9 yields the capacitance

$$C = \frac{\varepsilon_0 \pi (0.05\ \text{m})^2}{0.003\ \text{m}} = 2.318\times10^{-11}\,\text{F},$$

so that the capacitive time constant is

$$\tau = (20.0 \times 10^6\ \Omega)(2.318 \times 10^{-11}\ \text{F}) = 4.636 \times 10^{-4}\ \text{s}.$$

At $t = 250 \times 10^{-6}$ s, the current is

$$i = \frac{12.0\ \text{V}}{20.0 \times 10^6\ \Omega}\ e^{-t/\tau} = 3.50 \times 10^{-7}\ \text{A}.$$

Since $i = i_d$ (see Eq. 32-15) and $r = 0.0300$ m, then (with plate radius $R = 0.0500$ m) we find

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.50\times10^{-7}\ \text{A})(0.030\ \text{m})}{2\pi(0.050\ \text{m})^2} = 8.40\times10^{-13}\ \text{T}.$$

23. (a) Using Eq. 27-10, we find $E = \rho J = \frac{\rho i}{A} = \frac{(1.62 \times 10^{-8}\,\Omega \cdot \text{m})(100\,\text{A})}{5.00 \times 10^{-6}\,\text{m}^2} = 0.324\,\text{V/m}$.

(b) The displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0 \rho \frac{di}{dt} = (8.85 \times 10^{-12}\,\text{F/m})(1.62 \times 10^{-8}\,\Omega)(2000\,\text{A/s})$$
$$= 2.87 \times 10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\frac{B(\text{due to}\, i_d)}{B(\text{due to}\, i)} = \frac{\mu_0 i_d / 2\pi r}{\mu_0 i / 2\pi r} = \frac{i_d}{i} = \frac{2.87 \times 10^{-16}\,\text{A}}{100\,\text{A}} = 2.87 \times 10^{-18}$.

24. (a) Fig. 32-35 indicates that $i = 4.0$ A when $t = 20$ ms. Thus,

$$B_i = \mu_o i/2\pi r = 0.089 \text{ mT}.$$

(b) Fig. 32-35 indicates that $i = 8.0$ A when $t = 40$ ms. Thus, $B_i \approx 0.18$ mT.

(c) Fig. 32-35 indicates that $i = 10$ A when $t > 50$ ms. Thus, $B_i \approx 0.220$ mT.

(d) Eq. 32-4 gives the displacement current in terms of the time-derivative of the electric field: $i_d = \varepsilon_o A(dE/dt)$, but using Eq. 26-5 and Eq. 26-10 we have $E = \rho i/A$ (in terms of the real current); therefore, $i_d = \varepsilon_o \rho(di/dt)$. For $0 < t < 50$ ms, Fig. 32-35 indicates that $di/dt =$ 200 A/s. Thus, $B_{id} = \mu_o i_d/2\pi r = 6.4 \times 10^{-22}$ T.

(e) As in (d), $B_{id} = \mu_o i_d/2\pi r = 6.4 \times 10^{-22}$ T.

(f) Here $di/dt = 0$, so (by the reasoning in the previous step) $B = 0$.

(g) By the right-hand rule, the direction of $\vec{B}_i$ at $t = 20$ s is out of page.

(h) By the right-hand rule, the direction of $\vec{B}_{id}$ at $t = 20$ s is out of page.

25. (a) Eq. 32-16 (with Eq. 26-5) gives, with $A = \pi R^2$,

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{\mu_0 J_d A r}{2\pi R^2} = \frac{\mu_0 J_d (\pi R^2) r}{2\pi R^2} = \frac{1}{2}\mu_0 J_d r$$
$$= \frac{1}{2}(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(6.00\ \mathrm{A/m^2})(0.0200\ \mathrm{m}) = 75.4\ \mathrm{nT}\,.$$

(b) Similarly, Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = \dfrac{\mu_0 J_d \pi R^2}{2\pi r} = 67.9\ \mathrm{nT}$.

26. (a) Eq. 32-16 gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 2.22\ \mu\text{T}$.

(b) Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = 2.00\ \mu\text{T}$.

27. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current, and if we related this to the displacement current density J_d, then

$$i_{d\ \text{enc}} = \int_0^r J_d 2\pi r\, dr = (4.00\ \text{A/m}^2)(2\pi)\int_0^r \left(1 - r/R\right) r\, dr = 8\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Now, we apply Eq. 32-17 (with i_d replaced with $i_{d,\text{enc}}$) or start from scratch with Eq. 32-11, to get $B = \dfrac{\mu_0 i_{d\ \text{enc}}}{2\pi r} = 27.9\ \text{nT}$.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$i_{d\ \text{enc}} = i_d = 8\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{4}{3}\pi R^2 .$$

Now Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = 15.1\ \text{nT}$.

28. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current. Thus Eq. 32-17 (which derives from Eq. 32-11) becomes, with i_d replaced with $i_{d,\text{enc}}$,

$$B = \frac{\mu_0 i_{d\ \text{enc}}}{2\pi r} = \frac{\mu_0 (3.00\ \text{A})(r/R)}{2\pi r}$$

which yields (after canceling r, and setting $R = 0.0300$ m) $B = 20.0\ \mu$T.

(b) Here $i_d = 3.00$ A, and we get $B = \dfrac{\mu_0 i_d}{2\pi r} = 12.0\ \mu\text{T}$.

29. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{max} = i_{d\,max} = 7.60\ \mu A$.

(b) Since $i_d = \varepsilon_0\,(d\Phi_E/dt)$,

$$\left(\frac{d\Phi_E}{dt}\right)_{max} = \frac{i_{d\,max}}{\varepsilon_0} = \frac{7.60\times10^{-6}\,A}{8.85\times10^{-12}\,F/m} = 8.59\times10^{5}\,V\cdot m/s.$$

(c) According to Problem 32-13, the displacement current is

$$i_d = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \mathcal{E}_m \sin \omega t$ and $dV/dt = \omega\mathcal{E}_m \cos \omega t$. Thus, $i_d = (\varepsilon_0 A\omega\mathcal{E}_m / d)\cos\omega t$ and $i_{d\,max} = \varepsilon_0 A\omega\mathcal{E}_m / d$. This means

$$d = \frac{\varepsilon_0 A\omega\mathcal{E}_m}{i_{d\,max}} = \frac{\left(8.85\times10^{-12}\,F/m\right)\pi\left(0.180\,m\right)^2\left(130\,rad/s\right)\left(220\,V\right)}{7.60\times10^{-6}\,A} = 3.39\times10^{-3}\,m,$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates, $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B}\cdot d\vec{s} = 2\pi rB$. Thus,

$$2\pi rB = \mu_0\left(\frac{r^2}{R^2}\right)i_d \quad\Rightarrow\quad B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

The maximum magnetic field is given by

$$B_{max} = \frac{\mu_0 i_{d\,max} r}{2\pi R^2} = \frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)\left(7.6\times10^{-6}\,A\right)\left(0.110m\right)}{2\pi\left(0.180\,m\right)^2} = 5.16\times10^{-12}\,T.$$

30. (a) The flux through Arizona is

$$\Phi = -B_r A = -\left(43\times10^{-6}\ \text{T}\right)\left(295{,}000\,\text{km}^2\right)\left(10^3\ \text{m/km}\right)^2 = -1.3\times10^7\ \text{Wb}\ ,$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3 \times 10^7$ Wb.

(b) The direction is outward.

31. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T}\ .$$

32. We use Eq. 32-27 to obtain

$$\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z},$$

where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-24 and 32-25). Thus,

$$\Delta U = -B\left[\mu_B - (-\mu_B)\right] = 2\mu_B B = 2\left(9.27\times10^{-24}\ \text{J/T}\right)\left(0.25\,\text{T}\right) = 4.6\times10^{-24}\ \text{J}\ .$$

33. We use Eq. 32-31: $\mu_{\text{orb},z} = -m_\ell \mu_B$.

(a) For $m_\ell = 1$, $\mu_{\text{orb},z} = -(1)(9.3 \times 10^{-24}\ \text{J/T}) = -9.3 \times 10^{-24}$ J/T.

(b) For $m_\ell = -2$, $\mu_{\text{orb},z} = -(-2)(9.3 \times 10^{-24}\ \text{J/T}) = 1.9 \times 10^{-23}$ J/T.

34. Combining Eq. 32-27 with Eqs. 32-22 and 32-23, we see that the energy difference is

$$\Delta U = 2\mu_B B$$

where μ_B is the Bohr magneton (given in Eq. 32-25). With ΔU = 6.00 × 10^{-25} J, we obtain B = 32.3 mT.

