Chapter 12

1. (a) The center of mass is given by

$$x_{\text{com}} = \frac{0 + 0 + 0 + (m)(2.00\text{ m}) + (m)(2.00\text{ m}) + (m)(2.00\text{ m})}{6m} = 1.00\text{ m}.$$

(b) Similarly, we have

$$y_{\text{com}} = \frac{0 + (m)(2.00\text{ m}) + (m)(4.00\text{ m}) + (m)(4.00\text{ m}) + (m)(2.00\text{ m}) + 0}{6m} = 2.00\text{ m}.$$

(c) Using Eq. 12-14 and noting that the gravitational effects are different at the different locations in this problem, we have

$$x_{\text{cog}} = \frac{\sum_{i=1}^{6} x_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{x_1 m_1 g_1 + x_2 m_2 g_2 + x_3 m_3 g_3 + x_4 m_4 g_4 + x_5 m_5 g_5 + x_6 m_6 g_6}{m_1 g_1 + m_2 g_2 + m_3 g_3 + m_4 g_4 + m_5 g_5 + m_6 g_6} = 0.987\text{ m}.$$

(d) Similarly, y_{cog} = [0 + (2.00)(m)(7.80) + (4.00)(m)(7.60) + (4.00)(m)(7.40) + (2.00)(m)(7.60) + 0]/(8.00m + 7.80m + 7.60m + 7.40m + 7.60m + 7.80m) = 1.97 m.

2. The situation is somewhat similar to that depicted for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin \theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). Setting $T = F$ therefore yields $\theta = 30°$. Since $\alpha = 180° - 2\theta$ is the angle between the two segments, then we find $\alpha = 120°$.

3. The object exerts a downward force of magnitude F = 3160 N at the midpoint of the rope, causing a "kink" similar to that shown for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are colinear). In this problem, we have

$$\theta = \tan^{-1}\left(\frac{0.35\,\text{m}}{1.72\,\text{m}}\right) = 11.5°.$$

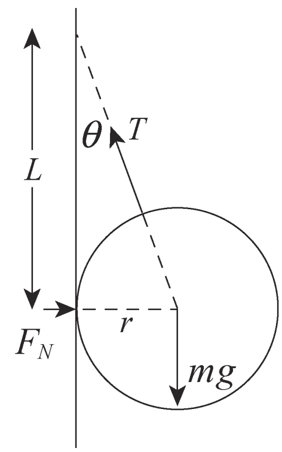
Therefore, $T = F/(2\sin\theta) = 7.92 \times 10^3$ N.

4. From $\vec{\tau} = \vec{r} \times \vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

(a) Among persons 1 through 4, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 2.

(b) Among persons 5 through 8, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 7.

5. Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives



vertical component : $T\cos\theta - mg = 0$

horizontal component: $F_N - T\sin\theta = 0$.

(a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)\sqrt{(0.080\text{ m})^2+(0.042\text{ m})^2}}{0.080\text{ m}} = 9.4\text{ N}.$$

(b) We solve the second equation for the normal force: $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)(0.042\text{ m})}{(0.080\text{ m})} = 4.4\text{ N}.$$

6. Our notation is as follows: M = 1360 kg is the mass of the automobile; L = 3.05 m is the horizontal distance between the axles; $\ell = (3.05 - 1.78)\ \text{m} = 1.27\ \text{m}$ is the horizontal distance from the rear axle to the center of mass; F_1 is the force exerted on each front wheel; and, F_2 is the force exerted on each back wheel.

(a) Taking torques about the rear axle, we find

$$F_1 = \frac{Mg\ell}{2L} = \frac{(1360\,\text{kg})(9.80\,\text{m/s}^2)(1.27\,\text{m})}{2(3.05\,\text{m})} = 2.77 \times 10^3\ \text{N}.$$

(b) Equilibrium of forces leads to $2F_1 + 2F_2 = Mg$, from which we obtain $F_2 = 3.89 \times 10^3\,\text{N}$.

7. We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$
$$F_1 d + W(L-d) = 0$$

(a) The second equation gives

$$F_1 = -\frac{L-d}{d}W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N}$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N.

(b) Since F_1 is negative, indicating that this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N}$

which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

8. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0-1.5)\ \text{m} = 3.5\ \text{m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

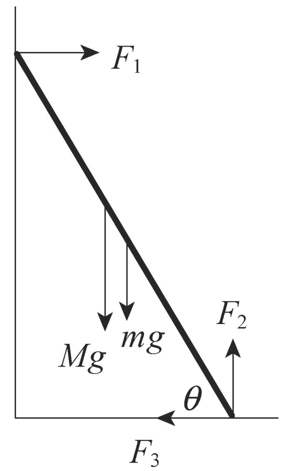
$$F_1 = \frac{m_w g\ell_2 + m_s g\ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4\times 10^2\ \text{N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4\times 10^3\ \text{N}$$

which (using our result from part (a)) yields $F_2 = 5.3\times 10^2\,\text{N}$.

9. The forces on the ladder are shown in the diagram on the right. F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder.



The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find θ = 60º. Here L is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).

(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then,

$$Mg\ell\cos\theta + mg\left(L/2\right)\cos\theta - F_1 L\sin\theta = 0,$$

and

$$F_1 = \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} = \frac{[(75\,\text{kg})(3.0\,\text{m}) + (10\,\text{kg})(2.5\,\text{m})](9.8\,\text{m/s}^2)\cos 60°}{(5.0\,\text{m})\sin 60°}$$
$$= 2.8\times 10^2\ \text{N}.$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$F_1 - F_3 = 0$$
$$F_2 - Mg - mg = 0$$

The first of these equations gives $F_3 = F_1 = 2.8\times 10^2\,\text{N}$ and the second gives

$$F_2 = (M+m)g = (75\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) = 8.3\times 10^2\,\text{N}$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^{\,2} + F_3^{\,2}} = \sqrt{(2.8\times 10^2\,\text{N})^2 + (8.3\times 10^2\,\text{N})^2} = 8.8\times 10^2\,\text{N}.$$

(c) The angle ϕ between the force and the horizontal is given by

$$\tan\phi = F_3/F_2 = 830/280 = 2.94,$$

so ϕ = 71º. The force points to the left and upward, 71º above the horizontal. We note that this force is not directed along the ladder.

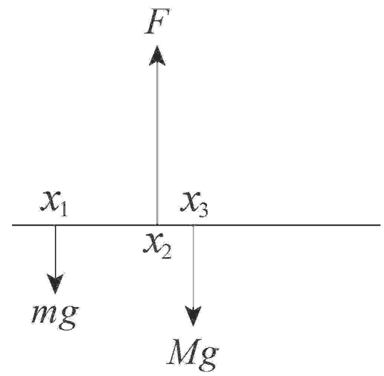
10. The angle of each half of the rope, measured from the dashed line, is

$$\theta = \tan^{-1}\left(\frac{0.30\,\text{m}}{9.0\,\text{m}}\right) = 1.9°.$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9°} = 8.3\times10^{3}\ \text{N}.$$

11. The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram below. The nickels are at $x = x_1 = 0.120$ m, and m is their total mass. The knife edge is at $x = x_2 = 0.455$ m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500$ m. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:



$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

Thus,

$$M = \frac{x_2 - x_1}{x_3 - x_2} m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}} \right) (10.0\text{g}) = 74.4 \text{ g}.$$

12. (a) Analyzing vertical forces where string 1 and string 2 meet, we find

$$T_1 = \frac{w_A}{\cos\phi} = \frac{40\,\text{N}}{\cos 35°} = 49\,\text{N}.$$

(b) Looking at the horizontal forces at that point leads to

$$T_2 = T_1 \sin 35° = (49\,\text{N})\sin 35° = 28\,\text{N}.$$

(c) We denote the components of T_3 as T_x (rightward) and T_y (upward). Analyzing horizontal forces where string 2 and string 3 meet, we find $T_x = T_2 = 28$ N. From the vertical forces there, we conclude $T_y = w_B$ =50 N. Therefore,

$$T_3 = \sqrt{T_x^2 + T_y^2} = 57\,\text{N}.$$

(d) The angle of string 3 (measured from vertical) is

$$\theta = \tan^{-1}\left(\frac{T_x}{T_y}\right) = \tan^{-1}\left(\frac{28}{50}\right) = 29°.$$

13. (a) Analyzing the horizontal forces (which add to zero) we find $F_h = F_3 = 5.0$ N.

(b) Equilibrium of vertical forces leads to $F_v = F_1 + F_2 = 30$ N.

(c) Computing torques about point O, we obtain

$$F_v d = F_2 b + F_3 a \Rightarrow d = \frac{(10\ \text{N})(3.0\,\text{m}) + (5.0\ \text{N})(2.0\,\text{m})}{30\ \text{N}} = 1.3\,\text{m}.$$

14. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude mg = 500 N, is a horizontal distance $\ell = 0.375\,\text{m}$ from C. The applied force of magnitude F = 350 N is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\,\text{N})(0.375\,\text{m})}{350\,\text{N}} = 0.536\,\text{m}.$$

15. Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\perp} = (40\,\mathrm{N})\frac{d}{L} = (40\,\mathrm{N})\frac{2.6\,\mathrm{cm}}{12\,\mathrm{cm}} = 8.7\,\mathrm{N}.$$

16. With pivot at the left end, Eq. 12-9 leads to

$$-m_s g \frac{L}{2} - Mgx + T_R L = 0$$

where m_s is the scaffold's mass (50 kg) and M is the total mass of the paint cans (75 kg). The variable x indicates the center of mass of the paint can collection (as measured from the left end), and T_R is the tension in the right cable (722 N). Thus we obtain $x = 0.702$ m.

17. The (vertical) forces at points A, B and P are F_A, F_B and F_P, respectively. We note that $F_P = W$ and is upward. Equilibrium of forces and torques (about point B) lead to

$$\begin{aligned} F_A + F_B + W &= 0 \\ bW - aF_A &= 0. \end{aligned}$$

(a) From the second equation, we find

$$F_A = bW/a = (15/5)W = 3W = 3(900\text{ N})=2.7\times10^3\text{ N}.$$

(b) The direction is upward since $F_A > 0$.

(c) Using this result in the first equation above, we obtain

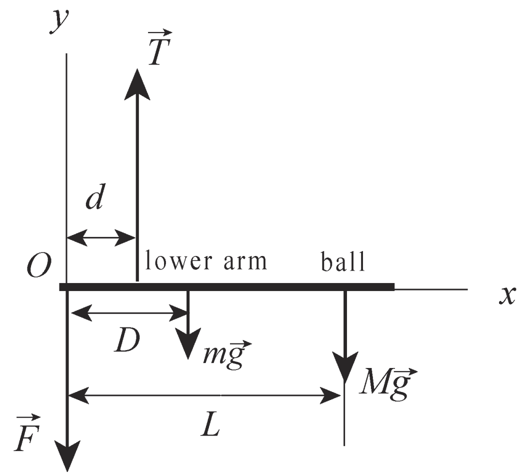
$$F_B = W - F_A = -4W = -4(900\text{ N}) = -3.6\times10^3\text{ N},$$

or $|F_B| = 3.6\times10^3\text{ N}$.

(d) F_B points downward, as indicated by the minus sign.

18. Our system consists of the lower arm holding a bowling ball. As shown in the free-body diagram, the forces on the lower arm consist of $\vec{T}$ from the biceps muscle, $\vec{F}$ from the bone of the upper arm, and the gravitational forces, $m\vec{g}$ and $M\vec{g}$. Since the system is in static equilibrium, the net force acting on the system is zero:

$$0 = \sum F_{\text{net},y} = T - F - (m+M)g$$



In addition, the net torque about O must also vanish:

$$0 = \sum_{O} \tau_{\text{net}} = (d)(T) + (0)F - (D)(mg) - L(Mg)\,.$$

(a) From the torque equation, we find the force on the lower arms by the biceps muscle to be

$$T = \frac{(mD+ML)g}{d} = \frac{[(1.8\text{ kg})(0.15\text{ m}) + (7.2\text{ kg})(0.33\text{ m})](9.8\text{ m/s}^2)}{0.040\text{ m}}$$
$$= 648\text{ N} \approx 6.5\times10^2\text{ N}.$$

(b) Substituting the above result into the force equation, we find F to be

$$F = T - (M+m)g = 648\text{ N} - (7.2\text{ kg} + 1.8\text{ kg})(9.8\text{ m/s}^2) = 560\text{ N} = 5.6\times10^2\text{ N}.$$

19. (a) With the pivot at the hinge, Eq. 12-9 gives $TL\cos\theta - mg\frac{L}{2} = 0$. This leads to $\theta = 78°$. Then the geometric relation $\tan\theta = L/D$ gives $D = 0.64$ m.

(b) A higher (steeper) slope for the cable results in a smaller tension. Thus, making D greater than the value of part (a) should prevent rupture.

20. With pivot at the left end of the lower scaffold, Eq. 12-9 leads to

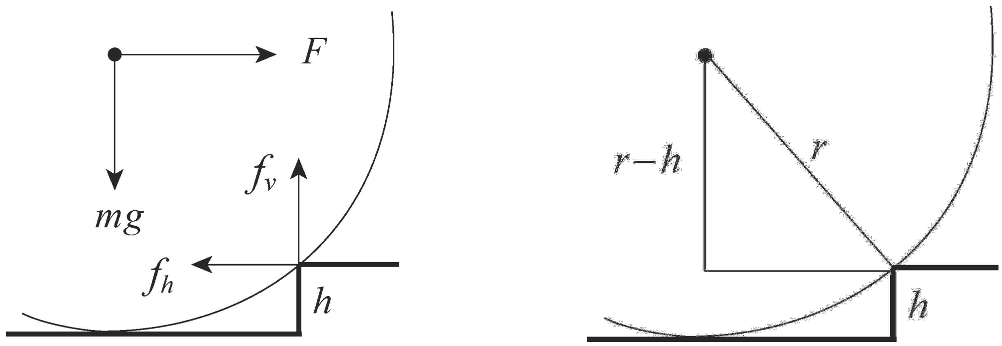
$$-m_2 g \frac{L_2}{2} - mgd + T_R L_2 = 0$$

where m_2 is the lower scaffold's mass (30 kg) and L_2 is the lower scaffold's length (2.00 m). The mass of the package (m = 20 kg) is a distance d = 0.50 m from the pivot, and T_R is the tension in the rope connecting the right end of the lower scaffold to the larger scaffold above it. This equation yields T_R = 196 N. Then Eq. 12-8 determines T_L (the tension in the cable connecting the right end of the lower scaffold to the larger scaffold above it): T_L = 294 N. Next, we analyze the larger scaffold (of length $L_1 = L_2 + 2d$ and mass m_1, given in the problem statement) placing our pivot at its left end and using Eq. 12-9:

$$-m_1 g \frac{L_1}{2} - T_L d - T_R (L_1 - d) + T L_1 = 0 .$$

This yields T = 457 N.

21. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.



We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step.

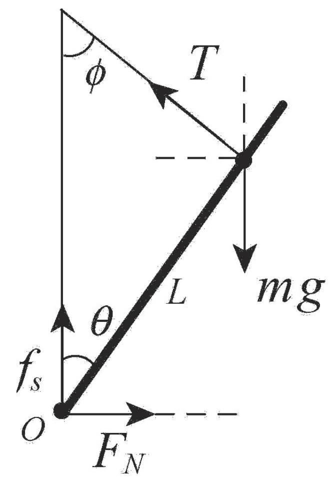
The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus,

$$F(r-h) - mg\sqrt{2rh - h^2} = 0\,.$$

The solution for F is

$$F = \frac{\sqrt{2rh-h^2}}{r-h}mg = \frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})}(0.800\text{ kg})(9.80\text{ m/s}^2)$$
$$= 13.6\text{ N}.$$

22. As shown in the free-body diagram, the forces on the climber consist of $\vec{T}$ from the rope, normal force $\vec{F}_N$ on her feet, upward static frictional force $\vec{f}_s$ and downward gravitational force $m\vec{g}$. Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0=\sum F_{\text{net},x}=F_N-T\sin\phi$$
$$0=\sum F_{\text{net},y}=T\cos\phi+f_s-mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0=\sum_O \tau_{\text{net}}=mgL\sin\theta-TL\sin(180°-\theta-\phi)$$

From the torque equation, we obtain $T=mg\sin\theta/\sin(180°-\theta-\phi)$. Substituting the expression into the force equations, and noting that $f_s=\mu_s F_N$, we find the coefficient of static friction to be

$$\mu_s=\frac{f_s}{F_N}=\frac{mg-T\cos\phi}{T\sin\phi}=\frac{mg-mg\sin\theta\cos\phi/\sin(180°-\theta-\phi)}{mg\sin\theta\sin\phi/\sin(180°-\theta-\phi)}$$
$$=\frac{1-\sin\theta\cos\phi/\sin(180°-\theta-\phi)}{\sin\theta\sin\phi/\sin(180°-\theta-\phi)}$$

With $\theta=40°$ and $\phi=30°$, the result is

$$\mu_s=\frac{1-\sin\theta\cos\phi/\sin(180°-\theta-\phi)}{\sin\theta\sin\phi/\sin(180°-\theta-\phi)}=\frac{1-\sin 40°\cos 30°/\sin(180°-40°-30°)}{\sin 40°\sin 30°/\sin(180°-40°-30°)}=1.19.$$

23. (a) All forces are vertical and all distances are measured along an axis inclined at $\theta = 30^\circ$. Thus, any trigonometric factor cancels out and the application of torques about the contact point (referred to in the problem) leads to

$$F_{\text{tripcep}} = \frac{(15\,\text{kg})(9.8\,\text{m/s}^2)(35\,\text{cm}) - (2.0\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{cm})}{2.5\,\text{cm}} = 1.9\times10^3\ \text{N}.$$

(b) The direction is upward since $F_{\text{tricep}} > 0$

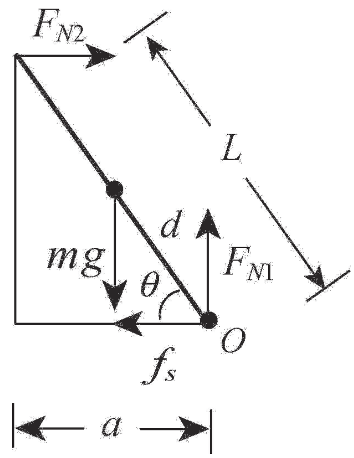
(c) Equilibrium of forces (with upwards positive) leads to

$$F_{\text{tripcep}} + F_{\text{humer}} + (15\,\text{kg})(9.8\,\text{m/s}^2) - (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 0$$

and thus to $F_{\text{humer}} = -2.1\times10^3\,\text{N}$, or $|F_{\text{humer}}| = 2.1\times10^3\,\text{N}$.

(d) The minus sign implies that F_{humer} points downward.

24. As shown in the free-body diagram, the forces on the climber consist of the normal forces F_{N1} on his hands from the ground and F_{N2} on his feet from the wall, static frictional force f_s and downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on him is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_{N2} - f_s$$
$$0 = \sum F_{\text{net},y} = F_{N1} - mg\,.$$

In addition, the net torque about O (contact point between his feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgd\cos\theta - F_{N2}L\sin\theta\,.$$

The torque equation gives $F_{N2} = mgd\cos\theta / L\sin\theta = mgd\cot\theta / L$. On the other hand, from the force equation we have $F_{N2} = f_s$ and $F_{N1} = mg$. These expressions can be combined to yield

$$f_s = F_{N2} = F_{N1}\cot\theta\frac{d}{L}\,.$$

On the other hand, the frictional force can also be written as $f_s = \mu_s F_{N1}$, where μ_s is the coefficient of static friction between his feet and the ground. From the above equation and the values given in the problem statement, we find μ_s to be

$$\mu_s = \cot\theta\frac{d}{L} = \frac{a}{\sqrt{L^2 - a^2}}\frac{d}{L} = \frac{0.914 \text{ m}}{\sqrt{(2.10 \text{ m})^2 - (0.914 \text{ m})^2}}\frac{0.940 \text{ m}}{2.10 \text{ m}} = 0.216\,.$$

25. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As shown in the figure, the beam makes an angle of 60º with the vertical and the wire makes an angle of 30º with the vertical.

(a) We calculate the torques around the hinge. Their sum is

$$TL \sin 30^\circ - W(L/2) \sin 60^\circ = 0.$$

Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W \sin 60^\circ}{2 \sin 30^\circ} = \frac{(222\,\mathrm{N}) \sin 60^\circ}{2 \sin 30^\circ} = 192\,\mathrm{N}.$$

(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T \sin 30^\circ = 0$ or

$$F_h = T \sin 30^\circ = (192.3\,\mathrm{N}) \sin 30^\circ = 96.1\,\mathrm{N}.$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T \cos 30^\circ - W = 0$ or

$$F_v = W - T \cos 30^\circ = 222\,\mathrm{N} - (192.3\,\mathrm{N}) \cos 30^\circ = 55.5\,\mathrm{N}.$$

26. (a) The problem asks for the person's pull (his force exerted on the rock) but since we are examining forces and torques *on the person*, we solve for the reaction force F_{N1} (exerted leftward on the hands by the rock). At that point, there is also an upward force of static friction on his hands f_1 which we will take to be at its maximum value $\mu_1 F_{N1}$. We note that equilibrium of horizontal forces requires $F_{N1} = F_{N2}$ (the force exerted leftward on his feet); on this feet there is also an upward static friction force of magnitude $\mu_2 F_{N2}$. Equilibrium of vertical forces gives

$$f_1 + f_2 - mg = 0 \Rightarrow F_{N1} = \frac{mg}{\mu_1 + \mu_2} = 3.4 \times 10^2 \text{ N}.$$

(b) Computing torques about the point where his feet come in contact with the rock, we find

$$mg(d+w) - f_1 w - F_{N1} h = 0 \;\Rightarrow\; h = \frac{mg(d+w) - \mu_1 F_{N1} w}{F_{N1}} = 0.88 \text{ m}.$$

(c) Both intuitively and mathematically (since both coefficients are in the denominator) we see from part (a) that F_{N1} would increase in such a case.

(d) As for part (b), it helps to plug part (a) into part (b) and simplify:

$$h = (d+w)\mu_2 + d\mu_1$$

from which it becomes apparent that h should decrease if the coefficients decrease.

27. (a) We note that the angle between the cable and the strut is

$$\alpha = \theta - \phi = 45° - 30° = 15°.$$

The angle between the strut and any vertical force (like the weights in the problem) is β = 90° – 45° = 45°. Denoting M = 225 kg and m = 45.0 kg, and ℓ as the length of the boom, we compute torques about the hinge and find

$$T = \frac{Mg\ell \sin\beta + mg\left(\frac{\ell}{2}\right)\sin\beta}{\ell \sin\alpha} = \frac{Mg\sin\beta + mg\sin\beta/2}{\sin\alpha}.$$

The unknown length ℓ cancels out and we obtain $T = 6.63 \times 10^3$ N.

(b) Since the cable is at 30° from horizontal, then horizontal equilibrium of forces requires that the horizontal hinge force be

$$F_x = T\cos 30° = 5.74 \times 10^3 \text{ N}.$$

(c) And vertical equilibrium of forces gives the vertical hinge force component:

$$F_y = Mg + mg + T\sin 30° = 5.96 \times 10^3 \text{ N}.$$

28. (a) The sign is attached in two places: at $x_1 = 1.00$ m (measured rightward from the hinge) and at $x_2 = 3.00$ m. We assume the downward force due to the sign's weight is equal at these two attachment points: each being *half* the sign's weight of mg. The angle where the cable comes into contact (also at x_2) is

$$\theta = \tan^{-1}(d_v/d_h) = \tan^{-1}(4.00\text{ m}/3.00\text{ m})$$

and the force exerted there is the tension T. Computing torques about the hinge, we find

$$T = \frac{\frac{1}{2}mgx_1 + \frac{1}{2}mgx_2}{x_2 \sin\theta} = \frac{\frac{1}{2}(50.0\text{ kg})\left(9.8\text{ m/s}^2\right)(1.00\text{ m}) + \frac{1}{2}(50.0\text{ kg})(9.8\text{m/s}^2)(3.00\text{ m})}{(3.00\text{ m})(0.800)}$$
$$= 408\text{ N}.$$

(b) Equilibrium of horizontal forces requires the horizontal hinge force be

$$F_x = T\cos\theta = 245\text{ N}.$$

(c) The direction of the horizontal force is rightward.