35. (a) Since $m_\ell = 0$, $L_{\text{orb},z} = m_\ell h/2\pi = 0$.

(b) Since $m_\ell = 0$, $\mu_{\text{orb},z} = -m_\ell \mu_B = 0$.

(c) Since $m_\ell = 0$, then from Eq. 32-32, $U = -\mu_{\text{orb},z}B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}} = 0$.

(d) Regardless of the value of m_ℓ, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times10^{-24}\ \text{J/T}\right)\left(35\,\text{mT}\right) = \pm3.2\times10^{-25}\ \text{J}\ .$$

(e) Now $m_\ell = -3$, so

$$L_{\text{orb},z} = \frac{m_\ell h}{2\pi} = \frac{(-3)\left(6.63\times10^{-27}\ \text{J}\cdot\text{s}\right)}{2\pi} = -3.16\times10^{-34}\ \text{J}\cdot\text{s} \approx -3.2\times10^{-34}\ \text{J}\cdot\text{s}$$

(f) and $\mu_{\text{orb},z} = -m_\ell\mu_B = -(-3)\left(9.27\times10^{-24}\ \text{J/T}\right) = 2.78\times10^{-23}\ \text{J/T} \approx 2.8\times10^{-23}\ \text{J/T}$.

(g) The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{\text{orb},z}B_{\text{ext}} = -\left(2.78\times10^{-23}\ \text{J/T}\right)\left(35\times10^{-3}\ \text{T}\right) = -9.7\times10^{-25}\ \text{J}.$$

(h) On the other hand, the potential energy associated with the electron spin, being independent of m_ℓ, remains the same: $\pm3.2 \times 10^{-25}$ J.

36. (a) The potential energy of the atom in association with the presence of an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-31 and 32-32:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}}.$$

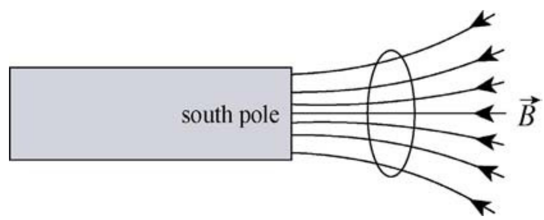
For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_\ell = 0$, meaning that $m_\ell = 0$ for this level.

(b) For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_ℓ. The middle one in the triplet is unshifted from the original value of E_2 so its m_ℓ must be equal to 0. The other two in the triplet then correspond to $m_\ell = -1$ and $m_\ell = +1$, respectively.

(c) For any pair of adjacent levels in the triplet $|\Delta m_\ell| = 1$. Thus, the spacing is given by

$$\Delta U = |\Delta(-m_\ell \mu_B B)| = |\Delta m_\ell| \mu_B B = \mu_B B = (9.27\times10^{-24}\,\text{J/T})(0.50\text{T}) = 4.64\times10^{-24}\ \text{J}.$$

37. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of §32-9 is two-fold: $\vec{u}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material towards regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch, or in the $+x$ direction.

(c) The direction of the current is clockwise (from the perspective of the bar magnet.)

(d) Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the right in our sketch, or in the $+x$ direction.

38. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t},$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e}.$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr.$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e}.$$

39. The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = M\mathcal{V}$, where M is the magnetization and $\mathcal{V}$ is the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). Thus,

$$\mu = M\pi r^2 L = \left(5.30\times 10^3 \text{ A/m}\right)\pi\left(0.500\times 10^{-2}\text{ m}\right)^2\left(5.00\times 10^{-2}\text{ m}\right) = 2.08\times 10^{-2}\text{ J/T}.$$

40. Reviewing Sample Problem 32-3 before doing this exercise is helpful. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - \left(-\vec{\mu}\cdot\vec{B}\right)\right| = 2\mu B$$

which leads to

$$T = \frac{4\mu B}{3k} = \frac{4\left(1.0\times10^{-23}\ \mathrm{J/T}\right)\left(0.50\ \mathrm{T}\right)}{3\left(1.38\times10^{-23}\ \mathrm{J/K}\right)} = 0.48\ \mathrm{K}\ .$$

41. For the measurements carried out, the largest ratio of the magnetic field to the temperature is (0.50 T)/(10 K) = 0.050 T/K. Look at Fig. 32-14 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