(d) Equilibrium of vertical forces requires the vertical hinge force be

$$F_y = mg - T\sin\theta = 163\text{ N}.$$

(e) The direction of the vertical force is upward.

29. The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let T_l be the tension force of the left–hand cord, T_r be the tension force of the right–hand cord, and m be the mass of the bar. The equations for equilibrium are:

$$
\begin{aligned}
\text{vertical force components} \quad & T_l \cos\theta + T_r \cos\phi - mg = 0 \\
\text{horizontal force components} \quad & -T_l \sin\theta + T_r \sin\phi = 0 \\
\text{torques} \quad & mgx - T_r L \cos\phi = 0.
\end{aligned}
$$

The origin was chosen to be at the left end of the bar for purposes of calculating the torque. The unknown quantities are T_l, T_r, and x. We want to eliminate T_l and T_r, then solve for x. The second equation yields $T_l = T_r \sin\phi / \sin\theta$ and when this is substituted into the first and solved for T_r the result is

$$T_r = \frac{mg\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}.$$

This expression is substituted into the third equation and the result is solved for x:

$$x = L\frac{\sin\theta\cos\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta} = L\frac{\sin\theta\cos\phi}{\sin(\theta+\phi)}.$$

The last form was obtained using the trigonometric identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$. For the special case of this problem $\theta + \phi = 90°$ and $\sin(\theta + \phi) = 1$. Thus,

$$x = L\sin\theta\cos\phi = (6.10 \text{ m}) \sin 36.9°\cos 53.1° = 2.20 \text{ m}.$$

30. (a) Computing torques about point A, we find

$$T_{\max} L \sin\theta = W x_{\max} + W_b \left(\frac{L}{2}\right).$$

We solve for the maximum distance:

$$x_{\max} = \left(\frac{T_{\max}\sin\theta - W_b/2}{W}\right) L = \left(\frac{(500\text{ N})\sin 30.0° - (200\text{ N})/2}{300\text{ N}}\right)(3.00\text{ m}) = 1.50\text{ m}.$$

(b) Equilibrium of horizontal forces gives $F_x = T_{\max}\cos\theta = 433\text{ N}$.

(c) And equilibrium of vertical forces gives $F_y = W + W_b - T_{\max}\sin\theta = 250\text{ N}$.

31. The problem states that each hinge supports half the door's weight, so each vertical hinge force component is $F_y = mg/2 = 1.3 \times 10^2$ N. Computing torques about the top hinge, we find the horizontal hinge force component (at the bottom hinge) is

$$F_h = \frac{(27\,\text{kg})(9.8\,\text{m/s}^2)\left(0.91\text{ m/2}\right)}{2.1\,\text{m} - 2(0.30\,\text{m})} = 80\,\text{N}.$$

Equilibrium of horizontal forces demands that the horizontal component of the top hinge force has the same magnitude (though opposite direction).

(a) In unit-vector notation, the force on the door at the top hinge is

$$F_{\text{top}} = (-80\text{ N})\hat{\text{i}} + (1.3\times 10^2\,\text{N})\hat{\text{j}}.$$

(b) Similarly, the force on the door at the bottom hinge is

$$F_{\text{bottom}} = (+80\text{ N})\hat{\text{i}} + (1.3\times 10^2\,\text{N})\hat{\text{j}}$$

32. (a) Computing torques about the hinge, we find the tension in the wire:

$$TL\sin\theta - Wx = 0 \Rightarrow T = \frac{Wx}{L\sin\theta}.$$

(b) The horizontal component of the tension is $T\cos\theta$, so equilibrium of horizontal forces requires that the horizontal component of the hinge force is

$$F_x = \left(\frac{Wx}{L\sin\theta}\right)\cos\theta = \frac{Wx}{L\tan\theta}.$$

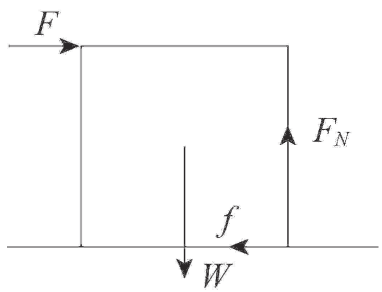
(c) The vertical component of the tension is $T\sin\theta$, so equilibrium of vertical forces requires that the vertical component of the hinge force is

$$F_y = W - \left(\frac{Wx}{L\sin\theta}\right)\sin\theta = W\left(1 - \frac{x}{L}\right).$$

33. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram below. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

$$F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\,\text{N}.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium

$$F_N = W = 890\text{ N and } f = F = 445\text{ N},$$

so

$$\mu_s = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

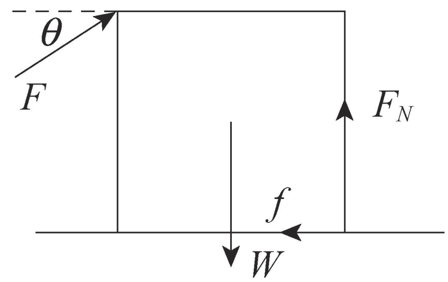
$$FL\cos\theta + FL\sin\theta - WL/2 = 0,$$

with the solution

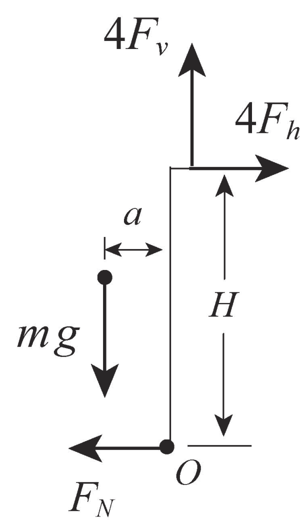
$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45°} = \frac{890\,\text{N}}{4\cos 45°} = 315\,\text{N}.$$



34. As shown in the free-body diagram, the forces on the climber consist of the normal force from the wall, the vertical component F_v and the horizontal component F_h of the force acting on her four fingertips, and the downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = 4F_h - F_N$$
$$0 = \sum F_{\text{net},y} = 4F_v - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = (mg)a - (4F_h)H\,.$$

(a) From the torque equation, we find the horizontal component of the force on her fingertip to be

$$F_h = \frac{mga}{4H} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(0.20\text{ m})}{4(2.0\text{ m})} \approx 17\text{ N}.$$

(b) From the y-component of the force equation, we obtain

$$F_v = \frac{mg}{4} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)}{4} \approx 1.7\times10^2\text{ N}.$$

35. (a) With the pivot at the hinge, Eq. 12-9 yields

$$TL\cos\theta - F_a y = 0 .$$

This leads to $T = (F_a/\cos\theta)(y/L)$ so that we can interpret $F_a/\cos\theta$ as the slope on the tension graph (which we estimate to be 600 in SI units). Regarding the F_h graph, we use Eq. 12–7 to get

$$F_h = T\cos\theta - F_a = (-F_a)(y/L) - F_a$$

after substituting our previous expression. The result implies that the slope on the F_h graph (which we estimate to be –300) is equal to $-F_a$, or $F_a = 300$ N and (plugging back in) $\theta = 60.0°$.

(b) As mentioned in the previous part, $F_a = 300$ N.

36. (a) With $F = ma = -\mu_k mg$ the magnitude of the deceleration is

$$|a| = \mu_k g = (0.40)(9.8 \text{ m/s}^2) = 3.92 \text{ m/s}^2.$$

(b) As hinted in the problem statement, we can use Eq. 12-9, evaluating the torques about the car's center of mass, and bearing in mind that the friction forces are acting horizontally at the bottom of the wheels; the total friction force there is $f_k = \mu_k gm = 3.92m$ (with SI units understood – and m is the car's mass), a vertical distance of 0.75 meter below the center of mass. Thus, torque equilibrium leads to

$$(3.92m)(0.75) + F_{Nr}(2.4) - F_{Nf}(1.8) = 0 .$$

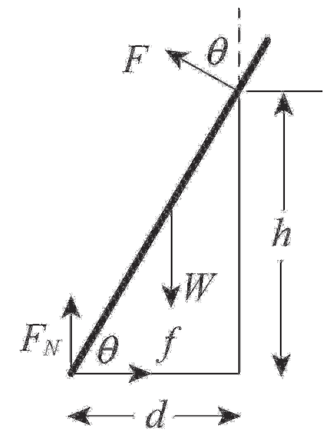
Eq. 12-8 also holds (the acceleration is horizontal, not vertical), so we have $F_{Nr} + F_{Nf} = mg$, which we can solve simultaneously with the above torque equation. The mass is obtained from the car's weight: $m = 11000/9.8$, and we obtain $F_{Nr} = 3929 \approx 4000$ N. Since each involves two wheels then we have (roughly) 2.0×10^3 N on each rear wheel.

(c) From the above equation, we also have $F_{Nf} = 7071 \approx 7000$ N, or 3.5×10^3 N on each front wheel, as the values of the individual normal forces.

(d) Eq. 6-2 directly yields (approximately) 7.9×10^2 N of friction on each rear wheel,

(e) Similarly, Eq. 6-2 yields 1.4×10^3 N on each front wheel.

37. The free-body diagram on the right shows the forces acting on the plank. Since the roller is frictionless the force it exerts is normal to the plank and makes the angle θ with the vertical. Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.



The equations of equilibrium are:

horizontal force components	$F\sin\theta - f = 0$
vertical force components	$F\cos\theta - W + F_N = 0$
torques	$F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70^\circ$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70^\circ$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have use $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\theta$. We use the trigonometric identity $1 + \tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

We substitute these expressions for f and F_N into $\mu_s = f/F_N$ and obtain

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70°$, we obtain

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70°\cos 70°}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70°\cos^2 70°} = 0.34.$$

38. The phrase "loosely bolted" means that there is no torque exerted by the bolt at that point (where A connects with B). The force exerted on A at the hinge has x and y components F_x and F_y. The force exerted on A at the bolt has components G_x and G_y and those exerted on B are simply $-G_x$ and $-G_y$ by Newton's third law. The force exerted on B at its hinge has components H_x and H_y. If a horizontal force is positive, it points rightward, and if a vertical force is positive it points upward.

(a) We consider the combined A∪B system, which has a total weight of Mg where M = 122 kg and the line of action of that downward force of gravity is x = 1.20 m from the wall. The vertical distance between the hinges is y = 1.80 m. We compute torques about the bottom hinge and find

$$F_x = -\frac{Mgx}{y} = -797\,\text{N}.$$

If we examine the forces on A alone and compute torques about the bolt, we instead find

$$F_y = \frac{m_A g x}{\ell} = 265\,\text{N}$$

where m_A = 54.0 kg and ℓ = 2.40 m (the length of beam A). Thus, in unit-vector notation, we have

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(b) Equilibrium of horizontal and vertical forces on beam A readily yields $G_x = -F_x$ = 797 N and $G_y = m_A g - F_y$ = 265 N. In unit-vector notation, we have

$$\vec{G} = G_x\hat{\text{i}} + G_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}$$

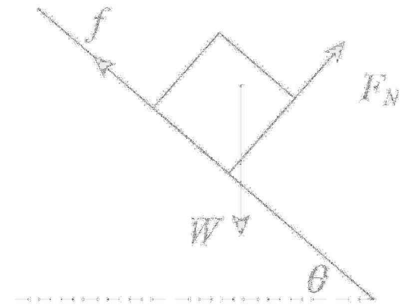
(c) Considering again the combined A∪B system, equilibrium of horizontal and vertical forces readily yields $H_x = -F_x$ = 797 N and $H_y = Mg - F_y$ = 931 N. In unit-vector notation, we have

$$\vec{H} = H_x\hat{\text{i}} + H_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (931\text{ N})\hat{\text{j}}$$

(d) As mentioned above, Newton's third law (and the results from part (b)) immediately provide $-G_x = -797$ N and $-G_y = -265$ N for the force components acting on B at the bolt. In unit-vector notation, we have

$$-\vec{G} = -G_x\hat{\text{i}} - G_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} - (265\text{ N})\hat{\text{j}}$$

39. The force diagram shown below depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, F_N is the normal force of the plane on the crate, and f is the force of friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W\sin\theta - f = 0 \text{ and } F_N - W\cos\theta = 0$$

respectively. The y equation gives $F_N = W\cos\theta$. Since the crate is about to slide

$$f = \mu_s F_N = \mu_s W\cos\theta,$$

where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \Rightarrow \tan\theta = \mu_s.$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.60 = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $F_N\ell/2$, where ℓ is the length of an edge. Since the total torque vanishes, $fh = F_N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $F_N = W\cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7°.$$

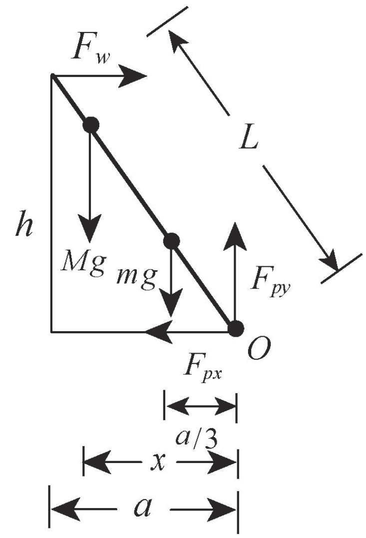
As θ is increased from zero the crate slides before it tips.

(b) It starts to slide when $\theta = 31°$.

(c) The crate begins to slide when $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.70 = 35.0°$ and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased.

(d) Tipping begins at $\theta = 33.7° \approx 34°$.

40. Let x be the horizontal distance between the firefighter and the origin O (see figure) that makes the ladder on the verge of sliding. The forces on the firefighter + ladder system consist of the horizontal force F_w from the wall, the vertical component F_{py} and the horizontal component F_{px} of the force $\vec{F}_p$ on the ladder from the pavement, and the downward gravitational forces Mg and mg, where M and m are the masses of the firefighter and the ladder, respectively. Since the system is in static equilibrium, the net force acting on the system is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_w - F_{px}$$
$$0 = \sum F_{\text{net},y} = F_{py} - (M+m)g\ .$$

Since the ladder is on the verge of sliding, $F_{px} = \mu_s F_{py}$. Therefore, we have

$$F_w = F_{px} = \mu_s F_{py} = \mu_s (M+m)g\ .$$

In addition, the net torque about O (contact point between the ladder and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = -h(F_w) + x(Mg) + \frac{a}{3}(mg) = 0\ .$$

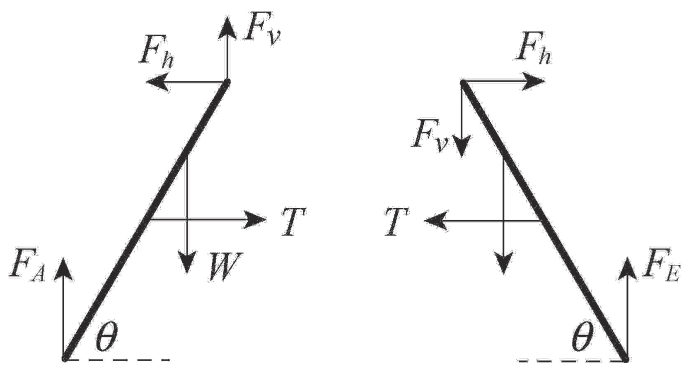
Solving for x, we obtain

$$x = \frac{hF_w - (a/3)mg}{Mg} = \frac{h\mu_s(M+m)g - (a/3)mg}{Mg} = \frac{h\mu_s(M+m) - (a/3)m}{M}$$

Substituting the values given in the problem statement (with $a = \sqrt{L^2 - h^2} = 7.58\ \text{m}$), the fraction of ladder climbed is

$$\frac{x}{a} = \frac{h\mu_s(M+m) - (a/3)m}{Ma} = \frac{(9.3\ \text{m})(0.53)(72\ \text{kg} + 45\ \text{kg}) - (7.58\ \text{m}/3)(45\ \text{kg})}{(72\ \text{kg})(7.58\ \text{m})}$$
$$= 0.848 \approx 85\%.$$

41. The diagrams below show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction.



Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $F_v + F_A - W = 0$. The horizontal components must sum to zero: $T - F_h = 0$. The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then

$$F_A L \cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0.$$

Here we recognize that the man is one–fourth the length of the ladder side from the top and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L \cos\theta - T(L/2)\sin\theta = 0$.

There are 5 different equations:

$$\begin{aligned}
F_v + F_A - W &= 0,\\
T - F_h &= 0\\
F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta &= 0\\
F_E - F_v &= 0\\
F_E L\cos\theta - T(L/2)\sin\theta &= 0.
\end{aligned}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$
\begin{aligned}
T - F_h &= 0 \\
WL\cos\theta - F_E L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta &= 0 \\
F_E L\cos\theta - T(L/2)\sin\theta &= 0.
\end{aligned}
$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{3W}{4\tan\theta}.$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\,\text{m})^2 - (0.381\,\text{m})^2} = 1.16\,\text{m}$. This means

$$\tan\theta = (1.16\text{m})/(0.381\text{m}) = 3.04.$$

Thus,

$$T = \frac{3(854\,\text{N})}{4(3.04)} = 211\,\text{N}.$$

(b) We now solve for F_A. Since $F_v = F_E$ and $F_E = T\sin\theta/2\cos\theta$, $F_v = 3W/8$. We substitute this into $F_v + F_A - W = 0$ and solve for F_A. We find

$$F_A = W - F_v = W - 3W/8 = 5W/8 = 5(884\,\text{N})/8 = 534\,\text{N}.$$

(c) We have already obtained an expression for F_E: $F_E = 3W/8$. Evaluating it, we get $F_E =$ 320 N.

42. (a) Eq. 12-9 leads to

$$TL\sin\theta - m_p g x - m_b g\left(\tfrac{L}{2}\right) = 0 \ .$$

This can be written in the form of a straight line (in the graph) with

$$T = (\text{“slope”})\,\tfrac{x}{L} + \text{“}y\text{-intercept”},$$

where “slope” = $m_p g/\sin\theta$ and “y-intercept” = $m_b g/2\sin\theta$. The graph suggests that the slope (in SI units) is 200 and the y-intercept is 500. These facts, combined with the given $m_p + m_b = 61.2$ kg datum, lead to the conclusion:

$$\sin\theta = 61.22g/1200 \Rightarrow \theta = 30.0^{\circ}.$$

(b) It also follows that $m_p = 51.0$ kg.

(c) Similarly, $m_b = 10.2$ kg.

43. (a) The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross–sectional area of the rod. In this case F is the weight of the object hung on the end: $F = mg$, where m is the mass of the object. If r is the radius of the rod then $A = \pi r^2$. Thus, the shear stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5\times10^6\,\text{N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F / A}{\Delta x / L}$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F / A)L}{G} = \frac{(6.5\times10^6\ \text{N/m}^2)(0.053\,\text{m})}{3.0\times10^{10}\ \text{N/m}^2} = 1.1\times10^{-5}\ \text{m}.$$

44. (a) The Young's modulus is given by

$$E = \frac{\text{stress}}{\text{strain}} = \text{slope of the stress-strain curve} = \frac{150\times10^{6}\,\text{N/m}^2}{0.002} = 7.5\times10^{10}\,\text{N/m}^2.$$

(b) Since the linear range of the curve extends to about 2.9×10^8 N/m^2, this is approximately the yield strength for the material.

45. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A / AE$, where A is the cross–sectional area of the wire and E is Young's modulus for steel (200×10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B / AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell.$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross–sectional area of a wire is

$$A = \pi r^2 = \pi\left(1.20\times 10^{-3}\,\text{m}\right)^2 = 4.52\times 10^{-6}\,\text{m}^2 .$$

Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$\begin{aligned} F_A &= \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m}) + (4.52\times 10^{-6}\,\text{m}^2)(200\times 10^9\,\text{N/m}^2)(2.0\times 10^{-3}\,\text{m})}{2.50\,\text{m} + 2.50\,\text{m}} \\ &= 866\,\text{N}. \end{aligned}$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143\,\text{N}.$$

(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$

46. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A \;(\text{differential strain}) L \;= AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (wire-area) × (wire-length) × (graph-area-under-curve). Since the area of a triangle (see the graph in the problem statement) is $\frac{1}{2}$(base)(height) then we determine the work done to be

$$W = (2.00 \text{ x } 10^{-6} \text{ m}^2)(0.800 \text{ m})\left(\tfrac{1}{2}\right)(1.0 \times 10^{-3})(7.0 \times 10^{7} \text{ N/m}^2) = 0.0560 \text{ J} .$$

47. (a) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{FA}{A_A E_A} \quad \text{and} \quad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B}$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)(2E_B)}{A_B E_B} = 4.$$

When we combine this ratio with the equation $F_A + F_B = W$, we find $F_A/W = 4/5 = 0.80$.

(b) This also leads to the result $F_B/W = 1/5 = 0.20$.

(c) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$ which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{1}{4} = 0.25.$$

48. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A\, (\text{differential strain}) L \;= AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (thread cross-sectional area) × (thread length) × (graph-area-under-curve). The area under the curve is

$$\begin{aligned}\text{graph area} &= \frac{1}{2}as_1 + \frac{1}{2}(a+b)(s_2 - s_1) + \frac{1}{2}(b+c)(s_3 - s_2) = \frac{1}{2}\left[as_2 + b(s_3 - s_1) + c(s_3 - s_2)\right] \\ &= \frac{1}{2}\left[(0.12\times10^9 \text{ N/m}^2)(1.4) + (0.30\times10^9 \text{ N/m}^2)(1.0) + (0.80\times10^9 \text{ N/m}^2)(0.60)\right] \\ &= 4.74\times10^8 \text{ N/m}^2.\end{aligned}$$

(a) The kinetic energy that would put the thread on the verge of breaking is simply equal to W:

$$K = W = AL(\text{graph area}) = (8.0\times10^{-12} \text{ m}^2)(8.0\times10^{-3} \text{ m})(4.74\times10^8 \text{ N/m}^2) = 3.03\times10^{-5} \text{ J}.$$

(b) The kinetic energy of the fruit fly of mass 6.00 mg and speed 1.70 m/s is

$$K_f = \frac{1}{2}m_f v_f^2 = \frac{1}{2}(6.00\times10^{-6} \text{ kg})(1.70 \text{ m/s})^2 = 8.67\times10^{-6} \text{ J}.$$

(c) Since $K_f < W$, the fruit fly will not be able to break the thread.

(d) The kinetic energy of a bumble bee of mass 0.388 g and speed 0.420 m/s is

$$K_b = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(3.99\times10^{-4} \text{ kg})0.420 \text{ m/s})^2 = 3.42\times10^{-5} \text{ J}.$$

(e) On the other hand, since $K_b > W$, the bumble bee will be able to break the thread.

49. The flat roof (as seen from the air) has area A = 150 m × 5.8 m = 870 m^2. The volume of material directly above the tunnel (which is at depth d = 60 m) is therefore

$$V = A \times d = (870\ \text{m}^2) \times (60\text{m}) = 52200\ \text{m}^3.$$

Since the density is ρ = 2.8 g/cm^3 = 2800 kg/m^3, we find the mass of material supported by the steel columns to be $m = \rho V$ = 1.46 × 10^8 m^3.

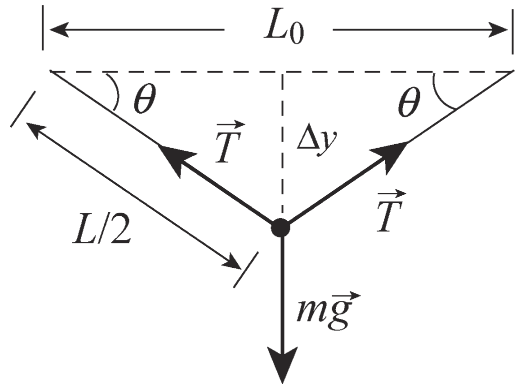
(a) The weight of the material supported by the columns is mg = 1.4 × 10^9 N.