42. (a) From Fig. 32-14 we estimate a slope of $B/T = 0.50$ T/K when $M/M_{max} = 50\%$. So

$$B = 0.50 \text{ T} = (0.50 \text{ T/K})(300 \text{ K}) = 1.5\times10^2 \text{ T}.$$

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 6.0\times10^2$ T.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

43. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r}.$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}.$$

Since the magnetic force with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so $r = m_e v / eB$. Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B}.$$

The magnetic force $-e\vec{v} \times \vec{B}$ must point toward the center of the circular path. If the magnetic field is directed out of the page (defined to be +z direction), the electron will travel counterclockwise around the circle. Since the electron is negative, the current is in the opposite direction, clockwise and, by the right-hand rule for dipole moments, the dipole moment is into the page, or in the $-z$ direction. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion.

(c) The direction of the dipole moment is $-z$, opposite to the magnetic field.

(d) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}\left(K_e + K_i\right) = \frac{5.3\times10^{21}\,\text{m}^{-3}}{1.2\,\text{T}}\left(6.2\times10^{-20}\,\text{J}+7.6\times10^{-21}\,\text{J}\right) = 3.1\times10^{2}\ \text{A/m}.$$

44. Section 32-10 explains the terms used in this problem and the connection between M and μ. The graph in Fig. 32-39 gives a slope of

$$\frac{M / M_{\max}}{B_{\text{ext}} / T} = \frac{0.15}{0.20 \text{ T/K}} = 0.75 \text{ K/T} \;.$$

Thus we can write

$$\frac{\mu}{\mu_{\max}} = (0.75 \text{ K/T}) \frac{0.800 \text{ T}}{2.00 \text{ K}} = 0.30 \,.$$

45. (a) We use the notation $P(\mu)$ for the probability of a dipole being parallel to $\vec{B}$, and $P(-\mu)$ for the probability of a dipole being antiparallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities:

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu) + P(-\mu)} = \frac{N\mu\left(e^{\mu B/KT} - e^{-\mu B/KT}\right)}{e^{\mu B/KT} + e^{-\mu B/KT}} = N\mu \tanh\left(\frac{\mu B}{kT}\right).$$

(b) For $\mu B \ll kT$ (that is, $\mu B / kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu\left[(1+\mu B/kT) - (1-\mu B/kT)\right]}{(1+\mu B/kT) + (1-\mu B/kT)} = \frac{N\mu^2 B}{kT}.$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so $M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx N\mu$.

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-14. By adjusting the parameters used in one's plot, the curve in Fig. 32-14 can reliably be fit with a tanh function.

46. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\mathrm{g/cm^3}\right)\left(5.0\,\mathrm{cm}\right)\left(1.0\,\mathrm{cm^2}\right)}{\left(55.847\,\mathrm{g/mol}\right)/\left(6.022\times 10^{23}/\mathrm{mol}\right)} = 4.3\times 10^{23}.$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1\times 10^{-23}\,\mathrm{J/T}\right)\left(4.3\times 10^{23}\right) = 8.9\,\mathrm{A\cdot m^2}.$$

(b) $\tau = \mu B \sin 90° = (8.9\ \mathrm{A\cdot m^2})(1.57\ \mathrm{T}) = 13\ \mathrm{N\cdot m}$.

47. (a) The field of a dipole along its axis is given by Eq. 30-29: $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$, where μ is the dipole moment and z is the distance from the dipole. Thus,

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m}/A\right)\left(1.5\times 10^{-23}\ \text{J/T}\right)}{2\pi\left(10\times 10^{-9}\ \text{m}\right)} = 3.0\times 10^{-6}\ \text{T}.$$

(b) The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi,$$

where ϕ is the angle between the dipole moment and the field. The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2\left(1.5\times 10^{-23}\ \text{J/T}\right)\left(3.0\times 10^{-6}\,\text{T}\right) = 9.0\times 10^{-29}\,\text{J} = 5.6\times 10^{-10}\ \text{eV}.$$

The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

48. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then $10°C + (30°C/km)x = 770°C$. Therefore,

$$x = \frac{770°\,C - 10°\,C}{30°\,C/km} = 25\,km.$$

49. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\text{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$n = \frac{\rho N_A}{M} = \frac{\left(8.90\,\text{g/cm}^3\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right)}{58.71\,\text{g/mol}} = 9.126\times10^{22}\,\text{atoms/cm}^3$$
$$= 9.126\times10^{28}\,\text{atoms/m}^3.$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\text{sat}}}{n} = \frac{4.70\times10^5\,\text{A/m}}{9.126\times10^{28}\,\text{m}^3} = 5.15\times10^{-24}\,\text{A}\cdot\text{m}^2.$$