(b) The number of columns needed is

$$n = \frac{1.43\times10^9\,\text{N}}{\frac{1}{2}(400\times10^6\,\text{N}/\text{m}^2)(960\times10^{-4}\,\text{m}^2)} = 75.$$

50. On the verge of breaking, the length of the thread is

$$L = L_0 + \Delta L = L_0(1+\Delta L / L_0) = L_0(1+2) = 3L_0\,,$$



where $L_0 = 0.020$ m is the original length, and $\text{strain} = \Delta L / L_0 = 2$, as given in the problem. The free-body diagram of the system is shown on the right. The condition for equilibrium is

$$mg = 2T\sin\theta$$

where m is the mass of the insect and $T = A(\text{stress})$. Since the volume of the thread remains constant is it is being stretched, we have $V = A_0 L_0 = AL$, or $A = A_0(L_0 / L) = A_0 / 3$. The vertical distance Δy is

$$\Delta y = \sqrt{(L/2)^2 - (L0/2)^2} = \sqrt{\frac{9L_0^2}{4} - \frac{L_0^2}{4}} = \sqrt{2}L_0\,.$$

Thus, the mass of the insect is

$$\begin{aligned} m &= \frac{2T\sin\theta}{g} = \frac{2(A_0/3)(\text{stress})\sin\theta}{g} = \frac{2A_0(\text{stress})}{3g}\frac{\Delta y}{3L_0/2} = \frac{4\sqrt{2}A_0(\text{stress})}{9g} \\ &= \frac{4\sqrt{2}(8.00\times10^{-12}\ \text{m}^2)(8.20\times10^{8}\ \text{N/m}^2)}{9(9.8\ \text{m/s}^2)} \\ &= 4.21\times10^{-4}\ \text{kg} \end{aligned}$$

or 0.421 g.

51. Let the forces that compress stoppers A and B be F_A and F_B, respectively. Then equilibrium of torques about the axle requires $FR = r_AF_A + r_BF_B$. If the stoppers are compressed by amounts $|\Delta y_A|$ and $|\Delta y_B|$ respectively, when the rod rotates a (presumably small) angle θ (in radians), then $|\Delta y_A| = r_A\theta$ and $|\Delta y_B| = r_B\theta$.

Furthermore, if their "spring constants" k are identical, then $k = |F/\Delta y|$ leads to the condition $F_A/r_A = F_B/r_B$ which provides us with enough information to solve.

(a) Simultaneous solution of the two conditions leads to

$$F_A = \frac{Rr_A}{r_A^2 + r_B^2}F = \frac{(5.0\text{ cm})(7.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 118\text{ N} \approx 1.2\times10^2\text{ N}.$$

(b) It also yields

$$F_B = \frac{Rr_B}{r_A^2 + r_B^2}F = \frac{(5.0\text{ cm})(4.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 68\text{ N}.$$

52. (a) With pivot at the hinge (at the left end), Eq. 12-9 gives

$$-mgx - Mg\frac{L}{2} + F_h h = 0$$

where m is the man's mass and M is that of the ramp; F_h is the leftward push of the right wall onto the right edge of the ramp. This equation can be written to be of the form (for a straight line in a graph)

$$F_h = (\text{"slope"})x + (\text{"y-intercept"}),$$

where the "slope" is mg/h and the "y-intercept" is $MgD/2h$. Since h = 0.480 m and D = 4.00 m, and the graph seems to intercept the vertical axis at 20 kN, then we find M = 500 kg.

(b) Since the "slope" (estimated from the graph) is (5000 N)/(4 m), then the man's mass must be m = 62.5 kg.

53. With the x axis parallel to the incline (positive uphill), then

$$\sum F_x = 0 \quad \Rightarrow \quad T\cos 25° - mg\sin 45° \ = \ 0.$$

Therefore, $T = 76$ N.

54. The beam has a mass $M = 40.0$ kg and a length $L = 0.800$ m. The mass of the package of tamale is $m = 10.0$ kg.

(a) Since the system is in static equilibrium, the normal force on the beam from roller A is equal to half of the weight of the beam:

$$F_A = Mg/2 = (40.0\text{ kg})(9.80\text{ m/s}^2)/2 = 196\text{ N}.$$

(b) The normal force on the beam from roller B is equal to half of the weight of the beam plus the weight of the tamale:

$$F_B = Mg/2 + mg = (40.0\text{ kg})(9.80\text{ m/s}^2)/2 + (10.0\text{ kg})(9.80\text{ m/s}^2) = 294\text{ N}.$$

(c) When the right-hand end of the beam is centered over roller B, the normal force on the beam from roller A is equal to the weight of the beam plus half of the weight of the tamale:

$$F_A = Mg + mg/2 = (40.0\text{ kg})(9.8\text{ m/s}^2) + (10.0\text{ kg})(9.80\text{ m/s}^2)/2 = 441\text{ N}.$$

(d) Similarly, the normal force on the beam from roller B is equal to half of the weight of the tamale:

$$F_B = mg/2 = (10.0\text{ kg})(9.80\text{ m/s}^2)/2 = 49.0\text{ N}.$$

(e) We choose the rotational axis to pass through roller B. When the beam is on the verge of losing contact with roller A, the net torque is zero. The balancing equation may be written as

$$mgx = Mg(L/4 - x) \quad \Rightarrow \quad x = \frac{L}{4}\frac{M}{M+m}.$$

Substituting the values given, we obtain $x = 0.160$ m.

55. (a) The forces acting on bucket are the force of gravity, down, and the tension force of cable A, up. Since the bucket is in equilibrium and its weight is

$$W_B = m_B g = (817\text{kg})(9.80\,\text{m/s}^2) = 8.01\times10^3\,\text{N},$$

the tension force of cable A is $T_A = 8.01\times10^3\,\text{N}$.

(b) We use the coordinates axes defined in the diagram. Cable A makes an angle of θ_2 = 66.0º with the negative y axis, cable B makes an angle of 27.0º with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means $T_B \cos 27.0° - T_A \cos 66.0° = 0$ and

$$T_B = \frac{\cos 66.0°}{\cos 27.0°} T_A = \left(\frac{\cos 66.0°}{\cos 27.0°}\right)(8.01\times10^3\,\text{N}) = 3.65\times10^3\text{N}.$$

(c) The x components must also sum to zero. This means

$$T_C + T_B \sin 27.0° - T_A \sin 66.0° = 0$$

Which yields

$$\begin{aligned} T_C &= T_A \sin 66.0° - T_B \sin 27.0° = (8.01\times10^3\,\text{N})\sin 66.0° - (3.65\times10^3\,\text{N})\sin 27.0° \\ &= 5.66\times10^3\,\text{N}. \end{aligned}$$

56. (a) Eq. 12-8 leads to $T_1 \sin 40° + T_2 \sin\theta = mg$. Also, Eq. 12-7 leads to

$$T_1 \cos 40° - T_2 \cos\theta = 0.$$

Combining these gives the expression

$$T_2 = \frac{mg}{\cos\theta \tan 40° + \sin\theta}.$$

To minimize this, we can plot it or set its derivative equal to zero. In either case, we find that it is at its minimum at $\theta = 50°$.

(b) At $\theta = 50°$, we find $T_2 = 0.77mg$.

57. The cable that goes around the lowest pulley is cable 1 and has tension $T_1 = F$. That pulley is supported by the cable 2 (so $T_2 = 2T_1 = 2F$) and goes around the middle pulley. The middle pulley is supported by cable 3 (so $T_3 = 2T_2 = 4F$) and goes around the top pulley. The top pulley is supported by the upper cable with tension T, so $T = 2T_3 = 8F$. Three cables are supporting the block (which has mass $m = 6.40$ kg):

$$T_1 + T_2 + T_3 = mg \Rightarrow F = \frac{mg}{7} = 8.96\,\text{N}.$$

Therefore, $T = 8(8.96\text{ N}) = 71.7$ N.

58. Since all surfaces are frictionless, the contact force $\vec{F}$ exerted by the lower sphere on the upper one is along that 45° line, and the forces exerted by walls and floors are "normal" (perpendicular to the wall and floor surfaces, respectively). Equilibrium of forces on the top sphere leads to the two conditions

$$F_{\text{wall}} = F\cos 45° \quad \text{and} \quad F\sin 45° = mg.$$

And (using Newton's third law) equilibrium of forces on the bottom sphere leads to the two conditions

$$F'_{\text{wall}} = F\cos 45° \quad \text{and} \quad F'_{\text{floor}} = F\sin 45° + mg.$$

(a) Solving the above equations, we find $F'_{\text{floor}} = 2mg$.

(b) We obtain for the left side of the container, $F'_{\text{wall}} = mg$.

(c) We obtain for the right side of the container, $F_{\text{wall}} = mg$.

(d) We get $F = mg/\sin 45° = \sqrt{2}mg$.

59. (a) The center of mass of the top brick cannot be further (to the right) with respect to the brick below it (brick 2) than $L/2$; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = L/2$ in the maximum case.

(b) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = L/4$.

(c) Now the total center of mass of bricks 1, 2 and 3 is one–third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{com}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6}$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$.

(d) A similar calculation

$$x'_{\text{com}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$.

(e) We find $h = \sum_{i=1}^{4} a_i = 25L/24$.

60. (a) If L (= 1500 cm) is the unstretched length of the rope and $\Delta L = 2.8\,\text{cm}$ is the amount it stretches then the strain is

$$\Delta L / L = (2.8\,\text{cm}) / (1500\,\text{cm}) = 1.9 \times 10^{-3}.$$

(b) The stress is given by F/A where F is the stretching force applied to one end of the rope and A is the cross–sectional area of the rope. Here F is the force of gravity on the rock climber. If m is the mass of the rock climber then $F = mg$. If r is the radius of the rope then $A = \pi r^2$. Thus the stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(95\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(4.8\times10^{-3}\ \text{m})^2} = 1.3\times10^7\ \text{N/m}^2.$$

(c) Young's modulus is the stress divided by the strain:

$$E = (1.3 \times 10^7\ \text{N/m}^2) / (1.9 \times 10^{-3}) = 6.9 \times 10^9\ \text{N/m}^2.$$

61. We denote the mass of the slab as *m*, its density as ρ, and volume as $V = LTW$. The angle of inclination is $\theta = 26°$.

(a) The component of the weight of the slab along the incline is

$$\begin{aligned} F_1 &= mg\sin\theta = \rho Vg\sin\theta \\ &= (3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\sin 26° \approx 1.8\times10^7\ \text{N}. \end{aligned}$$

(b) The static force of friction is

$$\begin{aligned} f_s &= \mu_s F_N = \mu_s mg\cos\theta = \mu_s \rho Vg\cos\theta \\ &= (0.39)(3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\cos 26° \approx 1.4\times10^7\ \text{N}. \end{aligned}$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77\times10^7\,\text{N} - 1.42\times10^7\ \text{N} = 3.5\times10^6\ \text{N}.$$

If the minimum number of bolts needed is *n*, then $F_2 / nA \le 3.6\times10^8\ \text{N/m}^2$, or

$$n \ge \frac{3.5\times10^6\ \text{N}}{(3.6\times10^8\ \text{N/m}^2)(6.4\times10^{-4}\ \text{m}^2)} = 15.2$$

Thus 16 bolts are needed.

62. The notation and coordinates are as shown in Fig. 12-6 in the textbook. Here, the ladder's center of mass is halfway up the ladder (unlike in the textbook figure). Also, we label the x and y forces at the ground f_s and F_N, respectively. Now, balancing forces, we have

$$\sum F_x = 0 \quad \Rightarrow \quad f_s = F_w$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_N = mg$$

Since $f_s = f_{s,\,\max}$, we divide the equations to obtain

$$\frac{f_{s,\max}}{F_N} = \mu_s = \frac{F_w}{mg} \quad .$$

Now, from $\Sigma\, \tau_z = 0$ (with axis at the ground) we have $mg(a/2) - F_w h = 0$. But from the Pythagorean theorem, $h = \sqrt{L^2 - a^2}$, where L = length of ladder. Therefore,

$$\frac{F_w}{mg} = \frac{a/2}{h} = \frac{a}{2\sqrt{L^2 - a^2}} \quad .$$

In this way, we find

$$\mu_s = \frac{a}{2\sqrt{L^2 - a^2}} \quad \Rightarrow \quad a = \frac{2\mu_s L}{\sqrt{1 + 4\mu_s^2}} = 3.4 \text{ m}.$$

63. Analyzing forces at the knot (particularly helpful is a graphical view of the vector right–triangle with horizontal "side" equal to the static friction force f_s and vertical "side" equal to the weight $m_B g$ of block B), we find $f_s = m_B g \tan \theta$ where $\theta = 30°$. For f_s to be at its maximum value, then it must equal $\mu_s m_A g$ where the weight of block A is $m_A g$= (10 kg)(9.8 m/s^2). Therefore,

$$\mu_s m_A g = m_B g \tan\theta \Rightarrow \mu_s = \frac{5.0}{10}\tan 30° = 0.29.$$

64. To support a load of $W = mg = (670 \text{ kg})(9.8 \text{ m/s}^2) = 6566$ N, the steel cable must stretch an amount proportional to its "free" length:

$$\Delta L = \left(\frac{W}{AY}\right)L \quad \text{where } A = \pi r^2$$

and $r = 0.0125$ m.

(a) If $L = 12$ m, then $\Delta L = \left(\dfrac{6566 \text{ N}}{\pi(0.0125 \text{ m})^2 (2.0\times10^{11} \text{ N/m}^2)}\right)(12 \text{ m}) = 8.0\times10^{-4} \text{ m}.$

(b) Similarly, when $L = 350$ m, we find $\Delta L = 0.023 \text{ m}$.

65. With the pivot at the hinge, Eq. 12-9 leads to

$$-mg\sin\theta_1\frac{L}{2} + TL\sin(180° - \theta_1 - \theta_2) = 0 .$$

where $\theta_1 = 60°$ and $T = mg/2$. This yields $\theta_2 = 60°$.

66. (a) Setting up equilibrium of torques leads to

$$F_{\text{far end}} L = (73\,\text{kg})(9.8\,\text{m/s}^2)\frac{L}{4} + (2700\,\text{N})\frac{L}{2}$$

which yields $F_{\text{far end}} = 1.5 \times 10^3$ N.

(b) Then, equilibrium of vertical forces provides

$$F_{\text{near end}} = (73)(9.8) + 2700 - F_{\text{far end}} = 1.9 \times 10^3\,\text{N}.$$

67. (a) and (b) With $+x$ rightward and $+y$ upward (we assume the adult is pulling with force $\vec{P}$ to the right), we have

$$\sum F_y = 0 \quad \Rightarrow \quad W = T\cos\theta = 270 \text{ N}$$
$$\sum F_x = 0 \quad \Rightarrow \quad P = T\sin\theta = 72 \text{ N}$$

where $\theta = 15°$.

(c) Dividing the above equations leads to

$$\frac{P}{W} = \tan\theta \; .$$

Thus, with $W = 270$ N and $P = 93$ N, we find $\theta = 19°$.

68. We denote the tension in the upper left string (*bc*) as T' and the tension in the lower right string (*ab*) as T. The supported weight is Mg = 19.6 N. The force equilibrium conditions lead to

$$T' \cos 60° = T \cos 20° \qquad \text{horizontal forces}$$
$$T' \sin 60° = W + T \sin 20° \qquad \text{vertical forces.}$$

(a) We solve the above simultaneous equations and find

$$T = \frac{W}{\tan 60° \cos 20° - \sin 20°} = 15 \text{N}.$$

(b) Also, we obtain $T' = T \cos 20° / \cos 60° = 29$ N.

69. (a) Because of Eq. 12-3, we can write

$$\vec{T} + (m_B\, g \ \angle -90^\circ) + (m_A\, g \ \angle -150^\circ) = 0 \, .$$

Solving the equation, we obtain $\vec{T} = (106.34 \ \angle\, 63.963^\circ)$. Thus, the magnitude of the tension in the upper cord is 106 N,

(b) and its angle (measured ccw from the $+x$ axis) is 64.0°.

70. (a) The angle between the beam and the floor is

$$\sin^{-1}(d/L) = \sin^{-1}(1.5/2.5) = 37°,$$

so that the angle between the beam and the weight vector $\vec{W}$ of the beam is 53°. With L = 2.5 m being the length of beam, and choosing the axis of rotation to be at the base,

$$\sum \tau_z = 0 \Rightarrow PL - W\left(\frac{L}{2}\right)\sin 53° = 0$$

Thus, $P = ½\, W \sin 53° = 200$ N.

(b) Note that

$$\vec{P} + \vec{W} = (200 \angle 90°) + (500 \angle -127°) = (360 \angle -146°)$$

using magnitude-angle notation (with angles measured relative to the beam, where "uphill" along the beam would correspond to 0°) with the unit Newton understood. The "net force of the floor" $\vec{F}_f$ is equal and opposite to this (so that the total net force on the beam is zero), so that $|\vec{F}_f| = 360$ N and is directed 34° counterclockwise from the beam.

(c) Converting that angle to one measured from true horizontal, we have $\theta = 34° + 37° = 71°$. Thus, $f_s = F_f \cos\theta$ and $F_N = F_f \sin\,\theta$. Since $f_s = f_{s,\,\max}$, we divide the equations to obtain

$$\frac{F_N}{f_{s,\max}} = \frac{1}{\mu_s} = \tan\theta\ .$$

Therefore, $\mu_s = 0.35$.

71. The cube has side length l and volume $V = l^3$. We use $p = B\Delta V / V$ for the pressure p. We note that

$$\frac{\Delta V}{V} = \frac{\Delta l^3}{l^3} = \frac{(l+\Delta l)^3 - l^3}{l^3} \approx \frac{3l^2\Delta l}{l^3} = 3\frac{\Delta l}{l}.$$

Thus, the pressure required is

$$p = \frac{3B\Delta l}{l} = \frac{3(1.4\times10^{11}\ \text{N/m}^2)(85.5\,\text{cm} - 85.0\,\text{cm})}{85.5\,\text{cm}} = 2.4\times10^9\ \text{N/m}^2.$$

72. Adopting the usual convention that torques that would produce counterclockwise rotation are positive, we have (with axis at the hinge)

$$\sum \tau_z = 0 \Rightarrow TL\sin 60^\circ - Mg\left(\frac{L}{2}\right) = 0$$

where L = 5.0 m and M = 53 kg. Thus, T = 300 N. Now (with F_p for the force of the hinge)

$$\sum F_x = 0 \Rightarrow F_{px} = -T\cos\theta = -150\text{N}$$
$$\sum F_y = 0 \Rightarrow F_{py} = Mg - T\sin\theta = 260\text{N}$$

where θ = 60°. Therefore, $\vec{F}_p = (-1.5\times10^2 \text{ N})\hat{\text{i}} + (2.6\times10^2 \text{ N})\hat{\text{j}}$.

73. (a) Choosing an axis through the hinge, perpendicular to the plane of the figure and taking torques that would cause counterclockwise rotation as positive, we require the net torque to vanish:

$$FL\sin 90° - Th\sin 65° = 0$$

where the length of the beam is L = 3.2 m and the height at which the cable attaches is h = 2.0 m. Note that the weight of the beam does not enter this equation since its line of action is directed towards the hinge. With F = 50 N, the above equation yields T = 88 N.

(b) To find the components of $\vec{F}_p$ we balance the forces:

$$\begin{aligned} \sum F_x = 0 \quad &\Rightarrow \quad F_{px} = T\cos 25° - F \\ \sum F_y = 0 \quad &\Rightarrow \quad F_{py} = T\sin 25° + W \end{aligned}$$

where W is the weight of the beam (60 N). Thus, we find that the hinge force components are F_{px} = 30 N rightward and F_{py} = 97 N upward. In unit-vector notation, $\vec{F}_p = (30\text{ N})\hat{\text{i}} + (97\text{ N})\hat{\text{j}}$.

74. (a) Computing the torques about the hinge, we have $TL\sin 40° = W\frac{L}{2}\sin 50°$ where the length of the beam is L = 12 m and the tension is T = 400 N. Therefore, the weight is $W = 671\text{ N}$, which means that the gravitational force on the beam is $\vec{F}_w = (-671\text{ N})\hat{\text{j}}$.

(b) Equilibrium of horizontal and vertical forces yields, respectively,

$$F_{\text{hinge}\,x} = T = 400\text{ N}$$
$$F_{\text{hinge}\,y} = W = 671\text{ N}$$

where the hinge force components are rightward (for x) and upward (for y). In unit-vector notation, we have $\vec{F}_{\text{hinge}} = (400\text{ N})\hat{\text{i}} + (671\text{ N})\hat{\text{j}}$

75. We locate the origin of the x axis at the edge of the table and choose rightwards positive. The criterion (in part (a)) is that the center of mass of the block above another must be no further than the edge of the one below; the criterion in part (b) is more subtle and is discussed below. Since the edge of the table corresponds to $x = 0$ then the total center of mass of the blocks must be zero.

(a) We treat this as three items: one on the upper left (composed of two bricks, one directly on top of the other) of mass $2m$ whose center is above the left edge of the bottom brick; a single brick at the upper right of mass m which necessarily has its center over the right edge of the bottom brick (so $a_1 = L/2$ trivially); and, the bottom brick of mass m. The total center of mass is

$$\frac{(2m)(a_2 - L) + ma_2 + m(a_2 - L/2)}{4m} = 0$$

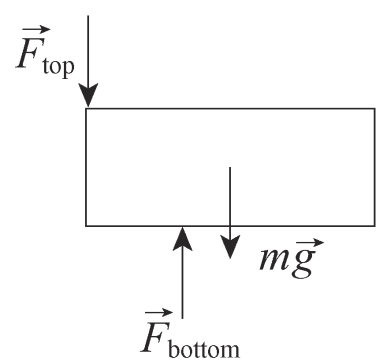
which leads to $a_2 = 5L/8$. Consequently, $h = a_2 + a_1 = 9L/8$.

(b) We have four bricks (each of mass m) where the center of mass of the top and the center of mass of the bottom one have the same value $x_{cm} = b_2 - L/2$. The middle layer consists of two bricks, and we note that it is possible for each of their centers of mass to be beyond the respective edges of the bottom one! This is due to the fact that the top brick is exerting downward forces (each equal to half its weight) on the middle blocks — and in the extreme case, this may be thought of as a pair of concentrated forces exerted at the innermost edges of the middle bricks. Also, in the extreme case, the support force (upward) exerted on a middle block (by the bottom one) may be thought of as a concentrated force located at the edge of the bottom block (which is the point about which we compute torques, in the following).

If (as indicated in our sketch, where $\vec{F}_{\text{top}}$ has magnitude $mg/2$) we consider equilibrium of torques on the rightmost brick, we obtain

$$mg\left(b_1 - \frac{1}{2}L\right) = \frac{mg}{2}(L - b_1)$$

which leads to $b_1 = 2L/3$. Once we conclude from symmetry that $b_2 = L/2$ then we also arrive at $h = b_2 + b_1 = 7L/6$.



76. One arm of the balance has length ℓ_1 and the other has length ℓ_2. The two cases described in the problem are expressed (in terms of torque equilibrium) as

$$m_1\ell_1 = m\ell_2 \quad \text{and} \quad m\ell_1 = m_2\ell_2.$$

We divide equations and solve for the unknown mass: $m = \sqrt{m_1 m_2}$.

77. Since *GA* exerts a leftward force *T* at the corner *A*, then (by equilibrium of horizontal forces at that point) the force F_{diag} in *CA* must be pulling with magnitude

$$F_{\text{diag}} = \frac{T}{\sin 45°} = T\sqrt{2}.$$

This analysis applies equally well to the force in *DB*. And these diagonal bars are pulling on the bottom horizontal bar exactly as they do to the top bar, so the bottom bar *CD* is the "mirror image" of the top one (it is also under tension *T*). Since the figure is symmetrical (except for the presence of the turnbuckle) under 90° rotations, we conclude that the side bars (*DA* and *BC*) also are under tension *T* (a conclusion that also follows from considering the vertical components of the pull exerted at the corners by the diagonal bars).

(a) Bars that are in tension are *BC*, *CD* and *DA*.

(b) The magnitude of the forces causing tension is $T = 535\ \text{N}$.

(c) The magnitude of the forces causing compression on *CA* and *DB* is

$$F_{\text{diag}} = \sqrt{2}T = (1.41)535\ \text{N} = 757\ \text{N}.$$

78. (a) For computing torques, we choose the axis to be at support 2 and consider torques which encourage counterclockwise rotation to be positive. Let m = mass of gymnast and M = mass of beam. Thus, equilibrium of torques leads to

$$Mg(1.96\,\mathrm{m}) - mg(0.54\,\mathrm{m}) - F_1(3.92\,\mathrm{m}) = 0.$$

Therefore, the upward force at support 1 is F_1 = 1163 N (quoting more figures than are significant — but with an eye toward using this result in the remaining calculation). In unit-vector notation, we have $\vec{F}_1 \approx (1.16\times10^3\ \mathrm{N})\hat{\mathrm{j}}$.

(b) Balancing forces in the vertical direction, we have $F_1 + F_2 - Mg - mg = 0$, so that the upward force at support 2 is $F_2 = 1.74 \times 10^3$ N. In unit-vector notation, we have $\vec{F}_2 \approx (1.74\times10^3\ \mathrm{N})\hat{\mathrm{j}}$.

79. (a) Let $d = 0.00600$ m. In order to achieve the same final lengths, wires 1 and 3 must stretch an amount d more than wire 2 stretches:

$$\Delta L_1 = \Delta L_3 = \Delta L_2 + d\ .$$

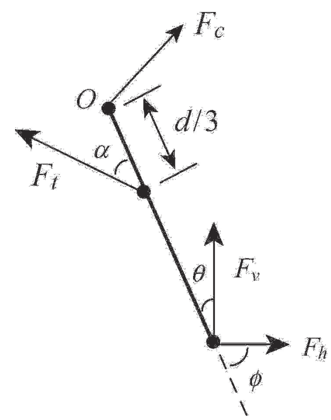
Combining this with Eq. 12-23 we obtain

$$F_1 = F_3 = F_2 + \frac{dAE}{L}\ .$$

Now, Eq. 12-8 produces $F_1 + F_3 + F_2 - mg = 0$. Combining this with the previous relation (and using Table 12-1) leads to $F_1 = 1380\text{ N} \approx 1.38\times10^3\text{ N}$.

(b) Similarly, $F_2 = 180$ N.

80. Our system is the second finger bone. Since the system is in static equilibrium, the net force acting on it is zero. In addition, the torque about any point must be zero. We set up the torque equation about point O where $\vec{F}_c$ act:



$$0=\sum_O \tau_{\text{net}} = -\left(\frac{d}{3}\right)F_t \sin\alpha + (d)F_v \sin\theta + (d)F_h \sin\phi .$$

Solving for F_t and substituting the values given, we obtain

$$F_t = \frac{3(F_v \sin\theta + F_h \sin\phi)}{\sin\alpha} = \frac{3[(162.4\text{ N})\sin 10° + (13.4\text{ N})\sin 80°]}{\sin 45°} = 175.6\text{ N}$$
$$\approx 1.8\times10^2\text{ N}.$$

81. When it is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point O where P is being applied. Thus, the line of action of $\vec{F}_N$ passes through point O and exerts no torque about O (of course, a similar observation applied to the pull P). Since $F_N = mg$ in this problem, we have $f_{s\max} = \mu mg$ applied a distance h away from O. And the line of action of force of gravity (of magnitude mg), which is best pictured as a concentrated force at the center of the cube, is a distance $L/2$ away from O. Therefore, equilibrium of torques about O produces

$$\mu mgh = mg\left(\frac{L}{2}\right) \Rightarrow \mu = \frac{L}{2h} = \frac{(8.0\text{ cm})}{2(7.0\text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

(a) For it to slide but not tip, a value of μ *less* than that derived above is needed, since then — static friction will be exceeded for a smaller value of P, before the pull is strong enough to cause it to tip. Thus, $\mu < L/2h = 0.57$ is required.

(b) And for it to tip but not slide, we need μ *greater* than that derived above is needed, since now — static friction will not be exceeded even for the value of P which makes the cube rotate about its front lower corner. That is, we need to have $\mu > L/2h = 0.57$ in this case.

82. The assumption stated in the problem (that the density does not change) is not meant to be realistic; those who are familiar with Poisson's ratio (and other topics related to the strengths of materials) might wish to think of this problem as treating a fictitious material (which happens to have the same value of E as aluminum, given in Table 12-1) whose density does not significantly change during stretching. Since the mass does not change, either, then the constant-density assumption implies the volume (which is the circular area times its length) stays the same:

$$(\pi r^2 L)_{\text{new}} = (\pi r^2 L)_{\text{old}} \quad \Rightarrow \quad \Delta L = L\left[(1000/999.9)^2 - 1\right] .$$

Now, Eq. 12-23 gives

$$F = \pi r^2 E\, \Delta L/L \;=\; \pi r^2 (7.0 \times 10^9\ \text{N/m}^2)\left[(1000/999.9)^2 - 1\right] .$$

Using either the new or old value for r gives the answer $F = 44$ N.

83. Where the crosspiece comes into contact with the beam, there is an upward force of $2F$ (where F is the upward force exerted by each man). By equilibrium of vertical forces, $W = 3F$ where W is the weight of the beam. If the beam is uniform, its center of gravity is a distance $L/2$ from the man in front, so that computing torques about the front end leads to

$$W\frac{L}{2} = 2Fx = 2\left(\frac{W}{3}\right)x$$

which yields $x = 3L/4$ for the distance from the crosspiece to the front end. It is therefore a distance $L/4$ from the rear end (the "free" end).

84. (a) Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\text{catch}} = (11\,\text{kg})\,(9.8\,\text{m/s}^2)\,\frac{(91/2-10)\,\text{cm}}{91\,\text{cm}} = 42\,\text{N}.$$

(b) Then, equilibrium of vertical forces provides

$$F_{\text{hinge}} = (11\,\text{kg})\,(9.8\,\text{m/s}^2) - F_{\text{catch}} = 66\,\text{N}.$$

85. We choose an axis through the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0\text{ m})/5 = 1.6\text{ m}$; in other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is $r_\perp = (4/5)(8.0\text{ m}) = 6.4\text{ m}$. The moment arm for the weight is half the horizontal distance from the wall to the base of the ladder; this works out to be $\sqrt{(10\text{ m})^2 - (8\text{ m})^2}/2 = 3.0\text{ m}$. Similarly, the moment arms for the x and y components of the force at the ground $\left(\vec{F}_g\right)$ are 8.0 m and 6.0 m, respectively. Thus, with lengths in meters, we have

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - F_{gy}(6.0\text{ m}) = 0.$$

In addition, from balancing the vertical forces we find that $W = F_{gy}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - W(6.0\text{ m}) = 0.$$

(a) With F = 50 N and W = 200 N, the above equation yields F_{gx} = 35 N. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(b) With F = 150 N and W = 200 N, the above equation yields F_{gx} = –45 N. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(c) Note that the phrase “start to move towards the wall” implies that the friction force is pointed away from the wall (in the $-\hat{\text{i}}$ direction). Now, if $f = -F_{gx}$ and $F_N = F_{gy}$ = 200 N are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with μ_s = 0.38, then we find F_{gx} = –76 N. Returning this to the above equation, we obtain

$$F = \frac{(200\text{ N})(3.0\text{ m}) + (76\text{ N})(8.0\text{ m})}{6.4\text{ m}} = 1.9\times10^2\text{ N}.$$

86. The force F exerted on the beam is F = 7900 N, as computed in the Sample Problem. Let $F/A = S_u/6$, where $S_u = 50\times10^6$ N/m^2 is the ultimate strength (see Table 12-1), then

$$A = \frac{6F}{S_u} = \frac{6(7900 \text{ N})}{50\times10^6 \text{ N/m}^2} = 9.5\times10^{-4}\text{m}^2.$$

Thus the thickness is $\sqrt{A} = \sqrt{9.5\times10^{-4} \text{ m}^2} = 0.031\text{m}$.

Chapter 13

1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.2\mathrm{kg}\right)\left(2.4\mathrm{kg}\right)}{2.3\times10^{-12}\,\mathrm{N}}} = 19\,\mathrm{m}\,.$$

2. We use subscripts s, e, and m for the Sun, Earth and Moon, respectively. Plugging in the numerical values (say, from Appendix C) we find

$$\frac{F_{sm}}{F_{em}} = \frac{Gm_s m_m / r_{sm}^2}{Gm_e m_m / r_{em}^2} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2 = \frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\left(\frac{3.82\times10^{8}\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.16.$$

3. The gravitational force between the two parts is

$$F = \frac{Gm(M - m)}{r^2} = \frac{G}{r^2}(mM - m^2)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M - 2m) \Rightarrow M = 2m .$$

This leads to the result $m/M = 1/2$.

4. The gravitational force between you and the moon at its initial position (directly opposite of Earth from you) is

$$F_0 = \frac{GM_m m}{(R_{ME} + R_E)^2}$$

where M_m is the mass of the moon, R_{ME} is the distance between the moon and the Earth, and R_E is the radius of the Earth. At its final position (directly above you), the gravitational force between you and the moon is

$$F_1 = \frac{GM_m m}{(R_{ME} - R_E)^2}.$$

(a) The ratio of the moon's gravitational pulls at the two different positions is

$$\frac{F_1}{F_0} = \frac{GM_m m/(R_{ME} - R_E)^2}{GM_m m/(R_{ME} + R_E)^2} = \left(\frac{R_{ME} + R_E}{R_{ME} - R_E}\right)^2 = \left(\frac{3.82\times10^8 \text{ m} + 6.37\times10^6 \text{ m}}{3.82\times10^8 \text{ m} - 6.37\times10^6 \text{ m}}\right)^2 = 1.06898.$$

Therefore, the increase is 0.06898, or approximately, 6.9%.

(b) The change of the gravitational pull may be approximated as

$$F_1 - F_0 = \frac{GM_m m}{(R_{ME} - R_E)^2} - \frac{GM_m m}{(R_{ME} + R_E)^2} \approx \frac{GM_m m}{R_{ME}^2}\left(1 + 2\frac{R_E}{R_{ME}}\right) - \frac{GM_m m}{R_{ME}^2}\left(1 - 2\frac{R_E}{R_{ME}}\right) = \frac{4GM_m mR_E}{R_{ME}^3}.$$

On the other hand, your weight, as measured on a scale on Earth is

$$F_g = mg_E = \frac{GM_E m}{R_E^2}.$$

Since the moon pulls you "up," the percentage decrease of weight is

$$\frac{F_1 - F_0}{F_g} = 4\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_{ME}}\right)^3 = 4\left(\frac{7.36\times10^{22} \text{ kg}}{5.98\times10^{24} \text{ kg}}\right)\left(\frac{6.37\times10^6 \text{ m}}{3.82\times10^8 \text{ m}}\right)^3 = 2.27\times10^{-7} \approx (2.3\times10^{-5})\%.$$

5. We require the magnitude of force (given by Eq. 13-1) exerted by particle C on A be equal to that exerted by B on A. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2} \; .$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "m_A") solve for r. We find $r = 5d$. Thus, particle C is placed on the x axis, to left of particle A (so it is at a negative value of x), at $x = -5.00d$.

6. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8} \angle 60.6°\right).$$

(a) The magnitude of the force is 2.13×10^{-8} N.

(b) The direction of the force relative to the $+x$ axis is $60.6°$.

7. At the point where the forces balance $GM_e m/r_1^2 = GM_s m/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d - r_1)^2}.$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s} + \sqrt{M_e}} = \frac{(150\times10^9\ \text{m})\sqrt{5.98\times10^{24}\ \text{kg}}}{\sqrt{1.99\times10^{30}\ \text{kg}} + \sqrt{5.98\times10^{24}\ \text{kg}}} = 2.60\times10^8\ \text{m}.$$

Values for M_e, M_s, and d can be found in Appendix C.

8. The gravitational forces on m_5 from the two 5.00g masses m_1 and m_4 cancel each other. Contributions to the net force on m_5 come from the remaining two masses:

$$F_{\text{net}} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.50\times10^{-3}\ \text{kg}\right)\left(3.00\times10^{-3}\ \text{kg}-1.00\times10^{-3}\ \text{kg}\right)}{\left(\sqrt{2}\times10^{-1}\ \text{m}\right)^2}$$

$$=1.67\times10^{-14}\ \text{N}.$$

The force is directed along the diagonal between m_2 and m_3, towards m_2. In unit-vector notation, we have

$$\vec{F}_{\text{net}} = F_{\text{net}}(\cos 45^\circ\hat{\text{i}}+\sin 45^\circ\hat{\text{j}}) = (1.18\times10^{-14}\,\text{N})\,\hat{\text{i}} + (1.18\times10^{-14}\,\text{N})\,\hat{\text{j}}$$

9. The gravitational force from Earth on you (with mass m) is

$$F_g = \frac{GM_E m}{R_E^2} = mg$$

where $g = GM_E / R_E^2 = 9.8\ \text{m/s}^2$. If r is the distance between you and a tiny black hole of mass $M_b = 1\times10^{11}\ \text{kg}$ that has the same gravitational pull on you as the Earth, then

$$F_g = \frac{GM_b m}{r^2} = mg.$$

Combining the two equations, we obtain

$$mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow r = \sqrt{\frac{GM_b}{g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1\times10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

10. (a) We are told the value of the force when particle C is removed (that is, as its position x goes to infinity), which is a situation in which any force caused by C vanishes (because Eq. 13-1 has r^2 in the denominator). Thus, this situation only involves the force exerted by A on B:

$$\frac{Gm_A\, m_B}{(0.20\text{ m})^2} = 4.17 \times 10^{-10}\text{ N}\,.$$

Since $m_B = 1.0$ kg, then this yields $m_A = 0.25$ kg.

(b) We note (from the graph) that the net force on B is zero when $x = 0.40$ m. Thus, at that point, the force exerted by C must have the same magnitude (but opposite direction) as the force exerted by A (which is the one discussed in part (a)). Therefore

$$\frac{Gm_C\, m_B}{(0.40\text{ m})^2} = 4.17 \times 10^{-10}\text{ N} \quad \Rightarrow \quad m_C = 1.00\text{ kg}.$$

11. (a) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell / 2\cos 30° = \ell / \sqrt{3}$$

where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4 m}{r^2}\right)\sin 30° = 3\frac{Gm_4 m}{\ell^2}.$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4 m}{\ell^2} = \frac{Gm_4 m}{\left(\ell / \sqrt{3}\right)^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

12. All the forces are being evaluated at the origin (since particle *A* is there), and all forces (except the net force) are along the location-vectors $\vec{r}$ which point to particles *B* and *C*. We note that the angle for the location-vector pointing to particle *B* is 180º – 30.0º = 150º (measured ccw from the +*x* axis). The component along, say, the *x* axis of one of the force-vectors $\vec{F}$ is simply Fx/r in this situation (where *F* is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned *x* component would have the form $GmMx/r^3$; similarly for the other components. With m_A = 0.0060 kg, m_B = 0.0120 kg, and m_C = 0.0080 kg, we therefore have

$$F_{\text{net }x} = \frac{Gm_A m_B x_B}{r_B^3} + \frac{Gm_A m_C x_C}{r_C^3} = (2.77 \times 10^{-14}\text{ N})\cos(-163.8^\circ)$$

and

$$F_{\text{net }y} = \frac{Gm_A m_B y_B}{r_B^3} + \frac{Gm_A m_C y_C}{r_C^3} = (2.77 \times 10^{-14}\text{ N})\sin(-163.8^\circ)$$

where $r_B = d_{AB} = 0.50$ m, and $(x_B, y_B) = (r_B\cos(150^\circ), r_B\sin(150^\circ))$ (with SI units understood). A fairly quick way to solve for r_C is to consider the vector difference between the net force and the force exerted by *A*, and then employ the Pythagorean theorem. This yields r_C = 0.40 m.

(a) By solving the above equations, the *x* coordinate of particle *C* is $x_C = -0.20$ m.

(b) Similarly, the *y* coordinate of particle *C* is $y_C = -0.35$ m.

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere. That is $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}.$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d-R/2)^2}.$$

The force of the hollowed sphere on m is

$$\begin{aligned} F = F_1 - F_2 &= GMm\left(\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1-R/2d)^2}\right) \\ &= \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(2.95\ \text{kg})(0.431\ \text{kg})}{(9.00\times10^{-2}\,\text{m})^2}\left(1 - \frac{1}{8[1-(4\times10^{-2}\,\text{m})/(2\cdot9\times10^{-2}\,\text{m})]^2}\right) \\ &= 8.31\times10^{-9}\,\text{N}. \end{aligned}$$

14. Using Eq. 13-1, we find

$$\vec{F}_{AB} = \frac{2Gm_A^2}{d^2}\,\hat{\mathrm{j}} \qquad \text{and} \qquad \vec{F}_{AC} = -\frac{4Gm_A^2}{3d^2}\,\hat{\mathrm{i}} \;.$$

Since the vector sum of all three forces must be zero, we find the third force (using magnitude-angle notation) is

$$\vec{F}_{AD} = \frac{Gm_A^2}{d^2}\,(2.404 \;\;\angle\;\; -56.3^\circ)\;.$$

This tells us immediately the direction of the vector $\vec{r}$ (pointing from the origin to particle D), but to find its magnitude we must solve (with $m_D = 4m_A$) the following equation:

$$2.404\left(\frac{Gm_A^2}{d^2}\right) = \frac{Gm_A m_D}{r^2} \quad .$$

This yields $r = 1.29d$. In magnitude-angle notation, then, $\vec{r} = (1.29 \;\angle\; -56.3^\circ)$, with SI units understood. The "exact" answer without regard to significant figure considerations is

$$\vec{r} = \left(\, 2\sqrt{\frac{6}{13\sqrt{13}}}\;,\; -3\sqrt{\frac{6}{13\sqrt{13}}} \,\right).$$

(a) In (x, y) notation, the x coordinate is $x = 0.716d$.

(b) Similarly, the y coordinate is $y = -1.07d$.

15. All the forces are being evaluated at the origin (since particle A is there), and all forces are along the location-vectors $\vec{r}$ which point to particles B, C and D. In three dimensions, the Pythagorean theorem becomes $r = \sqrt{x^2 + y^2 + z^2}$. The component along, say, the x axis of one of the force-vectors $\vec{F}$ is simply Fx/r in this situation (where F is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned x component would have the form $GmMx/r^3$; similarly for the other components. For example, the z component of the force exerted on particle A by particle B is

$$\frac{Gm_A m_B z_B}{r_B^{\,3}} = \frac{Gm_A(2m_A)(2d)}{((2d)^2 + d^2 + (2d)^2)^3} = \frac{4Gm_A^{\,2}}{27\, d^2}.$$

In this way, each component can be written as some multiple of $Gm_A^{\,2}/d^2$. For the z component of the force exerted on particle A by particle C, that multiple is $-9\sqrt{14}/196$. For the x components of the forces exerted on particle A by particles B and C, those multiples are $4/27$ and $-3\sqrt{14}/196$, respectively. And for the y components of the forces exerted on particle A by particles B and C, those multiples are $2/27$ and $3\sqrt{14}/98$, respectively. To find the distance r to particle D one method is to solve (using the fact that the vector add to zero)

$$\left(\frac{Gm_A m_D}{r^2}\right)^2 = [(4/27 - 3\sqrt{14}/196)^2 + (2/27 + 3\sqrt{14}/98)^2 + (4/27 - 9\sqrt{14}/196)^2]\left(\frac{Gm_A^{\,2}}{d^2}\right)^2$$

(where $m_D = 4m_A$) for r. This gives $r = 4.357d$. The individual values of x, y and z (locating the particle D) can then be found by considering each component of the $Gm_A m_D/r^2$ force separately.

(a) The x component of $\vec{r}$ would be

$$Gm_A\, m_D\, x/r^3 = -(4/27 - 3\sqrt{14}/196)Gm_A^{\,2}/d^2,$$

which yields $x = -1.88d$.

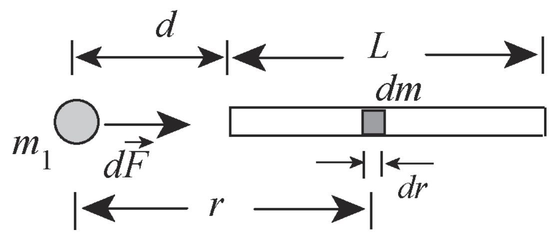
(b) Similarly, $y = -3.90d$,

(c) and $z = 0.489d$.

In this way we are able to deduce that $(x, y, z) = (1.88d, 3.90d, 0.49d)$.

16. Since the rod is an extended object, we cannot apply Equation 13-1 directly to find the force. Instead, we consider a small differential element of the rod, of mass dm of thickness dr at a distance r from m_1. The gravitational force between dm and m_1 is

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1 (M/L)dr}{r^2},$$



where we have substituted $dm = (M/L)dr$ since mass is uniformly distributed. The direction of $d\vec{F}$ is to the right (see figure). The total force can be found by integrating over the entire length of the rod:

$$F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d}\frac{dr}{r^2} = -\frac{Gm_1 M}{L}\left(\frac{1}{L+d} - \frac{1}{d}\right) = \frac{Gm_1 M}{d(L+d)}.$$

Substituting the values given in the problem statement, we obtain

$$F = \frac{Gm_1 M}{d(L+d)} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(0.67\ \text{kg})(5.0\ \text{kg})}{(0.23\ \text{m})(3.0\ \text{m}+0.23\ \text{m})} = 3.0\times10^{-10}\ \text{N}.$$

17. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(4.9\,\text{m}/\text{s}^2\right)}} - 6.37\times10^6\,\text{m} = 2.6\times10^6\,\text{m}.$$

18. We follow the method shown in Sample Problem 13-3. Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}\,dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

But since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}\,dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600\text{ N}\right)\frac{1.61\times10^3\text{ m}}{6.37\times10^6\text{ m}} = -0.303\text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

19. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}} = 1.67$ m/s^2 (see Appendix C). The ratio of weights (for a given mass) is the ratio of g-values, so

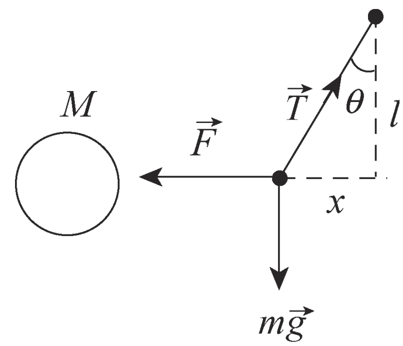
$$W_{\text{moon}} = (100 \text{ N})(1.67/9.8) = 17 \text{ N}.$$

(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g = 1.67$ m/s^2. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5 \times 10^7 \text{ m}$$

so the object would need to be a distance of $r/R_E = 2.4$ "radii" from Earth's center.

20. The free-body diagram of the force acting on the plumb line is shown on the right. The mass of the sphere is



$$M = \rho V = \rho\left(\frac{4\pi}{3}R^3\right) = \frac{4\pi}{3}(2.6\times10^3\ \text{kg/m}^3)(2.00\times10^3\ \text{m})^3$$
$$= 8.71\times10^{13}\ \text{kg}.$$

The force between the "spherical" mountain and the plumb line is $F = GMm/r^2$. Suppose at equilibrium the line makes an angle θ with the vertical and the net force acting on the line is zero. Therefore,

$$0 = \sum F_{\text{net},\,x} = T\sin\theta - F = T\sin\theta - \frac{GMm}{r^2}$$
$$0 = \sum F_{\text{net},\,y} = T\cos - mg$$

The two equations can be combined to give $\tan\theta = \dfrac{F}{mg} = \dfrac{GM}{gr^2}$. The distance the lower end moves toward the sphere is

$$x = l\tan\theta = l\frac{GM}{gr^2} = (0.50\ \text{m})\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(8.71\times10^{13}\ \text{kg})}{(9.8)(3\times2.00\times10^3\ \text{m})^2}.$$
$$= 8.2\times10^{-6}\ \text{m}.$$

21. (a) The gravitational acceleration is $a_g = \frac{GM}{R^2} = 7.6\ \text{m/s}^2$.

(b) Note that the total mass is 5*M*. Thus, $a_g = \frac{G(5M)}{(3R)^2} = 4.2\ \text{m/s}^2$.

22. (a) Plugging $R_h = 2GM_h/c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{(1.001R_h)^2} = \frac{GM_h}{(1.001)^2\left(2GM_h/c^2\right)^2} = \frac{c^4}{(2.002)^2 G}\frac{1}{M_h}$$

which yields $a_g = (3.02 \times 10^{43}\ \text{kg·m/s}^2)/M_h$.

(b) Since M_h is in the denominator of the above result, a_g decreases as M_h increases.

(c) With $M_h = (1.55 \times 10^{12})(1.99 \times 10^{30}\ \text{kg})$, we obtain $a_g = 9.82\ \text{m/s}^2$.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for M in Eq. 13–16, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2\frac{GM}{\left(2.002GM/c^2\right)^3}dr = -\frac{2c^6}{(2.002)^3(GM)^2}dr$$

where $dr \to 1.70$ m as in Sample Problem 13-3. This yields (in absolute value) an acceleration difference of $7.30 \times 10^{-15}\ \text{m/s}^2$.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

23. From Eq. 13-14, we see the extreme case is when "g" becomes zero, and plugging in Eq. 13-15 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

Thus, with R = 20000 m and ω = 2π rad/s, we find $M = 4.7 \times 10^{24}$ kg ≈ 5×10^{24} kg.