50. From Eq. 29-37 (see also Eq. 29-36) we write the torque as $\tau = -\mu B_h \sin\theta$ where the minus indicates that the torque opposes the angular displacement θ (which we will assume is small and in radians). The small angle approximation leads to $\tau \approx -\mu B_h \theta$, which is an indicator for simple harmonic motion (see section 16-5, especially Eq. 16-22). Comparing with Eq. 16-23, we then find the period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{\mu B_h}}$$

where I is the rotational inertial that we asked to solve for. Since the frequency is given as 0.312 Hz, then the period is $T = 1/f = 1/(0.312 \text{ Hz}) = 3.21$ s. Similarly, $B_h = 18.0 \times 10^{-6}$ T and $\mu = 6.80 \times 10^{-4}$ J/T. The above relation then yields $I = 3.19 \times 10^{-9}\ \text{kg·m}^2$.

51. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius (r_{avg} = 5.5 cm) to calculate n:

$$n = \frac{N}{2\pi r_{avg}} = \frac{400 \text{ turns}}{2\pi(5.5\times10^{-2}\text{m})} = 1.16\times10^3 \text{ turns/m} .$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\text{ T}}{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(1.16\times10^3\text{ /m})} = 0.14 \text{ A} .$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\varepsilon = N(d\Phi/dt)$ and the current in the secondary is $i_s = \varepsilon/R$, where R is the resistance of the coil. Thus,

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt}.$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\, dt = \frac{N}{R}\int \frac{d\Phi}{dt}dt = \frac{N}{R}\int_0^{\Phi} d\Phi = \frac{N\Phi}{R} .$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus,

$$\Phi = 801\pi r^2 B_0 .$$

The radius r is (6.0 cm – 5.0 cm)/2 = 0.50 cm and

$$\Phi = 801\pi(0.50\times10^{-2}\text{ m})^2(0.20\times10^{-3}\text{T}) = 1.26\times10^{-5}\text{ Wb} .$$

Consequently,

$$q = \frac{50(1.26\times10^{-5}\text{ Wb})}{8.0\ \Omega} = 7.9\times10^{-5}\text{ C} .$$

52. (a) Eq. 29-36 gives

$$\tau = \mu_{\text{rod}} B \sin\theta = (2700 \text{ A/m})(0.06 \text{ m})\pi(0.003 \text{ m})^2(0.035 \text{ T})\sin(68°) = 1.49 \times 10^{-4} \text{ N}\cdot\text{m}.$$

We have used the fact that the volume of a cylinder is its length times its (circular) cross sectional area.

(b) Using Eq. 29-38, we have

$$\begin{aligned}\Delta U &= -\mu_{\text{rod}} B(\cos\theta_f - \cos\theta_i)\\ &= -(2700 \text{ A/m})(0.06 \text{ m})\pi(0.003\text{m})^2(0.035\text{T})[\cos(34°) - \cos(68°)]\\ &= -72.9\ \mu\text{J}.\end{aligned}$$

53. (a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}.$$

We substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} \quad\Rightarrow\quad R = \left(\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right)^{1/3}.$$

The mass of an iron atom is $m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\,\text{kg/u}) = 9.30\times10^{-26}\,\text{kg}$. Therefore,

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\,\text{J/T})}{4\pi(14\times10^{3}\,\text{kg/m}^3)(2.1\times10^{-23}\,\text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}.$$

(b) The volume of the sphere is $V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82\times10^{5}\,\text{m})^3 = 2.53\times10^{16}\,\text{m}^3$ and the volume of the Earth is

$$V_e = \frac{4\pi}{3}(6.37\times10^{6}\,\text{m})^3 = 1.08\times10^{21}\,\text{m}^3,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}.$$

54. (a) Inside the gap of the capacitor, $B_1 = \mu_o i_d r_1/2\pi R^2$ (Eq. 32-16); outside the gap the magnetic field is $B_2 = \mu_o i_d/2\pi r_2$ (Eq. 32-17). Consequently, $B_2 = B_1 R^2/r_1 r_2 = 16.7$ nT.

(b) The displacement current is $i_d = 2\pi B_1 R^2/\mu_o r_1 = 5.00$ mA.

55. (a) The Pythagorean theorem leads to

$$B=\sqrt{B_h^2+B_v^2}=\sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2+\left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2}=\frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m+4\sin^2\lambda_m}$$

$$=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m},$$

where $\cos^2\lambda_m+\sin^2\lambda_m=1$ was used.

(b) We use Eq. 3-6:

$$\tan\phi_i=\frac{B_v}{B_h}=\frac{\left(\mu_0\mu/2\pi r^3\right)\sin\lambda_m}{\left(\mu_0\mu/4\pi r^3\right)\cos\lambda_m}=2\tan\lambda_m\ .$$

56. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B = \frac{\mu_0 \mu}{4\pi r^3} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\ \text{A}\cdot\text{m}^2\right)}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^3} = 3.10\times10^{-5}\ \text{T}.$$

(b) $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0°$.

(c) At $\lambda_m = 60.0°$, we find

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \left(3.10\times10^{-5}\right)\sqrt{1+3\sin^2 60.0°} = 5.59\times10^{-5}\ \text{T}.$$

(d) $\phi_i = \tan^{-1}(2\tan 60.0°) = 73.9°$.

(e) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \left(3.10\times10^{-5}\right)\sqrt{1+3(1.00)^2} = 6.20\times10^{-5}\ \text{T}.$$

(f) $\phi_i = \tan^{-1}(2\tan 90.0°) = 90.0°$.

57. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\ \mathrm{J/T}}{\pi(6.37\times10^6\ \mathrm{m})^2} = 6.3\times10^8\,\mathrm{A}\ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

58. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} \ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3} \ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{(R_e+h)^3} \ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3}-1\right)R_e = \left(2^{1/3}-1\right)(6370\,\text{km}) = 1.66\times 10^3\,\text{km}.$$

(b) We use the expression for B obtained in Problem 32-55, part (a). For maximum B, we set $\sin\lambda_m = 1.00$. Also, $r = 6370$ km – 2900 km = 3470 km. Thus,

$$B_{\text{max}} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times 10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi\left(3.47\times 10^6\,\text{m}\right)^3}\sqrt{1+3(1.00)^2}$$

$$= 3.83\times 10^{-4}\,\text{T}.$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at Earth's geographic north pole. Also $r = R_e = 6370$ km. Thus,

$$B = \frac{\mu_0\mu}{4\pi R_E^3}\sqrt{1+3\sin^2\lambda_m} = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.0\times 10^{22}\ \text{J/T}\right)\sqrt{1+3\sin^2 78.5°}}{4\pi\left(6.37\times 10^6\,\text{m}\right)^3}$$

$$= 6.11\times 10^{-5}\,\text{T}.$$

(d) $\phi_i = \tan^{-1}\left(2\tan 78.5°\right) = 84.2°$.

(e) A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we obtained in Problem 32-55 are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

59. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt},$$

and for $r \ge R$, it is

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}.$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\max} = \frac{\mu_0 \varepsilon_0 R}{2} \frac{dE}{dt}.$$

There are two values of r for which $B = B_{\max}/2$: one less than R and one greater.

(a) To find the one that is less than R, we solve

$$\frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = R/2 = (55.0 \text{ mm})/2 = 27.5 \text{ mm}$.

(b) To find the one that is greater than R, we solve

$$\frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = 2R = 2(55.0 \text{ mm}) = 110 \text{ mm}$.

60. (a) The period of rotation is $T = 2\pi/\omega$ and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2}q\omega r^2 \; .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

61. (a) For a given value of ℓ, m_ℓ varies from $-\ell$ to $+\ell$. Thus, in our case ℓ = 3, and the number of different m_ℓ's is $2\ell + 1 = 2(3) + 1 = 7$. Thus, since $L_{\text{orb},z} \propto m_\ell$, there are a total of seven different values of $L_{\text{orb},z}$.

(b) Similarly, since $\mu_{\text{orb},z} \propto m_\ell$, there are also a total of seven different values of $\mu_{\text{orb},z}$.

(c) Since $L_{\text{orb},z} = m_\ell h/2\pi$, the greatest allowed value of $L_{\text{orb},z}$ is given by $|m_\ell|_{\max} h/2\pi = 3h/2\pi$.