24. (a) What contributes to the GmM/r^2 force on m is the (spherically distributed) mass M contained within r (where r is measured from the center of M). At point A we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{G(M_1 + M_2)m}{a^2}.$$

(b) In the case $r = b$, only M_1 is contained within that radius, so the force on m becomes GM_1m/b^2.

(c) If the particle is at C, then no other mass is at smaller radius and the gravitational force on it is zero.

25. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2} = 9.83\ \text{m/s}^2.$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to Fig. 13-43). The total mass is

$$M = (1.93 \times 10^{24}\ \text{kg} + 4.01 \times 10^{24}\ \text{kg}) = 5.94 \times 10^{24}\ \text{kg}.$$

The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.94\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.84\ \text{m/s}^2.$$

(c) A point 25 km below the surface is at the mantle-crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345\times10^{6}\ \text{m}}{6.37\times10^{6}\ \text{m}}\right)^3 \left(5.98\times10^{24}\ \text{kg}\right) = 5.91\times10^{24}\ \text{kg}.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.91\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.79\ \text{m/s}^2.$$

26. (a) Using Eq. 13-1, we set GmM/r^2 equal to $\frac{1}{2}GmM/R^2$, and we find $r = R\sqrt{2}$. Thus, the distance from the surface is $(\sqrt{2} - 1)R = 0.414R$.

(b) Setting the density ρ equal to M/V where $V = \frac{4}{3}\pi R^3$, we use Eq. 13-19:

$$F = \frac{4\pi Gmr\rho}{3} = \frac{4\pi Gmr}{3}\left(\frac{M}{4\pi R^3/3}\right) = \frac{GMmr}{R^3} = \frac{1}{2}\frac{GMm}{R^2} \quad \Rightarrow \quad r = R/2.$$

27. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4\,\text{kg}}{\frac{4}{3}\pi\left(1.0\text{ m}\right)^3} = 2.4\times10^3\,\text{kg/m}^3.$$

When the particle of mass m (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius r (measured from the center of the sphere), then whatever mass M is at a radius less than r must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5$ m, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{GmM_{\text{total}}}{r^2} = m\left(3.0\times10^{-7}\,\text{N/kg}\right).$$

(b) At $r = 0.50$ m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3\text{ kg}.$$

Thus, the force on m has magnitude $GMm/r^2 = m\,(3.3\times10^{-7}\text{ N/kg})$.

(c) Pursuing the calculation of part (b) algebraically, we find

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

28. The difference between free-fall acceleration g and the gravitational acceleration a_g at the equator of the star is (see Equation 13.14):

$$a_g - g = \omega^2 R$$

where

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.041\,\text{s}} = 153\,\text{rad/s}$$

is the angular speed of the star. The gravitational acceleration at the equator is

$$a_g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.98\times10^{30}\ \text{kg})}{(1.2\times10^4\ \text{m})^2} = 9.17\times10^{11}\ \text{m/s}^2.$$

Therefore, the percentage difference is

$$\frac{a_g - g}{a_g} = \frac{\omega^2 R}{a_g} = \frac{(153\,\text{rad/s})^2(1.2\times10^4\ \text{m})}{9.17\times10^{11}\ \text{m/s}^2} = 3.06\times10^{-4} \approx 0.031\%.$$

29. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_g M = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2} a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2 \left(9.8\ \text{m/s}^2\right) = 3.8\ \text{m/s}^2.$$

(c) If v is the escape speed, then, for a particle of mass m

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

For Mars, the escape speed is

$$v = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.11)\left(5.98\times10^{24}\ \text{kg}\right)}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

30. (a) The gravitational potential energy is

$$U=-\frac{GMm}{r}=-\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.2\ \text{kg}\right)\left(2.4\ \text{kg}\right)}{19\ \text{m}}=-\ 4.4\times10^{-11}\ \text{J}.$$

(b) Since the change in potential energy is

$$\Delta U=-\frac{GMm}{3r}-\left(-\frac{GMm}{r}\right)=-\frac{2}{3}\left(-4.4\times10^{-11}\ \text{J}\right)=2.9\times10^{-11}\ \text{J},$$

the work done by the gravitational force is $W=-\ \Delta U=-2.9\times10^{-11}$ J.

(c) The work done by you is $W'=\Delta U=2.9\times10^{-11}$ J.

31. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass m on a planet of mass M and radius R needs $K = GmM/R$ in order to (barely) escape.
(a) Setting up the ratio, we find

$$\frac{K_m}{K_E} = \frac{M_m}{M_E}\frac{R_E}{R_m} = 0.0451$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_J} = 28.5.$$

32. (a) The potential energy at the surface is (according to the graph) -5.0×10^9 J, so (since U is inversely proportional to r – see Eq. 13-21) at an r-value a factor of 5/4 times what it was at the surface then U must be a factor of 4/5 what it was. Thus, at $r = 1.25R_s$ $U = -4.0 \times 10^9$ J. Since mechanical energy is assumed to be conserved in this problem, we have $K + U = -2.0 \times 10^9$ J at this point. Since $U = -4.0 \times 10^9$ J here, then $K = 2.0 \times 10^9 \text{ J}$ at this point.

(b) To reach the point where the mechanical energy equals the potential energy (that is, where $U = -2.0 \times 10^9$ J) means that U must reduce (from its value at $r = 1.25R_s$) by a factor of 2 – which means the r value must increase (relative to $r = 1.25R_s$) by a corresponding factor of 2. Thus, the turning point must be at $r = 2.5R_s$.

33. The equation immediately preceding Eq. 13-28 shows that $K = -U$ (with U evaluated at the planet's surface: -5.0×10^9 J) is required to "escape." Thus, $K = 5.0 \times 10^9$ J.

34. The gravitational potential energy is

$$U = -\frac{Gm(M-m)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$ which leads to the ratio $m/M = 1/2$ to obtain the least potential energy.

Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m − which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

35. (a) The work done by you in moving the sphere of mass m_B equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_A m_B}{d} - \frac{Gm_A m_C}{L} - \frac{Gm_B m_C}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_A m_B}{L-d} - \frac{Gm_A m_C}{L} - \frac{Gm_B m_C}{d}.$$

The work done is

$$\begin{aligned}
W = U_f - U_i &= Gm_B\left[m_A\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_C\left(\frac{1}{L-d} - \frac{1}{d}\right)\right] \\
&= Gm_B\left[m_A\frac{L-2d}{d(L-d)} + m_C\frac{2d-L}{d(L-d)}\right] = Gm_B(m_A - m_C)\frac{L-2d}{d(L-d)} \\
&= (6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.010\ \text{kg})(0.080\ \text{kg} - 0.020\ \text{kg})\frac{0.12\ \text{m} - 2(0.040\ \text{m})}{(0.040\ \text{m})(0.12 - 0.040\ \text{m})} \\
&= +5.0\times10^{-13}\ \text{J}.
\end{aligned}$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0\times10^{-13}$ J.

36. (a) From Eq. 13-28, we see that $v_0 = \sqrt{GM / 2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2}mv_o^2 - GMm/R_E = -GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

37. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})} = 1.7\times10^3\ \text{m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)}.$$

The solution for h is

$$h = \frac{2a_gR^2}{2a_gR - v^2} - R = \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - (1000\ \text{m/s})^2} - (500\times10^3\ \text{m})$$
$$= 2.5\times10^5\ \text{m}.$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{(500\times10^3\ \text{m})+(1000\times10^3\ \text{m})}}$$
$$= 1.4\times10^3\ \text{m/s}.$$

38. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}.$$

where $M = 5.0 \times 10^{23}$ kg, $r_1 = R = 3.0 \times 10^6$ m and $m = 10$ kg.

(a) If $K_1 = 5.0 \times 10^7$ J and $r_2 = 4.0 \times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7 \text{ J}.$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7 \text{ J}.$$

39. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\ \text{kg})}{10^{10}\ \text{m}}} = 8.2\times10^4\ \text{m/s}.$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$v = \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} = \sqrt{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\ \text{kg})\left(\frac{1}{2\times10^5\ \text{m}} - \frac{1}{10^{10}\ \text{m}}\right)}$$
$$= 1.8\times10^7\ \text{m/s}.$$

40. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_A M_B}{r_i} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.80\ \text{m}}$$
$$= -1.67\times10^{-8}\ \text{J} \approx -1.7\times10^{-8}\ \text{J}.$$

(b) We use conservation of energy (with $K_i = 0$):

$$U_i = K + U \quad \Rightarrow \quad -1.7\times10^{-8} = K - \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.60\ \text{m}}$$

which yields $K = 5.6 \times 10^{-9}$ J. Note that the value of r is the difference between 0.80 m and 0.20 m.

41. Let m = 0.020 kg and d = 0.600 m (the original edge-length, in terms of which the final edge-length is $d/3$). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}\,d} \; .$$

Since U is inversely proportional to r then reducing the size by 1/3 means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f = 3U_i \;\Rightarrow\; \Delta U = 2U_i = 2(4+\sqrt{2}\,)\left(-\frac{Gm^2}{d}\right) = -4.82 \times 10^{-13}\ \text{J} \; .$$

42. (a) Applying Eq. 13-21 and the Pythagorean theorem leads to

$$U = -\left(\frac{GM^2}{2D} + \frac{2GmM}{\sqrt{y^2 + D^2}}\right)$$

where M is the mass of particle B (also that of particle C) and m is the mass of particle A. The value given in the problem statement (for infinitely large y, for which the second term above vanishes) determines M, since D is given. Thus $M = 0.50$ kg.

(b) We estimate (from the graph) the $y = 0$ value to be $U_o = -3.5 \times 10^{-10}$ J. Using this, our expression above determines m. We obtain $m = 1.5$ kg.

43. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4 \text{ s}\right)^2} = 6.5 \times 10^{23} \text{ kg}.$$

44. From Eq. 13-37, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius r) around a planet of mass M. In this case, $M = 5.98 \times 10^{24}$ kg and

$$r = (700 + 6370)\text{m} = 7070 \text{ km} = 7.07 \times 10^6 \text{ m}.$$

The speed is found to be $v = 7.51 \times 10^3$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^4$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^4$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^4$ km/h.

45. Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N M$ and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(7.88 \times 10^{15}\ \text{s})^2 (2.0 \times 10^{30}\ \text{kg})} = 5.1 \times 10^{10}.$$

46. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \Rightarrow (1.52)^3 = \left(\frac{T_M}{1\,\text{y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semi-major axis ratio. This yields T_M = 1.87 y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semi-major axis ratio is a_M/a_E = 1.523 which does lead to T_M = 1.88 y using Kepler's law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semi-major axes, but this requires a more lengthy discussion of what is meant by a "mean distance" than is appropriate here.

47. (a) The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = (6.37 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m}) = 6.73 \times 10^6 \text{ m}$$
$$R_p = (6.37 \times 10^6 \text{ m} + 180 \times 10^3 \text{ m}) = 6.55 \times 10^6 \text{ m}.$$

Here 6.37×10^6 m is the radius of Earth. From Fig. 13-13, we see that the semi-major axis is

$$a = \frac{R_a + R_p}{2} = \frac{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}}{2} = 6.64 \times 10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73 \times 10^6 \text{ m} - 6.55 \times 10^6 \text{ m}}{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}} = 0.0136.$$

48. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1 \text{ lunar month}}\right)^2$$

which yields $T_s = 0.35$ lunar month for the period of the satellite.

49. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m the orbit radius is $r = (6.37 \times 10^6 \text{ m} + 160 \times 10^3 \text{ m}) = 6.53 \times 10^6$ m. The solution for v is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.53 \times 10^6 \text{ m}}} = 7.82 \times 10^3 \text{ m/s}.$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.53 \times 10^6 \text{ m})}{7.82 \times 10^3 \text{ m/s}} = 5.25 \times 10^3 \text{ s}.$$

This is equivalent to 87.5 min.

50. (a) The distance from the center of an ellipse to a focus is *ae* where *a* is the semimajor axis and *e* is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50 \times 10^{11}\ \text{m}\right)\left(0.0167\right) = 5.01 \times 10^{9}\ \text{m}.$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01 \times 10^{9}\ \text{m}}{6.96 \times 10^{8}\ \text{m}} = 7.20.$$

51. (a) The period of the comet is 1420 years (and one month), which we convert to T = 4.48×10^{10} s. Since the mass of the Sun is 1.99×10^{30} kg, then Kepler's law of periods gives

$$(4.48 \times 10^{10} \text{ s})^2 = \left(\frac{4\pi^2}{(6.67\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{30} \text{ kg})} \right) a^3 \Rightarrow a = 1.89 \times 10^{13} \text{ m}.$$

(b) Since the distance from the focus (of an ellipse) to its center is ea and the distance from center to the aphelion is a, then the comet is at a distance of

$$ea + a = (0.11+1)\,(1.89\times10^{13} \text{ m}) = 2.1\times10^{13} \text{ m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{2.1 \times 10^{13}}{5.9 \times 10^{12}} \right) R_P = 3.6 R_P.$$

52. To “hover” above Earth ($M_E = 5.98 \times 10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler’s law of periods,

$$(86400)^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3 \Rightarrow r = 4.225 \times 10^7 \text{ m}.$$

Its altitude is therefore $r - R_E$ (where $R_E = 6.37 \times 10^6$ m) which yields 3.58×10^7 m.

53. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log(4\pi^2/GM) + 3\log(a) \Rightarrow \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log(4\pi^2/GM)$$

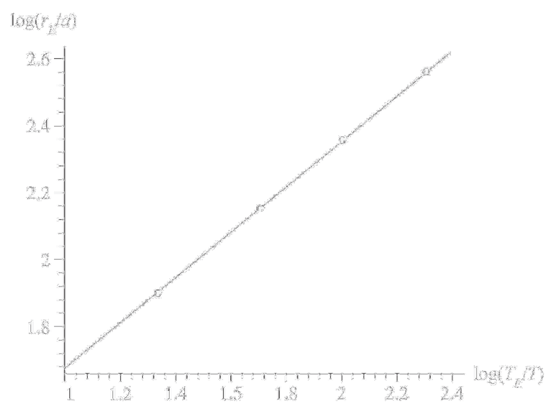
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter-moon system, where M is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass M_o), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where T_E = 365.25 days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive) which is the way we plot the data ($\log(r_E/a)$ on the vertical axis and $\log(T_E/T)$ on the horizontal axis).



(b) When we perform a least-squares fit to the data, we obtain

$$\log(r_E/a) = 0.666\log(T_E/T) + 1.01,$$

which confirms the expectation of slope = 2/3 based on the above equation.

(c) And the 1.01 intercept corresponds to the term $1/3\log(M_o/M)$ which implies

$$\frac{M_o}{M} = 10^{3.03} \Rightarrow M = \frac{M_o}{1.07 \times 10^3}.$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value 1.90×10^{27} kg found in Appendix C.

54. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius $r = 100000$ m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \Rightarrow M = 6.3 \times 10^{16} \text{ kg}.$$

(b) Dividing the mass M by the given volume yields an average density equal to

$$\rho = 6.3 \times 10^{16}/1.41 \times 10^{13} = 4.4 \times 10^3 \text{ kg/m}^3,$$

which is about 20% less dense than Earth.

55. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, 1.99×10^{30} kg). With $r = 2r_1$ in this system (so r_1 is one-half the Earth-to-Sun distance r), and $v = \pi r/T$ for the speed, we have

$$\frac{Gm_1m_2}{r^2} = m_1\frac{(\pi r/T)^2}{r/2} \Rightarrow T = \sqrt{\frac{2\pi^2 r^3}{GM}}.$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\,\text{y}}\right)(1\,\text{y}) = \left(\frac{2.2\times10^7\ \text{s}}{3.156\times10^7\ \text{s}}\right)(1\ \text{y}) = 0.71\ \text{y}.$$

56. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as *O*), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where r is the distance between the centers of the stars. To find the relation between r and r_1, we locate the center of mass relative to m_1. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2} = \frac{m_2r}{m_1+m_2} \quad \Rightarrow \quad r = \frac{m_1+m_2}{m_2}r_1 .$$

On the other hand, since the orbital speed of m_1 is $v = 2\pi r_1/T$, then $r_1 = vT/2\pi$ and the expression for r can be rewritten as

$$r = \frac{m_1+m_2}{m_2}\frac{vT}{2\pi} .$$

Substituting r and r_1 into the force equation, we obtain

$$F = \frac{4\pi^2Gm_1m_2^3}{(m_1+m_2)^2v^2T^2} = \frac{2\pi m_1v}{T}$$

or

$$\begin{aligned}\frac{m_2^3}{(m_1+m_2)^2} &= \frac{v^3T}{2\pi G} = \frac{(2.7\times10^5\ \text{m/s})^3(1.70\ \text{days})(86400\ \text{s/day})}{2\pi(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times10^{30}\ \text{kg} \\ &= 3.467M_s ,\end{aligned}$$

where $M_s = 1.99\times10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for α:

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2/M_s \approx 9$.

57. From Kepler's law of periods (where T = 2.4(3600) = 8640 s), we find the planet's mass M:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)(8.0\times10^6\ \text{m})^3 \Rightarrow M = 4.06\times10^{24}\ \text{kg}.$$

But we also know $a_g = GM/R^2 = 8.0\ \text{m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = 5.8\times10^6\ \text{m}.$$

58. (a) We make use of

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3 T}{2\pi G}$$

where $m_1 = 0.9M_{\text{Sun}}$ is the estimated mass of the star. With $v = 70$ m/s and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}} + m_2)^2} = 1.06 \times 10^{23} \text{ kg} .$$

Since $M_{\text{Sun}} \approx 2.0 \times 10^{30}$ kg, we find $m_2 \approx 7.0 \times 10^{27}$ kg. Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

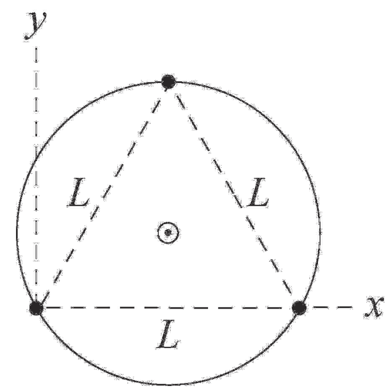
$$r_1 = \frac{vT}{2\pi} = \frac{(70\,\text{m/s})(1.3 \times 10^8\,\text{s})}{2\pi} = 1.4 \times 10^9 \text{ m}$$

for the star's orbital radius. If r is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet, and is given by

$$r_2 = r_1 \left(\frac{m_1 + m_2}{m_2} - 1 \right) = r_1 \frac{m_1}{m_2} = 3.7 \times 10^{11} \text{ m} .$$

Dividing this by 1.5×10^{11} m (Earth's orbital radius, r_E) gives $r_2 = 2.5r_E$.

59. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.



The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\left(\sqrt{3}/2\right)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L + L/2)/3 = L/2$ and the y coordinate is $y_c = \left(\sqrt{3}L/2\right)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is

$$R = \sqrt{x_c^2 + y_c^2} = \sqrt{\left(L^2/4\right) + \left(L^2/12\right)} = L/\sqrt{3}\,.$$

Once the substitution for R is made Newton's second law becomes $\left(2GM^2/L^2\right)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified somewhat by recognizing that $\cos 30° = \sqrt{3}/2$, and we divide the equation by M. Then, $GM/L^2 = v^2/L$ and $v = \sqrt{GM/L}$.

60. Although altitudes are given, it is the orbital radii which enter the equations. Thus, r_A = (6370 + 6370) km = 12740 km, and r_B = (19110 + 6370) km = 25480 km

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-GmM / r_B}{-GmM / r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{GmM / 2r_B}{GmM / 2r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of r has the smallest value of $|E|$ (since r is in the denominator). And since the values of E are negative, then the smallest value of $|E|$ corresponds to the largest energy E. Thus, satellite B has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the r values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass M of Earth is found in Appendix C.

61. (a) We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_e = 2(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300 \times 10^9 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(1.99 \times 10^{30} \text{kg})}} = 8.96 \times 10^7 \text{ s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T = 2.8$ y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e = (m/m_e)(r_e/r)$. We substitute $m = 2.0 \times 10^{-4} m_e$ and $r = 2r_e$ to obtain $K/K_e = 1.0 \times 10^{-4}$.

62. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}.$$

Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination mv^2 by multiplying both sides by $r = 2.0 \times 10^7$ m. Thus, $mv^2 = (2.0 \times 10^7$ m) (80 N) $= 1.6 \times 10^9$ J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6 \times 10^9 \text{ J}\right) = 8.0 \times 10^8 \text{ J}.$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2.$$

Thus, $F' = (80 \text{ N}) (2/3)^2 = 36$ N.

63. The energy required to raise a satellite of mass *m* to an altitude *h* (at rest) is given by

$$E_1 = \Delta U = GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = \frac{GM_E m}{2(R_E + h)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right].$$

(a) Solving the above equation, the height h_0 at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \quad \Rightarrow h_0 = \frac{R_E}{2} = 3.19\times10^6 \text{ m}.$$

(b) For greater height $h > h_0$, $\Delta E > 0$ implying $E_1 > E_2$. Thus, the energy of lifting is greater.

64. (a) From Eq. 13-40, we see that the energy of each satellite is $-GM_Em/2r$. The total energy of the two satellites is twice that result:

$$E = E_A + E_B = -\frac{GM_E m}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})(125\ \text{kg})}{7.87\times10^{6}\ \text{m}}$$
$$= -6.33\times10^{9}\,\text{J}.$$

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing m with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is

$$E = -\frac{GM_E(2m)}{2r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})2(125\ \text{kg})}{2(7.87\times10^{6}\ \text{m})} = -6.33\times10^{9}\,\text{J}.$$

(c) An object with zero speed at that distance from Earth will simply fall towards the Earth, its trajectory being toward the center of the planet.

65. (a) From Kepler's law of periods, we see that T is proportional to $r^{3/2}$.

(b) Eq. 13-38 shows that K is inversely proportional to r.

(c) and (d) From the previous part, knowing that K is proportional to v^2, we find that v is proportional to $1/\sqrt{r}$. Thus, by Eq. 13-31, the angular momentum (which depends on the product rv) is proportional to $r/\sqrt{r} = \sqrt{r}$.

66. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4\left(\frac{GM_E m}{2r}\right) = \frac{2(6.67\times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)\left(5.98\times 10^{24}\ \text{kg}\right)\left(0.0040\ \text{kg}\right)}{(6370+500)\times 10^3\ \text{m}} = 4.6\times 10^5\ \text{J}.$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6\times 10^5\ \text{J}}{\frac{1}{2}\left(0.0040\,\text{kg}\right)\left(950\,\text{m/s}\right)^2} = 2.6\times 10^2.$$

67. (a) The force acting on the satellite has magnitude GMm/r^2, where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r, where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite: $r = (6.37 \times 10^6 + 640 \times 10^3)$ m $= 7.01 \times 10^6$ m. Thus,

$$v = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{7.01\times10^6\ \text{m}}} = 7.54\times10^3\ \text{m/s}.$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01\times10^6\ \text{m})/(7.54\times10^3\ \text{m/s}) = 5.84\times10^3\ \text{s} \approx 97\ \text{min}.$$

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4 \times 10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.
The initial energy is

$$E_0 = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)(220\ \text{kg})}{2(7.01\times10^6\ \text{m})} = -6.26\times10^9\ \text{J},$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26\times10^9\ \text{J} - (1500\ \text{orbit})\left(1.4\times10^5\ \text{J/orbit}\right) = -6.47\times10^9\ \text{J},$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)(220\ \text{kg})}{2(-6.47\times10^9\ \text{J})} = 6.78\times10^6\ \text{m}.$$

The altitude is $h = r - R = (6.78 \times 10^6\ \text{m} - 6.37 \times 10^6\ \text{m}) = 4.1 \times 10^5$ m. Here R is the radius of Earth. This torque is internal to the satellite-Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{6.78\times10^6\ \text{m}}} = 7.67\times10^3\ \text{m/s} \approx 7.7\ \text{km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78\times10^6\ \text{m})}{7.67\times10^3\ \text{m/s}} = 5.6\times10^3\ \text{s} \approx 93\ \text{min}.$$

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7$ m, and $\Delta E = -1.4 \times 10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite-Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370\text{ km} + 400\text{ km} = 6770\text{ km} = 6.77\times 10^6\text{ m}$.