(d) Similar to part (c), since $\mu_{\text{orb},z} = -m_\ell \mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_\ell|_{\max}\mu_B = 3eh/4\pi m_e$.

(e) From Eqs. 32-23 and 32-29 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_\ell h}{2\pi} + \frac{m_s h}{2\pi}.$$

For the maximum value of $L_{\text{net},z}$ let $m_\ell = [m_\ell]_{\max} = 3$ and $m_s = \frac{1}{2}$. Thus

$$\left[L_{\text{net},z}\right]_{\max} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}.$$

(f) Since the maximum value of $L_{\text{net},z}$ is given by $[m_J]_{\max} h/2\pi$ with $[m_J]_{\max} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{\text{net},z}$ is given by $2[m_J]_{\max} + 1 = 2(3.5) + 1 = 8$.

62. (a) Eq. 30-22 gives $B = \dfrac{\mu_0 ir}{2\pi R^2} = 222\ \mu\text{T}$.

(b) Eq. 30-19 (or Eq. 30-6) gives $B = \dfrac{\mu_0 i}{2\pi r} = 167\ \mu\text{T}$.

(c) As in part (b), we obtain a field of $B = \dfrac{\mu_0 i}{2\pi r} = 22.7\ \mu\text{T}$.

(d) Eq. 32-16 (with Eq. 32-15) gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 1.25\ \mu\text{T}$.

(e) As in part (d), we get $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 3.75\ \mu\text{T}$.

(f) Eq. 32-17 yields $B = 22.7\ \mu\text{T}$.

(g) Because the displacement current in the gap is spread over a larger cross-sectional area, values of B within that area are relatively small. Outside that cross-sectional area, the two values of B are identical. See Fig. 32-22*b*.

63. (a) The complete set of values are

$$\{-4, -3, -2, -1, 0, +1, +2, +3, +4\} \quad \Rightarrow \quad \text{nine values in all.}$$

(b) The maximum value is $4\mu_B = 3.71 \times 10^{-23}$ J/T.

(c) Multiplying our result for part (b) by 0.250 T gives $U = +9.27 \times 10^{-24}$ J.

(d) Similarly, for the lower limit, $U = -9.27 \times 10^{-24}$ J.

64. (a) Using Eq. 32-31, we find

$$\mu_{\text{orb},z} = -3\mu_B = -2.78 \times 10^{-23} \text{ J/T}.$$

(That these are acceptable units for magnetic moment is seen from Eq. 32-32 or Eq. 32-27; they are equivalent to $A \cdot m^2$).

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z} = 3.71 \times 10^{-23}$ J/T.

65. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is

$$U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e\cos\theta\,,$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ

$$U(\theta) = -\mu B_e\cos\theta \approx -\mu B_e\left(1-\frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\,,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}}\,,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})\left(4.0\times10^{-2}\,\text{m}\right)^2(45\,\text{rad/s})^2}{12\left(16\times10^{-6}\,\text{T}\right)} = 8.4\times10^{2}\ \text{J/T}\,.$$

66. The definition of displacement current is Eq. 32-10, and the formula of greatest convenience here is Eq. 32-17:

$$i_d = \frac{2\pi r\, B}{\mu_0} = \frac{2\pi\left(0.0300\,\mathrm{m}\right)\left(2.00\times10^{-6}\,\mathrm{T}\right)}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 0.300\ \mathrm{A}\ .$$

67. (a) Using Eq. 32-13 but noting that the capacitor is being *discharged*, we have

$$\frac{d\,|\vec{E}\,|}{dt} = -\frac{i}{\varepsilon_0 A} = -\frac{5.0\text{ A}}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.0080\text{ m})^2} = -8.8\times10^{15}\text{ V/m}\cdot\text{s} .$$

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in §32-4), we follow part (a) of Sample Problem 32-2 and relate the (absolute value of the) line integral to the portion of displacement current enclosed:

$$\left|\oint \vec{B}\cdot d\vec{s}\right| = \mu_0 i_{d,\text{enc}} = \mu_0\left(\frac{WH}{L^2}i\right) = 5.9\times10^{-7}\text{ Wb/m}.$$

68. (a) From Eq. 32-1, we have

$$\left(\Phi_B\right)_{\text{in}} = \left(\Phi_B\right)_{\text{out}} = 0.0070\,\text{Wb} + \left(0.40\,\text{T}\right)\left(\pi r^2\right) = 9.2\times10^{-3}\,\text{Wb}.$$

Thus, the magnetic of the magnetic flux is 9.2 mWb.