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2\,(6.77\times 10^6\text{ m})^3}{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})\,(5.98\times 10^{24}\text{kg})}} = 5.54\times 10^3\text{ s} \approx 92.3\text{ min}.$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77\times 10^6\text{ m})}{5.54\times 10^3\text{ s}} = 7.68\times 10^3\text{ m/s}^2.$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000\text{ kg})(0.99)^2(7.68\times 10^3\text{ m/s})^2 = 5.78\times 10^{10}\text{ J}.$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})\,(5.98\times 10^{24}\text{kg})(2000\text{ kg})}{6.77\times 10^6\text{ m}} = -1.18\times 10^{11}\text{ J}.$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78\times 10^{10}\text{ J} + (-1.18\times 10^{11}\text{ J}) = -6.02\times 10^{10}\text{ J}.$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where a is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})\,(5.98\times 10^{24}\text{kg})(2000\text{ kg})}{2(-6.02\times 10^{10}\text{ J})} = 6.63\times 10^6\text{ m}.$$

(g) To find the period, we use Eq. 13-34 but replace r with a. The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2\,(6.63\times 10^6\text{ m})^3}{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})\,(5.98\times 10^{24}\text{kg})}} = 5.37\times 10^3\text{ s} \approx 89.5\text{ min}.$$

(h) The orbital period T for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540\text{ s} - 5370\text{ s} = 170\text{ s}.$$

Thus, Picard will arrive back at point P ahead of Igor by 170 s – 90 s = 80 s.

69. We define the "effective gravity" in his environment as $g_{eff} = 220/60 = 3.67$ m/s^2. Thus, using equations from Chapter 2 (and selecting downwards as the positive direction), we find the "fall-time" to be

$$\Delta y = v_0 t + \frac{1}{2} g_{eff} t^2 \Rightarrow t = \sqrt{\frac{2(2.1\text{ m})}{3.67\text{ m/s}^2}} = 1.1\text{ s.}$$

70. We estimate the planet to have radius $r = 10$ m. To estimate the mass m of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$):

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \quad \Rightarrow \quad m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6 \times 10^{24}$ kg and $R_E \approx 6.4 \times 10^6$ m) $m = 2.3 \times 10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(2.3\times10^7\ \text{kg}\right)}{(10\ \text{m})^2} = 1.5\times10^{-5}\ \text{m/s}^2 \approx 2\times10^{-5}\ \text{m/s}^2 .$$

(b) Eq. 13-28 gives the escape speed:

$$v = \sqrt{\frac{2Gm}{r}} \approx 0.02\ \text{m/s} .$$

71. Using energy conservation (and Eq. 13-21) we have

$$K_1 - \frac{GMm}{r_1} = K_2 - \frac{GMm}{r_2} \ .$$

Plugging in two pairs of values (for (K_1, r_1) and (K_2, r_2)) from the graph and using the value of G and M (for earth) given in the book, we find

(a) $m \approx 1.0 \times 10^3$ kg.

(b) Similarly, $v = (2K/m)^{1/2} \approx 1.5 \times 10^3$ m/s (at $r = 1.945 \times 10^7$ m).

72. (a) The gravitational acceleration a_g is defined in Eq. 13-11. The problem is concerned with the difference between a_g evaluated at $r = 50R_h$ and a_g evaluated at $r = 50R_h + h$ (where h is the estimate of your height). Assuming h is much smaller than $50R_h$ then we can approximate h as the dr which is present when we consider the differential of Eq. 13-11:

$$|da_g| = \frac{2GM}{r^3}\,dr \approx \frac{2GM}{50^3 R_h{}^3}\,h = \frac{2GM}{50^3(2GM/c^2)^3}\,h\ .$$

If we approximate $|da_g| = 10\ \text{m/s}^2$ and $h \approx 1.5$ m, we can solve this for M. Giving our results in terms of the Sun's mass means dividing our result for M by 2×10^{30} kg. Thus, admitting some tolerance into our estimate of h we find the "critical" black hole mass should in the range of 105 to 125 solar masses.

(b) Interestingly, this turns out to be lower limit (which will surprise many students) since the above expression shows $|da_g|$ is inversely proportional to M. It should perhaps be emphasized that a distance of $50R_h$ from a small black hole is much smaller than a distance of $50R_h$ from a large black hole.

73. The magnitudes of the individual forces (acting on m_C, exerted by m_A and m_B respectively) are

$$F_{AC} = \frac{Gm_A m_C}{r_{AC}^2} = 2.7 \times 10^{-8}\ \text{N} \quad \text{and} \quad F_{BC} = \frac{Gm_B m_C}{r_{BC}^2} = 3.6 \times 10^{-8}\ \text{N}$$

where r_{AC} = 0.20 m and r_{BC} = 0.15 m. With r_{AB} = 0.25 m, the angle $\vec{F}_A$ makes with the x axis can be obtained as

$$\theta_A = \pi + \cos^{-1}\left(\frac{r_{AC}^2 + r_{AB}^2 - r_{BC}^2}{2r_{AC}r_{AB}}\right) = \pi + \cos^{-1}(0.80) = 217°.$$

Similarly, the angle $\vec{F}_B$ makes with the x axis can be obtained as

$$\theta_B = -\cos^{-1}\left(\frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2r_{AB}r_{BC}}\right) = -\cos^{-1}(0.60) = -53°.$$

The net force acting on m_C then becomes

$$\begin{aligned}\vec{F}_C &= F_{AC}(\cos\theta_A\,\hat{\text{i}} + \sin\theta_A\,\hat{\text{j}}) + F_{BC}(\cos\theta_B\,\hat{\text{i}} + \sin\theta_B\,\hat{\text{j}}) \\ &= (F_{AC}\cos\theta_A + F_{BC}\cos\theta_B)\hat{\text{i}} + (F_{AC}\sin\theta_A + F_{BC}\sin\theta_B)\hat{\text{j}} \\ &= (-4.4\times10^{-8}\ \text{N})\hat{\text{j}}\end{aligned}$$

74. The key point here is that angular momentum is conserved:

$$I_p \omega_p = I_a \omega_a$$

which leads to $\omega_p = (r_a / r_p)^2 \omega_a$, but $r_p = 2a - r_a$ where a is determined by Eq. 13-34 (particularly, see the paragraph after that equation in the textbook). Therefore,

$$\omega_p = \frac{r_a^{\,2}\,\omega_a}{(2(GMT^2/4\pi^2)^{1/3} - r_a)^2} = 9.24 \times 10^{-5} \text{ rad/s} .$$

75. (a) Using Kepler's law of periods, we obtain

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right)r^3} = 2.15 \times 10^4 \text{ s} .$$

(b) The speed is constant (before she fires the thrusters), so $v_o = 2\pi r/T = 1.23 \times 10^4$ m/s.

(c) A two percent reduction in the previous value gives $v = 0.98v_o = 1.20 \times 10^4$ m/s.

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = 2.17 \times 10^{11}$ J.

(e) The potential energy is $U = -GmM/r = -4.53 \times 10^{11}$ J.

(f) Adding these two results gives $E = K + U = -2.35 \times 10^{11}$ J.

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = \frac{-GMm}{2E} = 4.04 \times 10^7 \text{ m} .$$

(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period is

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right)a^3} = 2.03 \times 10^4 \text{ s} .$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

(i) Elliptical orbit has a smaller period.

76. (a) With $M = 2.0\times10^{30}$ kg and r = 10000 m, we find

$$a_g = \frac{GM}{r^2} = 1.3\times10^{12}\ \text{m/s}^2\ .$$

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

where $K_o = 0$, $K = \frac{1}{2}mv^2$ and U given by Eq. 13-21. Thus, with r_o = 10001 m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6\times10^{6}\ \text{m/s}\ .$$

77. We note that r_A (the distance from the origin to sphere A, which is the same as the separation between A and B) is 0.5, $r_C = 0.8$, and $r_D = 0.4$ (with SI units understood). The force $\vec{F}_k$ that the k^{th} sphere exerts on m_B has magnitude $Gm_k m_B / r_k^2$ and is directed from the origin towards m_k so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_k m_B}{r_k^2}\left(\frac{x_k}{r_k}\hat{\mathrm{i}} + \frac{y_k}{r_k}\hat{\mathrm{j}}\right) = \frac{Gm_k m_B}{r_k^3}\left(x_k\hat{\mathrm{i}} + y_k\hat{\mathrm{j}}\right).$$

Consequently, the vector addition (where k equals A,B and D) to obtain the net force on m_B becomes

$$\vec{F}_{\text{net}} = \sum_k \vec{F}_k = Gm_B\left(\left(\sum_k \frac{m_k x_k}{r_k^3}\right)\hat{\mathrm{i}} + \left(\sum_k \frac{m_k y_k}{r_k^3}\right)\hat{\mathrm{j}}\right) = (3.7\times10^{-5}\,\mathrm{N})\hat{\mathrm{j}}.$$

78. (a) We note that r_C (the distance from the origin to sphere C, which is the same as the separation between C and B) is 0.8, $r_D = 0.4$, and the separation between spheres C and D is $r_{CD} = 1.2$ (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2}-\frac{GM_BM_D}{r_D^2}-\frac{GM_CM_D}{r_{CD}^2}=-1.3\times10^{-4}\text{ J}$$

using the mass-values given in the previous problem.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere A (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere A back in, I do negative work, since I am causing the system energy to become more negative.

79. We use $F = Gm_s m_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15$ m + 3 m = 18 m. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(20\mathrm{kg}\right)\left(7.0\mathrm{kg}\right)}{\left(18\mathrm{m}\right)^2} = 2.9\times10^{-11}\,\mathrm{N}.$$

80. (a) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3}.$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M which is at radius less than r is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \le R$ then we can write this result as

$$\frac{G\left(\dfrac{3M_{\text{total}}}{4\pi R^3}\right)\left(\dfrac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \quad\Rightarrow\quad r = \frac{R}{3}.$$

(b) Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \quad\Rightarrow\quad r = \sqrt{3}R.$$

81. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$ m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that m cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is $r_2 - R = 2.5 \times 10^7$ m.

82. (a) Because it is moving in a circular orbit, F/m must equal the centripetal acceleration:

$$\frac{80\ \text{N}}{50\ \text{kg}} = \frac{v^2}{r}.$$

But $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\,\text{m/s}^2 = \frac{4\pi^2}{T^2}r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6\ \text{m/s}^2)r$ which leads to $v^2 = 3.0 \times 10^7\ \text{m}^2/\text{s}^2$. Thus, $K = \frac{1}{2}mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in § 13-4, F/m also tells us the gravitational acceleration:

$$a_g = 1.6\ \text{m/s}^2 = \frac{GM}{r^2}.$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

83. (a) We write the centripetal acceleration (which is the same for each, since they have identical mass) as $r\omega^2$ where ω is the unknown angular speed. Thus,

$$\frac{G(M)(M)}{(2r)^2} = \frac{GM^2}{4r^2} = Mr\omega^2$$

which gives $\omega = \frac{1}{2}\sqrt{MG/r^3} = 2.2\times10^{-7}$ rad/s.

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4 \text{ m/s}.$$

84. See Appendix C. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of g, we divide by 9.8 m/s^2.

(a) The acceleration associated with Earth's spin (T = 24 h = 86400 s) is

$$a = g\frac{4\pi^2 (6.37\times 10^6 \text{ m})}{(86400\,\text{s})^2 (9.8\,\text{m/s}^2)} = 3.4\times 10^{-3} g\ .$$

(b) The acceleration associated with Earth's motion around the Sun (T = 1 y = 3.156 × 10^7 s) is

$$a = g\frac{4\pi^2 (1.5\times 10^{11} \text{ m})}{(3.156\times 10^7 \text{ s})^2 (9.8\,\text{m/s}^2)} = 6.1\times 10^{-4} g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center (T = 2.5 × 10^8 y = 7.9 × 10^{15} s) is

$$a = g\frac{4\pi^2 (2.2\times 10^{20} \text{ m})}{(7.9\times 10^{15} \text{ s})^2 (9.8\,\text{m/s}^2)} = 1.4\times 10^{-11} g\ .$$

85. We use m_1 for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), m_2 for the 40 kg of the sphere at $(x_2, y_2) = (-1.0, -1.0)$, and m_3 for the 60 kg of the sphere at $(x_3, y_3) = (0, -0.5)$. The mass of the 20 kg object at the origin is simply denoted m. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the n^{th} sphere exerts on m has magnitude $Gm_n m / r_n^2$ and is directed from the origin towards m_n, so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_n m}{r_n^2}\left(\frac{x_n}{r_n}\hat{\mathrm{i}} + \frac{y_n}{r_n}\hat{\mathrm{j}}\right) = \frac{Gm_n m}{r_n^3}\left(x_n\hat{\mathrm{i}} + y_n\hat{\mathrm{j}}\right).$$

Consequently, the vector addition to obtain the net force on m becomes

$$\vec{F}_{\text{net}} = \sum_{n=1}^{3}\vec{F}_n = Gm\left(\left(\sum_{n=1}^{3}\frac{m_n x_n}{r_n^3}\right)\hat{\mathrm{i}} + \left(\sum_{n=1}^{3}\frac{m_n y_n}{r_n^3}\right)\hat{\mathrm{j}}\right) = -9.3\times10^{-9}\,\hat{\mathrm{i}} - 3.2\times10^{-7}\,\hat{\mathrm{j}}$$

in SI units. Therefore, we find the net force magnitude is $\left|\vec{F}_{\text{net}}\right| = 3.2\times10^{-7}\,\mathrm{N}$.

86. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$ (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0 (-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8\text{ m/s}^2$ on the surface of Earth is given by

$$a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2,$$

where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(6.37\times10^6\text{ m})} = 7.9\times10^3\text{ m/s}.$$

87. (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) Since the change is small, we use differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \left(\frac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\left(5.98\times10^{24}\right)}{\left(1.5\times10^{11}\right)^2}\right)\left(5\times10^{9}\right)$$

which yields $\Delta U \approx 1.8 \times 10^{32}$ J. A more direct subtraction of the values of the potential energies leads to the same result.

(c) From the previous two parts, we see that the variation in the kinetic energy ΔK must also equal 1.8×10^{32} J.

(d) With $\Delta K \approx dK = mv\, dv$, where $v \approx 2\pi R/T$, we have

$$1.8\times10^{32} \approx \left(5.98\times10^{24}\right)\left(\frac{2\pi\left(1.5\times10^{11}\right)}{3.156\times10^{7}}\right)\Delta v$$

which yields a difference of $\Delta v \approx 0.99$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

88. Let the distance from Earth to the spaceship be r. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{(R_{em} - r)^2} = F_E = \frac{GM_e m}{r^2},$$

where m is the mass of the spaceship. Solving for r, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m / M_e} + 1} = \frac{3.82 \times 10^8 \text{ m}}{\sqrt{(7.36 \times 10^{22} \text{ kg})/(5.98 \times 10^{24} \text{ kg})} + 1} = 3.44 \times 10^8 \text{ m}.$$

89. We integrate Eq. 13-1 with respect to r from $3R_E$ to $4R_E$ and obtain the work equal to $-GM_Em(1/(4R_E) - 1/(3R_E)) = GM_Em/12R_E$.

90. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass m at its surface is given by $F = GmM / R^2$, where M is the mass of the planet and R is its radius. According to Newton's second law this must equal mv^2 / R, where v is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R}.$$

Replacing M with $(4\pi/3)\,\rho R^3$ (where ρ is the density of the planet) and v with $2\pi R/T$ (where T is the period of revolution), we find

$$\frac{4\pi}{3} G\rho R = \frac{4\pi^2 R}{T^2}.$$

We solve for T and obtain

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

(b) The density is 3.0×10^3 kg/m^3. We evaluate the equation for T:

$$T = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg}\right)\left(3.0\times10^3\,\mathrm{kg/m^3}\right)}} = 6.86\times10^3\,\mathrm{s} = 1.9\,\mathrm{h}.$$

91. (a) It is possible to use $v^2 = v_0^2 + 2a\,\Delta\,y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \;\Rightarrow\; v = \sqrt{\frac{2GM(r_0 - r)}{r_0\, r}}$$

which yields $v = 1.4 \times 10^6$ m/s.

(b) We estimate the height of the apple to be h = 7 cm = 0.07 m. We may find the answer by evaluating Eq. 13-11 at the surface (radius r in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation — setting dr equal to h. We illustrate the latter procedure:

$$|\,da_g\,| = \left|-2\frac{GM}{r^3}dr\right| \approx 2\frac{GM}{r^3}h = 3 \times 10^6 \text{ m/s}^2 .$$

92. (a) The gravitational force exerted on the baby (denoted with subscript b) by the obstetrician (denoted with subscript o) is given by

$$F_{bo} = \sqrt{\frac{Gm_o m_b}{r_{bo}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)(70\,\mathrm{kg})(3\,\mathrm{kg})}{(1\,\mathrm{m})^2}} = 1\times10^{-8}\,\mathrm{N}.$$

(b) The maximum (minimum) forces exerted by Jupiter on the baby occur when it is separated from the Earth by the shortest (longest) distance $r_{\min}$ ($r_{\max}$), respectively. Thus

$$F_{bJ}^{\max} = \sqrt{\frac{Gm_J m_b}{r_{\min}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(2\times10^{27}\,\mathrm{kg}\right)(3\,\mathrm{kg})}{\left(6\times10^{11}\,\mathrm{m}\right)^2}} = 1\times10^{-6}\,\mathrm{N}.$$

(c) And we obtain

$$F_{bJ}^{\min} = \sqrt{\frac{Gm_J m_b}{r_{\max}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(2\times10^{27}\,\mathrm{kg}\right)(3\,\mathrm{kg})}{\left(9\times10^{11}\,\mathrm{m}\right)^2}} = 5\times10^{-7}\,\mathrm{N}.$$

(d) No. The gravitational force exerted by Jupiter on the baby is greater than that by the obstetrician by a factor of up to 1×10^{-6} N/1×10^{-8} N = 100.

93. The magnitude of the net gravitational force on one of the smaller stars (of mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}\left(M + \frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M + \frac{m}{4}\right) = m\frac{v^2}{r} \quad \text{where } v = \frac{2\mathrm{p}r}{T}.$$

Plugging in for speed v, we arrive at an equation for period T:

$$T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/4)}}.$$

94. (a) We note that *height* = $R - R_{\text{Earth}}$ where $R_{\text{Earth}} = 6.37 \times 10^6$ m. With $M = 5.98 \times 10^{24}$ kg, $R_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2}m\,(3.70 \times 10^3)^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2}m\,(3.70 \times 10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km ≈ 1.03×10^3 km above the Earth's surface.

95. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$ and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \quad\Rightarrow\quad v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s}.$$

96. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

(a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

$$K_i + U_i = K + U \Rightarrow 0 + 0 = K - 2\frac{GmM}{r}$$

which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

(b) Since the y-component of each force will cancel, the net force points in the $-x$ direction, with a magnitude $2F_x = 2\,(GmM/r^2)\cos\theta$, where $\theta = \tan^{-1}(4/3) = 53°$. Thus, the result is $\vec{F}_{\text{net}} = (-6.4\times10^{-8}\ \text{N})\hat{\text{i}}$.

97. The kinetic energy in its circular orbit is $\frac{1}{2}mv^2$ where $v = 2\pi r/T$. Using the values stated in the problem and using Eq. 13-41, we directly find $E = -1.87 \times 10^9$ J.

98. (a) From Ch. 2, we have $v^2 = v_0^2 + 2a\Delta x$, where a may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v = 11000$ m/s and $\Delta x = 220$ m, we find $a = 2.75 \times 10^5$ m/s^2. Therefore,

$$a = \left(\frac{2.75 \times 10^5 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right) g = 2.8 \times 10^4 g .$$

(b) The acceleration is certainly deadly enough to kill the passengers.

(c) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{(7000 \text{ m/s})^2}{2(3500 \text{ m})} = 7000 \text{ m/s}^2 = 714g .$$

(d) Energy conservation gives the craft's speed v (in the absence of friction and other dissipative effects) at altitude $h = 700$ km after being launched from $R = 6.37 \times 10^6$ m (the surface of Earth) with speed $v_0 = 7000$ m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07 \times 10^6$ m.

$$\frac{1}{2} m v_0^2 - \frac{GMm}{R} = \frac{1}{2} m v^2 - \frac{GMm}{r} .$$

With $M = 5.98 \times 10^{24}$ kg (the mass of Earth) we find $v = 6.05 \times 10^3$ m/s. But to orbit at that radius requires (by Eq. 13-37)

$$v' = \sqrt{GM/r} = 7.51 \times 10^3 \text{ m/s}.$$

The difference between these is $v' - v = 1.46 \times 10^3$ m/s $\approx 1.5 \times 10^3$ m/s, which presumably is accounted for by the action of the rocket engine.

99. (a) All points on the ring are the same distance ($r = \sqrt{x^2 + R^2}$) from the particle, so the gravitational potential energy is simply $U = -GMm/\sqrt{x^2 + R^2}$, from Eq. 13-21. The corresponding force (by symmetry) is expected to be along the x axis, so we take a (negative) derivative of U (with respect to x) to obtain it (see Eq. 8-20). The result for the magnitude of the force is $GMmx(x^2 + R^2)^{-3/2}$.

(b) Using our expression for U, then the magnitude of the loss in potential energy as the particle falls to the center is $GMm(1/R - 1/\sqrt{x^2 + R^2}\,)$. This must "turn into" kinetic energy ($\frac{1}{2}mv^2$), so we solve for the speed and obtain

$$v = [2GM(R^{-1} - (R^2 + x^2)^{-1/2})]^{1/2} .$$

100. Consider that we are examining the forces on the mass in the lower left-hand corner of the square. Note that the mass in the upper right-hand corner is $20\sqrt{2} = 28$ cm $= 0.28$ m away. Now, the *nearest* masses each pull with a force of $GmM / r^2 = 3.8 \times 10^{-9}$ N, one upward and the other rightward. The net force caused by these two forces is $(3.8 \times 10^{-9}, 3.8 \times 10^{-9}) \rightarrow (5.3 \times 10^{-9} \angle 45°)$, where the rectangular components are shown first -- and then the polar components (magnitude-angle notation). Now, the mass in the upper right-hand corner also pulls at 45°, so its force-magnitude (1.9×10^{-9}) will simply add to the magnitude just calculated. Thus, the final result is 7.2×10^{-9} N.

101. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \Rightarrow K_{\text{total}} = \frac{Gm^2}{R_i}.$$

(b) They have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2}K_{\text{total}} = \frac{Gm^2}{2R_i}.$$

(c) With $K = ½\, mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$.

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$. This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body A acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$ and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $½\, mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$.

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body A). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

102. Gravitational acceleration is defined in Eq. 13-11 (which we are treating as a positive quantity). The problem, then, is asking for the magnitude difference of $a_{g\text{ net}}$ when the contributions from the Moon and the Sun are in the same direction ($a_{g\text{ net}} = a_{g\text{Sun}} + a_{g\text{Moon}}$) as opposed to when they are in opposite directions ($a_{g\text{ net}} = a_{g\text{Sun}} - a_{g\text{Moon}}$). The difference (in absolute value) is clearly $2a_{g\text{Moon}}$. In specifically wanting the *percentage* change, the problem is requesting us to divide this difference by the average of the two $a_{g\text{ net}}$ values being considered (that average is easily seen to be equal to $a_{g\text{Sun}}$), and finally multiply by 100% in order to quote the result in the right format. Thus,

$$\frac{2a_{g\text{Moon}}}{a_{g\text{Sun}}} = 2\left(\frac{M_{\text{Moon}}}{M_{\text{Sun}}}\right)\left(\frac{r_{\text{Sun to Eearth}}}{r_{\text{Moon to Earth}}}\right)^2 = 2\left(\frac{7.36 \times 10^{22}}{1.99 \times 10^{30}}\right)\left(\frac{1.50 \times 10^{11}}{3.82 \times 10^{8}}\right)^2 = 0.011 = 1.1\%.$$

103. (a) Kepler's law of periods is

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \ .$$

With $M = 6.0 \times 10^{30}$ kg and $T = 300(86400) = 2.6 \times 10^7$ s, we obtain $r = 1.9 \times 10^{11}$ m.