(b) The flux is inward.

69. (a) We use the result of part (a) in Sample Problem 32-1:

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} \qquad (\text{for } r \le R),$$

where $r = 0.80R$, and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d} e^{-t/\tau} .$$

Here $V_0 = 100$ V. Thus,

$$\begin{aligned} B(t) &= \left(\frac{\mu_0 \varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d} e^{-t/\tau}\right) = -\frac{\mu_0 \varepsilon_0 V_0 r}{2\tau d} e^{-t/\tau} \\ &= -\frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(100\ \text{V})(0.80)(16\ \text{mm})}{2\left(12\times 10^{-3}\ \text{s}\right)(5.0\ \text{mm})} e^{-t/12\ \text{ms}} \\ &= -\left(1.2\times 10^{-13}\ \text{T}\right) e^{-t/12\ \text{ms}} . \end{aligned}$$

The magnitude is $|B(t)| = \left(1.2\times 10^{-13}\ \text{T}\right) e^{-t/12\ \text{ms}}$.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\ \text{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}$ T, with a magnitude $|B(t)| = 5.9 \times 10^{-15}$ T.

70. (a) Again from Fig. 32-14, for M/M_{max} = 50% we have B/T = 0.50. So $T = B/0.50$ = 2/0.50 = 4 K.

(b) Now B/T = 2.0, so T = 2/2.0 = 1 K.

71. Let the area of each circular plate be A and that of the central circular section be a, then

$$\frac{A}{a} = \frac{\pi R^2}{\pi (R/2)^2} = 4 \; .$$

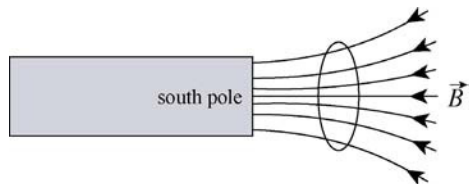
Thus, from Eqs. 32-14 and 32-15 the total discharge current is given by $i = i_d = 4(2.0 \text{ A}) = 8.0 \text{ A}$.

72. Ignoring points where the determination of the slope is problematic, we find the interval of largest $\Delta|\vec{E}|/\Delta t$ is 6 μs < t < 7 μs. During that time, we have, from Eq. 32-14,

$$i_d = \varepsilon_0 A \frac{\Delta|\vec{E}|}{\Delta t} = \varepsilon_0 (2.0\,\text{m}^2)(2.0 \times 10^6\ \text{V/m})$$

which yields $i_d = 3.5 \times 10^{-5}$ A.

73. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) For paramagnetic materials, the dipole moment $\vec{\mu}$ is in the same direction as $\vec{B}$. From the above figure, $\vec{\mu}$ points in the $-x$ direction.

(c) Form the right-hand rule, since $\vec{\mu}$ points in the $-x$ direction, the current flows counterclockwise, from the perspective of the bar magnet.

(d) The effect of $\vec{F}$ is to move the material towards regions of larger $|\vec{B}|$ values. Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the left, or $-x$.

74. (a) From Eq. 21-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(5.2\times10^{-11}\ \text{m}\right)^2} = 5.3\times10^{11}\ \text{N/C}\ .$$

(b) We use Eq. 29-28:

$$B = \frac{\mu_0}{2\pi}\frac{\mu_p}{r^3} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.4\times10^{-26}\ \text{J/T}\right)}{2\pi\left(5.2\times10^{-11}\ \text{m}\right)^3} = 2.0\times10^{-2}\ \text{T}\ .$$

(c) From Eq. 32-30,

$$\frac{\mu_{\text{orb}}}{\mu_p} = \frac{eh/4\pi m_e}{\mu_p} = \frac{\mu_B}{\mu_p} = \frac{9.27\times10^{-24}\ \text{J/T}}{1.4\times10^{-26}\ \text{J/T}} = 6.6\times10^{2}\ .$$

75. (a) Since the field lines of a bar magnet point towards its South pole, then the $\vec{B}$ arrows in one's sketch should point generally towards the left and also towards the central axis.

(b) The sign of $\vec{B}\cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

(c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \vec{B}\cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.