(b) That its orbit is circular suggests that its speed is constant, so

$$v = \frac{2\pi r}{T} = 4.6 \times 10^4 \text{ m/s} \ .$$

104. Using Eq. 13-21, the potential energy of the dust particle is

$$U = -GmM_E/R - GmM_m/r = -Gm(M_E/R + M_m/r) .$$

Chapter 14

1. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus

$$\Delta p = (42\text{ N})/\pi(0.011\text{ m})^2 = 1.1 \times 10^5\text{ Pa}.$$

This is equivalent to 1.1 atm.

2. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that 1L = 1000 cm^3, we find the weight of the first liquid to be

$$W_1 = m_1 g = \rho_1 V_1 g = (2.6 \text{ g/cm}^3)(0.50 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 1.27\times 10^6 \text{ g}\cdot\text{cm/s}^2$$
$$= 12.7 \text{ N}.$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0 \text{ g/cm}^3)(0.25 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 2.5 \text{ N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80 \text{ g/cm}^3)(0.40 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 3.1 \text{ N}.$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18$ N.

3. The air inside pushes outward with a force given by p_iA, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by p_oA, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since 1 atm = 1.013×10^5 Pa,

$$F = (1.0\text{ atm} - 0.96\text{ atm})(1.013\times10^5\text{ Pa/atm})(3.4\text{ m})(2.1\text{ m}) = 2.9\times10^4\text{ N}.$$

4. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

(a) $$P = \left(28 \text{ lb/in.}^2\right)\left(\frac{1.01\times10^5 \text{ Pa}}{14.7 \text{ lb/in}^2}\right) = 190 \text{ kPa}.$$

(b) $$(120 \text{ mmHg})\left(\frac{1.01\times10^5 \text{ Pa}}{760 \text{ mmHg}}\right) = 15.9 \text{ kPa}, \quad (80 \text{ mmHg})\left(\frac{1.01\times10^5 \text{ Pa}}{760 \text{ mmHg}}\right) = 10.6 \text{ kPa}.$$

5. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08 \text{ g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00 \text{ g/cm}^3$$

where ρ_w is the density of the water. This implies

$$\rho_{\text{fish}} V = \rho_w (V + V_a) \text{ or } (V + V_a)/V = 1.08/1.00,$$

which gives $V_a/(V + V_a) = 0.074 = 7.4\%$.

6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\text{N/m}^2 = 1$ Pa, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0\times10^5 \text{ Pa} - \frac{480 \text{ N}}{77\times10^{-4} \text{ m}^2} = 3.8\times10^4 \text{ Pa}.$$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle θ. Its leftward component is $\Delta p \cos \theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R \sin \theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\, d\theta$ and its area is $dA = 2\pi R^2 \sin \theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \ \sin^2\theta \Big|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = 1.01×10^5 Pa to show that Δp = 0.90 atm = 9.09×10^4 Pa. The sphere radius is R = 0.30 m, so

$$F_h = \pi(0.30 \text{ m})^2(9.09 \times 10^4 \text{ Pa}) = 2.6 \times 10^4 \text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho g h = (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.83 \text{ m}) = 1.90 \times 10^4 \text{ Pa}.$$

9. Recalling that 1 atm = 1.01×10^5 Pa, Eq. 14-8 leads to

$$\rho g h = (1024 \text{ kg/m}^3)\ (9.80 \text{ m/s}^2)\ (10.9 \times 10^3 \text{ m}) \left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \right) \approx 1.08 \times 10^3 \text{ atm}.$$

10. Note that 0.05 atm equals 5065 Pa. Application of Eq. 14-7 with the notation in this problem leads to

$$d_{\max} = \frac{p}{\rho_{\text{liquid}} g} = \frac{0.05 \text{ atm}}{\rho_{\text{liquid}} g} = \frac{5065 \text{ Pa}}{\rho_{\text{liquid}} g}.$$

Thus the difference of this quantity between fresh water (998 kg/m^3) and Dead Sea water (1500 kg/m^3) is

$$\Delta d_{\max} = \frac{5065 \text{ Pa}}{g}\left(\frac{1}{\rho_{\text{fw}}} - \frac{1}{\rho_{\text{sw}}}\right) = \frac{5065 \text{ Pa}}{9.8 \text{ m/s}^2}\left(\frac{1}{998 \text{ kg/m}^3} - \frac{1}{1500 \text{ kg/m}^3}\right) = 0.17 \text{ m}.$$

11. The pressure p at the depth d of the hatch cover is $p_0 + \rho g d$, where ρ is the density of ocean water and p_0 is atmospheric pressure. The downward force of the water on the hatch cover is $(p_0 + \rho g d)A$, where A is the area of the cover. If the air in the submarine is at atmospheric pressure then it exerts an upward force of $p_0 A$. The minimum force that must be applied by the crew to open the cover has magnitude

$$F = (p_0 + \rho g d)A - p_0 A = \rho g d A = (1024\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(100\ \text{m})(1.2\ \text{m})(0.60\ \text{m})$$
$$= 7.2 \times 10^5\ \text{N}.$$

12. With $A = 0.000500 \text{ m}^2$ and $F = pA$ (with p given by Eq. 14-9), then we have $\rho g h A$ = 9.80 N. This gives $h \approx 2.0$ m, which means $d + h = 2.80$ m.

13. In this case, Bernoulli's equation reduces to Eq. 14-10. Thus,

$$p_g = \rho g(-h) = -(1800\,\text{kg/m}^3)(9.8\text{ m/s}^2)(1.5\text{ m}) = -2.6\times 10^4\text{ Pa} .$$

14. Using Eq. 14-7, we find the gauge pressure to be $p_{\text{gauge}} = \rho g h$, where ρ is the density of the fluid medium, and h is the vertical distance to the point where the pressure is equal to the atmospheric pressure.

The gauge pressure at a depth of 20 m in seawater is

$$p_1 = \rho_{\text{sw}} g d = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(20 \text{ m}) = 2.00 \times 10^5 \text{ Pa} .$$

On the other hand, the gauge pressure at an altitude of 7.6 km is

$$p_2 = \rho_{\text{air}} g h = (0.87 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(7600 \text{ m}) = 6.48 \times 10^4 \text{ Pa} .$$

Therefore, the change in pressure is

$$\Delta p = p_1 - p_2 = 2.00 \times 10^5 \text{ Pa} - 6.48 \times 10^4 \text{ Pa} \approx 1.4 \times 10^5 \text{ Pa} .$$

15. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the brain of the giraffe is

$$p_{\text{brain}} = p_{\text{heart}} - \rho g h = 250 \text{ torr} - (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\frac{1 \text{ torr}}{133.33 \text{ Pa}} = 94 \text{ torr}.$$

(b) The gauge pressure at the feet of the giraffe is

$$p_{\text{feet}} = p_{\text{heart}} + \rho g h = 250 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\frac{1 \text{ torr}}{133.33 \text{ Pa}} = 406 \text{ torr}$$
$$\approx 4.1\times10^2 \text{ torr}.$$

(c) The increase in the blood pressure at the brain as the giraffe lower is head to the level of its feet is

$$\Delta p = p_{\text{feet}} - p_{\text{brain}} = 406 \text{ torr} - 94 \text{ torr} = 312 \text{ torr} \approx 3.1\times10^2 \text{ torr}.$$

16. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

17. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the heart of the *Argentinosaurus* is

$$p_{\text{heart}} = p_{\text{brain}} + \rho gh = 80 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m} - 9.0 \text{ m})\frac{1 \text{ torr}}{133.33 \text{ Pa}}.$$
$$= 1.0\times10^3 \text{ torr}.$$

(b) The gauge pressure at the feet of the *Argentinosaurus* is

$$p_{\text{feet}} = p_{\text{brain}} + \rho gh' = 80 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m})\frac{1 \text{ torr}}{133.33 \text{ Pa}}$$
$$= 80 \text{ torr} + 1642 \text{ torr} = 1722 \text{ torr} \approx 1.7\times10^3 \text{ torr}.$$

18. At a depth h without the snorkel tube, the external pressure on the diver is

$$p = p_0 + \rho g h$$

where p_0 is the atmospheric pressure. Thus, with a snorkel tube of length *h*, the pressure difference between the internal air pressure and the water pressure against the body is

$$\Delta p = p = p_0 = \rho g h .$$

(a) If $h = 0.20$ m, then

$$\Delta p = \rho g h = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.20 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} = 0.019 \text{ atm} .$$

(b) Similarly, if $h = 4.0$ m, then

$$\Delta p = \rho g h = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.0 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \approx 0.39 \text{ atm} .$$

19. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is

$$W = \rho A(h_1 - h)g(h - h_2).$$

We substitute $h = (h_1 + h_2)/2$ to obtain

$$\begin{aligned} W &= \frac{1}{4}\rho g A\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\,\text{kg/m}^3)(9.80\text{ m/s}^2)(4.00\times10^{-4}\,\text{m}^2)(1.56\text{ m} - 0.854\text{ m})^2 \\ &= 0.635\text{ J} \end{aligned}.$$

20. To find the pressure at the brain of the pilot, we note that the inward acceleration can be treated from the pilot's reference frame as though it is an outward gravitational acceleration against which the heart must push the blood. Thus, with $a = 4g$, we have

$$\begin{aligned} p_{\text{brain}} &= p_{\text{heart}} - \rho a r = 120 \text{ torr} - (1.06\times10^3 \text{ kg/m}^3)(4\times9.8 \text{ m/s}^2)(0.30 \text{ m})\frac{1 \text{ torr}}{133 \text{ Pa}} \\ &= 120 \text{ torr} - 94 \text{ torr} = 26 \text{ torr}. \end{aligned}$$

21. Letting $p_a = p_b$, we find

$$\rho_c g(6.0 \text{ km} + 32 \text{ km} + D) + \rho_m(y - D) = \rho_c g(32 \text{ km}) + \rho_m y$$

and obtain

$$D = \frac{(6.0\,\text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0\,\text{km})(2.9\,\text{g/cm}^3)}{3.3\,\text{g/cm}^3 - 2.9\,\text{g/cm}^3} = 44\,\text{km}.$$

22. (a) The force on face A of area A_A due to the water pressure alone is

$$F_A = p_A A_A = \rho_w g h_A A_A = \rho_w g (2d) d^2 = 2\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(5.0\,\text{m}\right)^3$$
$$= 2.5\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\ \text{Pa})(5.0\ \text{m})^2 = 2.5\times10^6\ \text{N},$$

we have

$$F_A' = F_0 + F_A = 2.5\times10^6\ \text{N} + 2.5\times10^6\ \text{N} = 5.0\times10^6\ \text{N}.$$

(b) The force on face B due to water pressure alone is

$$F_B = p_{\text{avg}B} A_B = \rho_\omega g\left(\frac{5d}{2}\right) d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(5.0\,\text{m}\right)^3$$
$$= 3.1\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\ \text{Pa})(5.0\ \text{m})^2 = 2.5\times10^6\ \text{N},$$

we obtain

$$F_B' = F_0 + F_B = 2.5\times10^6\ \text{N} + 3.1\times10^6\ \text{N} = 5.6\times10^6\ \text{N}.$$

23. We can integrate the pressure (which varies linearly with depth according to Eq. 14-7) over the area of the wall to find out the net force on it, and the result turns out fairly intuitive (because of that linear dependence): the force is the "average" water pressure multiplied by the area of the wall (or at least the part of the wall that is exposed to the water), where "average" pressure is taken to mean $\frac{1}{2}$(pressure at surface + pressure at bottom). Assuming the pressure at the surface can be taken to be zero (in the gauge pressure sense explained in section 14-4), then this means the force on the wall is $\frac{1}{2}\rho gh$ multiplied by the appropriate area. In this problem the area is hw (where w is the 8.00 m width), so the force is $\frac{1}{2}\rho gh^2 w$, and the change in force (as h is changed) is

$$\tfrac{1}{2}\rho gw\,(h_f^{\,2} - h_i^{\,2}) = \tfrac{1}{2}(998\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(8.00\ \text{m})(4.00^2 - 2.00^2)\text{m}^2 = 4.69 \times 10^5\ \text{N}.$$

24. (a) At depth y the gauge pressure of the water is $p = \rho gy$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho gyW\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho gyW\,dy = \frac{1}{2}\rho gWD^2 = \frac{1}{2}\left(1.00\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(314\text{ m}\right)\left(35.0\text{ m}\right)^2$$
$$= 1.88\times10^9\text{ N}.$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is

$$d\tau = dF(D - y) = \rho gyW\,(D - y)dy$$

and the total torque of the water is

$$\tau = \int_0^D \rho gyW\left(D - y\right)dy = \rho gW\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho gWD^3$$
$$= \frac{1}{6}\left(1.00\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(314\text{ m}\right)\left(35.0\text{ m}\right)^3 = 2.20\times10^{10}\text{ N}\cdot\text{m}.$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho gWD^3}{\frac{1}{2}\rho gWD^2} = \frac{D}{3} = \frac{35.0\text{ m}}{3} = 11.7\text{ m}.$$

25. As shown in Eq. 14-9, the atmospheric pressure p_0 bearing down on the barometer's mercury pool is equal to the pressure ρgh at the base of the mercury column: $p_0 = \rho gh$. Substituting the values given in the problem statement, we find the atmospheric pressure to be

$$p_0 = \rho gh = (1.3608\times10^4 \text{ kg/m}^3)(9.7835 \text{ m/s}^2)(0.74035 \text{ m})\frac{1 \text{ torr}}{133.33 \text{ Pa}} = 739.26 \text{ torr}.$$

26. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(4.0\times10^{-2}\,\mathrm{m}\right)}{1.01\times10^{5}\ \mathrm{Pa/atm}} = -3.9\times10^{-3}\,\mathrm{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

27. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly 1.3 kg/m^3. Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 7.9 \times 10^3 \text{ m} = 7.9 \text{ km}.$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g \, dy .$$

Assuming $\rho = \rho_0 (1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8$ m/s^2 for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 g h.$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5 \text{ Pa})}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 16 \times 10^3 \text{ m} = 16 \text{ km}.$$

28. (a) According to Pascal's principle $F/A = f/a \rightarrow F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\ \text{cm})^2}{(53.0\ \text{cm})^2}(20.0 \times 10^3\ \text{N}) = 103\ \text{N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

29. Eq. 14-13 combined with Eq. 5-8 and Eq. 7-21 (in absolute value) gives

$$mg = kx\frac{A_1}{A_2} .$$

With $A_2 = 18A_1$ (and the other values given in the problem) we find $m = 8.50$ kg.

30. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{top} = L/2$ (corresponding to the top of the block) is

$$p_{top} = p_{atm} + \rho g h_{top} = 1.01\times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.300 \text{ m}) = 1.04\times 10^5 \text{ Pa}$$

where the unit Pa (Pascal) is equivalent to N/m^2. The force on the top surface (of area $A = L^2 = 0.36$ m^2) is

$$F_{top} = p_{top} A = 3.75\times 10^4 \text{ N}.$$

(b) The pressure at a depth of $h_{bot} = 3L/2$ (that of the bottom of the block) is

$$p_{bot} = p_{atm} + \rho g h_{bot} = 1.01\times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.900 \text{ m}) = 1.10\times 10^5 \text{ Pa}$$

where we recall that the unit Pa (Pascal) is equivalent to N/m^2. The force on the bottom surface is

$$F_{bot} = p_{bot} A = 3.96\times 10^4 \text{ N}.$$

(c) Taking the difference $F_{bot} - F_{top}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{bot} - F_{top} = \rho g (h_{bot} - h_{top}) A = \rho g L^3 = 2.18\times 10^3 \text{ N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{bot} - F_{top}) = (450 \text{ kg})(9.80 \text{ m/s}^2) - 2.18\times 10^3 \text{ N} = 2.23\times 10^3 \text{ N}.$$

(d) This has already been noted in the previous part: $F_b = 2.18\times 10^3$ N, and $T + F_b = mg$.

31. (a) The anchor is completely submerged in water of density ρ_w. Its effective weight is $W_{\text{eff}} = W - \rho_w gV$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200 \text{ N}}{\left(1000 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)} = 2.04 \times 10^{-2} \text{ m}^3 .$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 14-1). Its weight in air is

$$W = mg = \rho V g = \left(7870 \text{ kg/m}^3\right)(2.04 \times 10^{-2} \text{ m}^3)\left(9.80 \text{ m/s}^2\right) = 1.57 \times 10^3 \text{ N} .$$

32. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid which corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is $W = 35.6$ kN. In salt water of density $\rho' = 1100$ kg/m^3, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{3.56\times10^3\,\text{N}}{(1.10\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 3.30\ \text{m}^3\ .$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63$ m^3, where $\rho = 1000$ kg/m^3. The difference is $V - V' = 0.330$ m^3.

33. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\mathrm{N}) + N\rho_{\mathrm{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log: $V = \pi(0.15\ \mathrm{m})^2\ (1.80\ \mathrm{m}) = 0.13\ \mathrm{m}^3$. The buoyant force is $F_b = \rho_{\mathrm{water}}gV_{\mathrm{submerged}}$ where we require $V_{\mathrm{submerged}} \le NV$. The density of water is 1000 kg/m^3. To obtain the minimum value of N we set $V_{\mathrm{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\mathrm{N}) + N\rho_{\mathrm{wood}}gV = \rho_{\mathrm{water}}gNV \;\;\Rightarrow\;\; N = \frac{3(356\,\mathrm{N})}{gV\left(\rho_{\mathrm{water}} - \rho_{\mathrm{wood}}\right)}$$

which yields $N = 4.28 \rightarrow 5$ logs.

34. Taking "down" as the positive direction, then using Eq. 14-16 in Newton's second law, we have $5g - 3g = 5a$ (where "5" = 5.00 kg, and "3" = 3.00 kg and $g = 9.8$ m/s^2). This gives $a = \frac{2}{5}g$. Then (see Eq. 2-15) $\frac{1}{2}at^2 = 0.0784$ m (in the downward direction).

35. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 7.4 \times 10^2 \text{ kg/m}^3$.

36. Work is the integral of the force (over distance – see Eq. 7-32), and referring to the equation immediately preceding Eq. 14-7, we see the work can be written as

$$W = \int \rho_{\text{water}}\, gA(-y)\, dy$$

where we are using $y = 0$ to refer to the water surface (and the $+y$ direction is upward). Let $h = 0.500$ m. Then, the integral has a lower limit of $-h$ and an upper limit of y_f, with $y_f/h = -\rho_{\text{cylinder}}/\rho_{\text{water}} = -0.400$. The integral leads to

$$W = \tfrac{1}{2}\,\rho_{\text{water}} gAh^2(1 - 0.4^2) \;=\; 4.11 \text{ kJ} .$$

37. (a) The downward force of gravity *mg* is balanced by the upward buoyant force of the liquid: $mg = \rho g\ V_s$. Here *m* is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)\left(800\ \text{kg/m}^3\right)(0.090\ \text{m})^3 = 1.22\ \text{kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where *m* is the mass of the sphere and *V* is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090\ \text{m})^3 - (0.080\ \text{m})^3\right) = 9.09\times10^{-4}\ \text{m}^3 .$$

The density is

$$\rho_m = \frac{1.22\ \text{kg}}{9.09\times10^{-4}\ \text{m}^3} = 1.3\times10^3\ \text{kg/m}^3 .$$

38. If the alligator floats, by Archimedes' principle the buoyancy force is equal to the alligator's weight (see Eq. 14-17). Therefore,

$$F_b = F_g = m_{H_2O} g = (\rho_{H_2O} A h) g \, .$$

If the mass is to increase by a small amount $m \to m' = m + \Delta m$, then

$$F_b \to F_b' = \rho_{H_2O} A (h + \Delta h) g \, .$$

With $\Delta F_b = F_b' - F_b = 0.010 mg$, the alligator sinks by

$$\Delta h = \frac{\Delta F_b}{\rho_{H_2O} A g} = \frac{0.01 mg}{\rho_{H_2O} A g} = \frac{0.010(130 \text{ kg})}{(998 \text{ kg/m}^3)(0.20 \text{ m}^2)} = 6.5 \times 10^{-3} \text{ m} = 6.5 \text{ mm} \, .$$

39. Let V_i be the total volume of the iceberg. The non-visible portion is below water, and thus the volume of this portion is equal to the volume V_f of the fluid displaced by the iceberg. The fraction of the iceberg that is visible is

$$\text{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i}.$$

Since iceberg is floating, Eq. 14-18 applies:

$$F_g = m_i g = m_f g \quad \Rightarrow \quad m_i = m_f.$$

Since $m = \rho V$, the above equation implies

$$\rho_i V_i = \rho_f V_f \quad \Rightarrow \quad \frac{V_f}{V_i} = \frac{\rho_i}{\rho_f}.$$

Thus, the visible fraction is

$$\text{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f}$$

(a) If the iceberg ($\rho_i = 917\ \text{kg/m}^3$) floats in saltwater with $\rho_f = 1024\ \text{kg/m}^3$, then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \text{kg/m}^3}{1024\ \text{kg/m}^3} = 0.10 = 10\%.$$

(b) On the other hand, if the iceberg floats in fresh water ($\rho_f = 1000\ \text{kg/m}^3$), then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = 0.083 = 8.3\%.$$

40. (a) An object of the same density as the surrounding liquid (in which case the "object" could just be a packet of the liquid itself) is not going to accelerate up or down (and thus won't gain any kinetic energy). Thus, the point corresponding to zero K in the graph must correspond to the case where the density of the object equals ρ_{liquid}. Therefore, $\rho_{\text{ball}} = 1.5 \text{ g/cm}^3$ (or 1500 kg/m^3).

(b) Consider the $\rho_{\text{liquid}} = 0$ point (where $K_{\text{gained}} = 1.6$ J). In this case, the ball is falling through perfect vacuum, so that $v^2 = 2gh$ (see Eq. 2-16) which means that $K = \frac{1}{2}mv^2 = 1.6$ J can be used to solve for the mass. We obtain $m_{\text{ball}} = 4.082$ kg. The volume of the ball is then given by $m_{\text{ball}}/\rho_{\text{ball}} = 2.72 \times 10^{-3} \text{ m}^3$.

41. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60 \text{ cm}.$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}}g \Rightarrow \rho_{\text{water}} g V_{\text{submerged}} = \rho_{\text{iron}} g \left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

where r_i is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm^3) and iron (7.87 g/cm^3), we obtain the inner diameter:

$$2r_i = 2r_o\left(1 - \frac{1.0 \text{ g/cm}^3}{7.87 \text{ g/cm}^3}\right)^{1/3} = 57.3 \text{ cm}.$$

42. From the "kink" in the graph it is clear that $d = 1.5$ cm. Also, the $h = 0$ point makes it clear that the (true) weight is 0.25 N. We now use Eq. 14-19 at $h = d = 1.5$ cm to obtain

$$F_b = (0.25 \text{ N} - 0.10 \text{ N}) = 0.15 \text{ N}.$$

Thus, $\rho_{\text{liquid}}\, g\, V = 0.15$, where $V = (1.5 \text{ cm})(5.67 \text{ cm}^2) = 8.5 \times 10^{-6} \text{ m}^3$. Thus, $\rho_{\text{liquid}} = 1800 \text{ kg/m}^3 = 1.8 \text{ g/cm}^3$.

43. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{cav} = V_{cast} - V_{iron}$. The volume of the iron is given by $V_{iron} = W/g\rho_{iron}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{eff} = W - g\rho_w V_{cast}$. Thus, $V_{cast} = (W - W_{eff})/g\rho_w$ and

$$V_{cav} = \frac{W - W_{eff}}{g\rho_w} - \frac{W}{g\rho_{iron}} = \frac{6000\ \text{N} - 4000\ \text{N}}{(9.8\ \text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\ \text{N}}{(9.8\ \text{m/s}^2)(7.87\times10^3\ \text{kg/m}^3)}$$
$$= 0.126\ \text{m}^3\ .$$

44. Due to the buoyant force, the ball accelerates upward (while in the water) at rate a given by Newton's second law:

$$\rho_{\text{water}}Vg - \rho_{\text{ball}}Vg = \rho_{\text{ball}}Va \quad \Rightarrow \quad \rho_{\text{ball}} = \rho_{\text{water}}\,(1 + \text{“}a\text{”})$$

where – for simplicity – we are using in that last expression an acceleration "a" measured in "gees" (so that "a" = 2, for example, means that $a = 2(9.80\text{ m/s}^2) = 19.6\text{ m/s}^2$). In this problem, with $\rho_{\text{ball}} = 0.300\,\rho_{\text{water}}$, we find therefore that "a" = 7/3. Using Eq. 2-16, then the speed of the ball as it emerges from the water is

$$v = \sqrt{2a\Delta y}\ ,$$

were $a = (7/3)g$ and $\Delta y = 0.600$ m. This causes the ball to reach a maximum height h_{max} (measured above the water surface) given by $h_{\text{max}} = v^2/2g$ (see Eq. 2-16 again). Thus, $h_{\text{max}} = (7/3)\Delta y = 1.40$ m.

45. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{\text{car}}$, which leads to

$$V_1 = \frac{W_{\text{car}}}{\rho_w g} = \frac{(1800\,\text{kg})(9.8\,\text{m/s}^2)}{(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.80\ \text{m}^3.$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{\text{car}} + \rho_w V_2 g$$

which gives

$$V_2 = V - \frac{W_{\text{car}}}{\rho_w g} = \left(0.750\,\text{m}^3 + 5.00\,\text{m}^3 + 0.800\,\text{m}^3\right) - \frac{1800\,\text{kg}}{1000\,\text{kg/m}^3} = 4.75\ \text{m}^3.$$

46. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.900\,\rho_w V_{\text{wood}} g = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) = (m_{\text{wood}} + m_{\text{lead}})g.$$

Thus,

$$m_{\text{lead}} = 0.900\rho_w\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) - m_{\text{wood}} = \frac{(0.900)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\ \text{kg/m}^3} - 3.67\ \text{kg} = 1.84\ \text{kg}\ .$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.900\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right) m_{\text{lead}} g = (m_{\text{wood}} + m_{\text{lead}})g,$$

which leads to

$$m_{\text{lead}} = \frac{0.900(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} = \frac{1.84\ \text{kg}}{1 - (1.00\times10^3\ \text{kg/m}^3/1.13\times10^4\ \text{kg/m}^3)}$$
$$= 2.01\ \text{kg}.$$

47. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$F_g - (F_g - F_b) = \Delta mg$$

which leads to $F_b = \Delta mg$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\ \text{kg/m}} \approx 6.378 \times 10^{-4}\ \text{m}^3 .$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\ V_m = 5.102\ \text{m}^3\ .$$

(c) Using $\rho \approx \rho_w = 1000$ kg/m^3, we find

$$\rho = \frac{m_{\text{dino}}}{V_{\text{dino}}} \Rightarrow m_{\text{dino}} = (1000\,\text{kg/m}^3)\ (5.102\ \text{m}^3)$$

which yields 5.102×10^3 kg for the *T. rex* mass.

48. Let ρ be the density of the cylinder (0.30 g/cm^3 or 300 kg/m^3) and ρ_{Fe} be the density of the iron (7.9 g/cm^3 or 7900 kg/m^3). The volume of the cylinder is

$$V_c = (6\times12)\ \text{cm}^3 = 72\ \text{cm}^3 = 0.000072\ \text{m}^3,$$

and that of the ball is denoted V_b. The part of the cylinder that is submerged has volume

$$V_s = (4 \times 12)\ \text{cm}^3 = 48\ \text{cm}^3 = 0.000048\ \text{m}^3.$$

Using the ideas of section 14-7, we write the equilibrium of forces as

$$\rho g V_c + \rho_{Fe} g V_b = \rho_w g V_s + \rho_w g V_b \quad \Rightarrow \quad V_b = 3.8\ \text{cm}^3$$

where we have used $\rho_w = 998\ \text{kg/m}^3$ (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find $r = 9.7$ mm.

49. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 \left(N A_2 \right) \quad \Rightarrow \quad v_2 = \frac{A_1}{N A_2} v_1 = \frac{R^2}{N r^2} v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{\left(1.9\,\text{cm}\right)^2}{24\left(0.13\,\text{cm}\right)^2} \left(0.91\,\text{m/s}\right) = 8.1\,\text{m/s}.$$

50. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,\text{m})(3.4\,\text{m})(2.3\,\text{m/s})+(6.8\,\text{m})(3.2\,\text{m})(2.6\,\text{m/s})=h(10.5\,\text{m})(2.9\,\text{m/s})$$

which leads to $h = 4.0$ m.

51. This problem involves use of continuity equation (Eq. 14-23): $A_1 v_1 = A_2 v_2$.

(a) Initially the flow speed is $v_i = 1.5$ m/s and the cross-sectional area is $A_i = HD$. At point *a*, as can be seen from Fig. 14-47, the cross-sectional area is

$$A_a = (H-h)D - (b-h)d\ .$$

Thus, by continuity equation, the speed at point *a* is

$$v_a = \frac{A_i v_i}{A_a} = \frac{HDv_i}{(H-h)D-(b-h)d} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m}-0.80\text{ m})(55\text{ m})-(12\text{ m}-0.80\text{ m})(30\text{ m})} = 2.96\text{ m/s}$$
$$\approx 3.0\text{ m/s}.$$

(b) Similarly, at point *b*, the cross-sectional area is $A_b = HD - bd$, and therefore, by continuity equation, the speed at point *b* is

$$v_b = \frac{A_i v_i}{A_b} = \frac{HDv_i}{HD-bd} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m})(55\text{ m})-(12\text{ m})(30\text{ m})} = 2.8\text{ m/s}.$$

52. The left and right sections have a total length of 60.0 m, so (with a speed of 2.50 m/s) it takes 60.0/2.50 = 24.0 seconds to travel through those sections. Thus it takes (88.8 – 24.0) s = 64.8 s to travel through the middle section. This implies that the speed in the middle section is v_{mid} = (110 m)/(64.8 s) = 0.772 m/s. Now Eq. 14-23 (plus that fact that $A = \pi r^2$) implies $r_{mid} = r_A\sqrt{(2.5\text{ m/s})/(0.772\text{ m/s})}$ where r_A = 2.00 cm. Therefore, r_{mid} = 3.60 cm.

53. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by Δmgh, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta mv^2$, where v is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2}\Delta mv^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$ and

$$\rho_w A v = (1000\text{ kg/m}^3)\,(3.14 \times 10^{-4}\text{ m}^2)\,(5.00\text{ m/s}) = 1.57\text{ kg/s}.$$

Thus,

$$P = \rho A v\left(gh + \frac{1}{2}v^2\right) = (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\,\text{W}.$$

54. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

55. (a) We use the equation of continuity: $A_1v_1 = A_2v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus

$$v_2 = (A_1/A_2)v_1 = [(4.0\text{ cm}^2)/(8.0\text{ cm}^2)]\,(5.0\text{ m/s}) = 2.5\text{m/s}.$$

(b) We use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2\,,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\text{ Pa} + \frac{1}{2}(1000\text{ kg/m}^3)\left[(5.0\text{ m/s})^2 - (2.5\text{ m/s})^2\right] + (1000\text{ kg/m}^3)(9.8\text{ m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa.} \end{aligned}$$

56. We use Bernoulli's equation:

$$p_2 - p_i = \rho g D + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000$ kg/m^3, $D = 180$ m, $v_1 = 0.40$ m/s and $v_2 = 9.5$ m/s. Therefore, we find $\Delta p = 1.7 \times 10^6$ Pa, or 1.7 MPa. The SI unit for pressure is the Pascal (Pa) and is equivalent to N/m^2.

57. (a) The equation of continuity leads to

$$v_2 A_2 = v_1 A_1 \quad \Rightarrow \quad v_2 = v_1 \left(\frac{r_1^2}{r_2^2} \right)$$

which gives $v_2 = 3.9$ m/s.

(b) With $h = 7.6$ m and $p_1 = 1.7 \times 10^5$ Pa, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho g h + \frac{1}{2} \rho \left(v_1^2 - v_2^2 \right) = 8.8 \times 10^4 \text{ Pa}.$$

58. (a) We use Av = const. The speed of water is

$$v=\frac{(25.0\,\text{cm})^2-(5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s})=2.40\,\text{m/s}.$$

(b) Since $p+\frac{1}{2}\rho v^2=\text{const.},$ the pressure difference is

$$\Delta p=\frac{1}{2}\rho\Delta v^2=\frac{1}{2}\left(1000\,\text{kg/m}^3\right)\left[(2.50\,\text{m/s})^2-(2.40\,\text{m/s})^2\right]=245\,\text{Pa}.$$

59. (a) We use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \,,$$

where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \tfrac{1}{2}\rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g\left(h_1 - h_2\right)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.30\,\text{m}\right)} = 2.42\,\text{m/s}.$$

The flow rate is $A_2v_2 = (6.5 \times 10^{-4}\ \text{m}^2)(2.42\ \text{m/s}) = 1.6 \times 10^{-3}\ \text{m}^3/\text{s}$.

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3 = \tfrac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\ \text{m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\tfrac{1}{2}\rho v_2^2 + \rho g h_2 = \tfrac{1}{2}\rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{\left(4.84\,\text{m/s}\right)^2 - \left(2.42\,\text{m/s}\right)^2}{2\left(9.8\,\text{m/s}^2\right)} = 0.90\ \text{m}.$$

60. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho gh$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1 \sqrt{2gh} A_1 = \rho_2 \sqrt{2gh} A_2 \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2.$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2}$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$ Thus

$$h_2 = h_1/4 = (12.0 \text{ cm})/4 = 3.00 \text{ cm}.$$

61. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = (1.0 \times 10^5 \text{ Pa})\,(1.4 \text{ m}^3) = 1.4 \times 10^5 \text{ J}.$$

62. (a) The volume of water (during 10 minutes) is

$$V = (v_1 t) A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ and $h_1 = h_2, p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2\right]$$
$$= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8×10^4 Pa.

63. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3\ \text{kg/m}^3)\ (9.8\ \text{m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040\ \text{m})^2 = 74\ \text{N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}$. Thus, the volume of water flowing out of the pipe in t = 3.0 h is

$$V = Avt = \frac{\pi^2}{4}(0.040\ \text{m})^2\sqrt{2(9.8\ \text{m/s}^2)\ (6.0\ \text{m})}\ (3.0\ \text{h})\ (3600\ \text{s/h}) = 1.5\times10^2\,\text{m}^3.$$

64. (a) We note (from the graph) that the pressures are equal when the value of inverse-area-squared is 16 (in SI units). This is the point at which the areas of the two pipe sections are equal. Thus, if $A_1 = 1/\sqrt{16}$ when the pressure difference is zero, then A_2 is 0.25 m^2.

(b) Using Bernoulli's equation (in the form Eq. 14-30) we find the pressure difference may be written in the form a straight line: $mx + b$ where x is inverse-area-squared (the horizontal axis in the graph), m is the slope, and b is the intercept (seen to be –300 kN/m^2). Specifically, Eq. 14-30 predicts that b should be $-\frac{1}{2}\rho\, v_2^{\,2}$. Thus, with $\rho = 1000$ kg/m^3 we obtain $v_2 = \sqrt{600}$ m/s. Then the volume flow rate (see Eq. 14-24) is

$$R = A_2\, v_2 = (0.25 \text{ m}^2)(\sqrt{600} \text{ m/s}) = 6.12 \text{ m}^3\text{/s}.$$

If the more accurate value (see Table 14-1) $\rho = 998$ kg/m^3 is used, then the answer is 6.13 m^3/s.

65. (a) Since Sample Problem 14-8 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v=\sqrt{2gh} \;\Rightarrow\; v=v_0 \text{ for the projectile motion.}$$

The stream of water emerges horizontally (θ_0 = 0° in the notation of Chapter 4), and setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t=\sqrt{\frac{-2(H-h)}{-g}}=\sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where $x_0 = 0$ by choice of coordinate origin, we find

$$x=v_0 t=\sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}=2\sqrt{h(H-h)}=2\sqrt{(10\text{ cm})(40\text{ cm}-10\text{ cm})}=35\text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2-Hh+\frac{x^2}{4}=0 \Rightarrow h=\frac{H\pm\sqrt{H^2-x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1+h_2=\frac{H+\sqrt{H^2-x^2}}{2}+\frac{H-\sqrt{H^2-x^2}}{2}=H.$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$. Its numerical value is $h' = 40\text{cm} - 10\text{ cm} = 30\text{ cm}$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh}=4H-8h=0 \Rightarrow h=\frac{H}{2}$$

or h = (40 cm)/2 = 20 cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.

66. By Eq. 14-23, we note that the speeds in the left and right sections are $\frac{1}{4}\,v_{mid}$ and $\frac{1}{9}\,v_{mid}$, respectively, where v_{mid} = 0.500 m/s. We also note that 0.400 m^3 of water has a mass of 399 kg (see Table 14-1). Then Eq. 14-31 (and the equation below it) gives

$$W = \frac{1}{2} m \, v_{mid}{}^2 \left(\frac{1}{9^2} - \frac{1}{4^2}\right) = -2.50 \text{ J} .$$

67. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho\left(A/a\right)^2 v^2$. We solve for v. The result is

$$v = \sqrt{\frac{2\Delta p}{\rho\left((A/a)^2 - 1\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(A^2 - a^2\right)}}.$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32\times10^{-4}\,\text{m}^2)^2(55\times10^3\,\text{Pa} - 41\times10^3\,\text{Pa})}{(1000\,\text{kg/m}^3)\left((64\times10^{-4}\,\text{m}^2)^2 - (32\times10^{-4}\,\text{m}^2)^2\right)}} = 3.06\,\text{m/s}.$$

Consequently, the flow rate is

$$Av = (64\times10^{-4}\,\text{m}^2)(3.06\ \text{m/s}) = 2.0\times10^{-2}\,\text{m}^3/\text{s}.$$

68. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0$ atm. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{4(1.01\times10^5\ \text{Pa})}{(1000\ \text{kg/m}^3)\,[(5a/a)^2 - 1]}} = 4.1\,\text{m/s}.$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21$ m/s.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\,(5.0\times10^{-4}\ \text{m}^2)\ (4.1\ \text{m/s}) = 8.0\times10^{-3}\,\text{m}^3/\text{s}.$$

69. (a) This is similar to the situation treated in Sample Problem 14-7, and we refer to some of its steps (and notation). Combining Eq. 14-35 and Eq. 14-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2 \sqrt{\frac{2\Delta p}{\rho\left(A_1^2 - A_2^2\right)}}$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that this reduces to Eq. 14-38 for the case $A_2 = A_1/2$ treated in the Sample Problem. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66\times10^{-3}\ \text{m}^4$, so that the square root is well defined. Therefore, we obtain $R = 0.0776\ \text{m}^3/\text{s}$.

(b) The mass rate of flow is $\rho R = 69.8$ kg/s.

70. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}}v^2 \cdot$ But $\Delta p = p_A - p_B = \rho gh$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho gh = \frac{1}{2}\rho_{\text{air}}v^2$, or

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}} = \sqrt{\frac{2\left(810\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(0.260\,\text{m})}{1.03\,\text{kg/m}^3}} = 63.3\,\text{m/s}.$$

71. We use the formula for *v* obtained in the previous problem:

$$v=\sqrt{\frac{2\Delta p}{\rho_{\text{air}}}}=\sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}}=1.1\times10^2\,\text{m/s}.$$

72. We use Bernoulli's equation $p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$.

When the water level rises to height h_2, just on the verge of flooding, v_2, the speed of water in pipe M , is given by

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2 \quad \Rightarrow \; v_2 = \sqrt{2g(h_1 - h_2)} = 13.86 \text{ m/s}.$$

By continuity equation, the corresponding rainfall rate is

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \frac{\pi(0.030 \text{ m})^2}{(30 \text{ m})(60 \text{ m})}(13.86 \text{ m/s}) = 2.177\times10^{-5} \text{ m/s} \approx 7.8 \text{ cm/h}.$$

73. The normal force $\vec{F}_N$ exerted (upward) on the glass ball of mass m has magnitude 0.0948 N. The buoyant force exerted by the milk (upward) on the ball has magnitude

$$F_b = \rho_{\text{milk}}\, g\, V$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the ball. Its radius is $r = 0.0200$ m. The milk density is $\rho_{\text{milk}} = 1030$ kg/m^3. The (actual) weight of the ball is, of course, downward, and has magnitude $F_g = m_{\text{glass}}\, g$. Application of Newton's second law (in the case of zero acceleration) yields

$$F_N + \rho_{\text{milk}}\, g\, V - m_{\text{glass}}\, g = 0$$

which leads to $m_{\text{glass}} = 0.0442$ kg. We note the above equation is equivalent to Eq.14-19 in the textbook.

74. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2}.$$

(a) With $R = 7.0 \times 10^{-3}$ m^3/s and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where v_s = 343 m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

75. If we examine both sides of the U-tube at the level where the low-density liquid (with $\rho = 0.800 \text{ g/cm}^3 = 800 \text{ kg/m}^3$) meets the water (with $\rho_w = 0.998 \text{ g/cm}^3 = 998 \text{ kg/m}^3$), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where h = 8.00 cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is h_w = (800/998)(8.00 cm) = 6.41 cm. The volume of water in that column is therefore

$$V = \pi r^2 h_w = \pi(1.50 \text{ cm})^2(6.41 \text{ cm}) = 45.3 \text{ cm}^3.$$

76. Since (using Eq. 5-8) $F_g = mg = \rho_{\text{skier}}\, g\, V$ and (Eq. 14-16) the buoyant force is $F_b = \rho_{\text{snow}}\, g\, V$, then their ratio is

$$\frac{F_b}{F_g} = \frac{\rho_{\text{snow}}\, g\, V}{\rho_{\text{skier}}\, g\, V} = \frac{\rho_{\text{snow}}}{\rho_{\text{skier}}} = \frac{96}{1020} = 0.094 \quad \text{(or 9.4\%)}.$$

77. (a) We consider a point *D* on the surface of the liquid in the container, in the same tube of flow with points *A, B* and *C*. Applying Bernoulli's equation to points *D* and *C*, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8\ \text{m/s}^2)(0.40\ \text{m} + 0.12\ \text{m})} = 3.2\ \text{m/s}.$$

(b) We now consider points *B* and *C*:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C\ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g(h_1 + h_2 + d) \\ &= 1.0\times10^5\ \text{Pa} - (1.0\times10^3\,\text{kg/m}^3)(9.8\ \text{m/s}^2)(0.25\ \text{m} + 0.40\ \text{m} + 0.12\ \text{m}) \\ &= 9.2\times10^4\ \text{Pa}. \end{aligned}$$

(c) Since $p_B \geq 0$, we must let $p_{\text{air}} - \rho g(h_1 + d + h_2) \geq 0$, which yields

$$h_1 \leq h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \leq \frac{p_{\text{air}}}{\rho} = 10.3\ \text{m}.$$

78. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \Rightarrow \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it which is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5.0% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

79. We note that in "gees" (where acceleration is expressed as a multiple of g) the given acceleration is 0.225/9.8 = 0.02296. Using $m = \rho V$, Newton's second law becomes

$$\rho_{\text{wat}} Vg - \rho_{\text{bub}} Vg = \rho_{\text{bub}} Va \quad \Rightarrow \quad \rho_{\text{bub}} = \rho_{\text{wat}} \,(1 + \text{"}a\text{"})$$

where in the final expression "a" is to be understood to be in "gees." Using $\rho_{\text{wat}} = 998$ kg/m^3 (see Table 14-1) we find $\rho_{\text{bub}} = 975.6$ kg/m^3. Using volume $V = \frac{4}{3}\pi r^3$ for the bubble, we then find its mass: $m_{\text{bub}} = 5.11 \times 10^{-7}$ kg.

80. The downward force on the balloon is mg and the upward force is $F_b = \rho_{out}Vg$. Newton's second law (with $m = \rho_{in}V$) leads to

$$\rho_{out}\,Vg - \rho_{in}\,Vg = \rho_{in}\,Va \;\; \Rightarrow \left(\frac{\rho_{out}}{\rho_{in}} - 1\right)g = a\,.$$

The problem specifies $\rho_{out} / \rho_{in} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is $(1.39 - 1.00)(9.80 \text{ m/s}^2) = 3.82 \text{ m/s}^2$.

81. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be $V = 1200\ \text{cm}^3$. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\,g = \rho_w V g \Rightarrow m_\ell = (1\,\text{g/cm}^3)\ (1200\ \text{cm}^3) - 130\ \text{g}$$

which yields 1.07×10^3 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

82. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{Hg}gx$, which should be compensated for by the water pressure $p_w = \rho_w gh$, where $h = 11.2$ cm. In these units, $\rho_w = 1.00$ g/cm^3 and $\rho_{Hg} = 13.6$ g/cm^3 (see Table 14-1). We obtain

$$x = \frac{\rho_w gh}{2\rho_{\mathrm{Hg}} g} = \frac{(1.00\ \mathrm{g/cm^3})\ (11.2\ \mathrm{cm})}{2(13.6\ \mathrm{g/cm^3})} = 0.412\ \mathrm{cm}.$$

83. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight W of the object. The buoyant force of the water on the object is therefore (30 – 20) N = 10 N, which means

$$F_b = \rho_w V g \Rightarrow V = \frac{10\,\text{N}}{(1000\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.02 \times 10^{-3}\ \text{m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is (30 – 24) N = 6.0 N, which implies

$$\rho_2 = \frac{6.0\ \text{N}}{(9.8\ \text{m/s}^2)(1.02 \times 10^{-3}\ \text{m}^3)} = 6.0{\times}10^2\ \text{kg/m}^3\ .$$

84. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where V_{sub} is the portion of the object which is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{liquid}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A,

$$\frac{\rho}{\rho_A} = \frac{1}{2}$$

so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*,

$$\frac{\rho}{\rho_B} = \frac{1}{3}$$

so that $\rho_B/\rho_w = 3$.

(c) For liquid C, noting that one-fourth *above* means three-fourths *below*,

$$\frac{\rho}{\rho_C} = \frac{3}{4}$$

so that $\rho_C/\rho_w = 4/3$.

85. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\text{body}}\, g \Rightarrow \rho_{\text{liquid}}\, gV_{\text{submerged}} = \rho_{\text{body}} gV_{\text{total}}$$

which leads to

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}.$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with ρ_{body} = 0.98 g/cm^3, we find $\rho_{\text{liquid}} \approx$ 1.5 g/cm^3 — certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

Chapter 15

1. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi\left(6.60\text{ Hz}\right)\right)^2\left(0.0220\text{ m}\right) = 37.8\text{ m/s}^2.$$

2. (a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus

$$\omega = 2\pi/(1.00 \times 10^{-5} \text{ s}) = 6.28 \times 10^5 \text{ rad/s}.$$

(b) The maximum speed v_m and maximum displacement x_m are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00 \times 10^3 \text{ m/s}}{6.28 \times 10^5 \text{ rad/s}} = 1.59 \times 10^{-3} \text{ m.}$$

3. (a) The amplitude is half the range of the displacement, or $x_m = 1.0$ mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi (120\text{ Hz})\left(1.0\times10^{-3}\text{ m}\right) = 0.75\text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi (120\text{ Hz})\right)^2 \left(1.0\times10^{-3}\text{ m}\right) = 5.7\times10^2\text{ m/s}^2.$$

4. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{max} = m\omega^2 x_m = (0.12 \text{ kg})(10\pi \text{ rad/s})^2 (0.085 \text{ m}) = 10 \text{ N}.$$

(b) Using Eq. 15-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.12\text{kg})(10\pi \text{ rad/s})^2 = 1.2\times10^2 \text{ N/m}.$$

5. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$ must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

6. (a) Since the problem gives the frequency f = 3.00 Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 15-12 leads to

$$\omega = \sqrt{\frac{k}{m_{\text{car}}/4}} \quad \Rightarrow \quad k = \frac{1}{4}(1450\text{kg})(6\pi \text{ rad/s})^2 = 1.29\times10^5 \text{ N/m}.$$

(b) If the new mass being supported by the four springs is m_{total} = [1450 + 5(73)] kg = 1815 kg, then Eq. 15-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{m_{\text{total}}/4}} \quad \Rightarrow f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29\times10^5 \text{ N/m}}{(1815/4)\text{ kg}}} = 2.68\text{Hz}.$$

7. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500 \text{ s}) = 2.00$ Hz.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00 \text{ Hz}) = 12.6$ rad/s.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) Let x_m be the amplitude. The maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6$ N.

8. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

(c) Since 2π radians are equivalent to a cycle, the angular frequency ω (in radians-per-second) is related to frequency f by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

9. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g / x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{1.0\times 10^{-6}\text{ m}}} = 498 \text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

10. We note (from the graph) that $x_m = 6.00$ cm. Also the value at $t = 0$ is $x_o = -2.00$ cm. Then Eq. 15-3 leads to

$$\phi = \cos^{-1}(-2.00/6.00) = +1.91 \text{ rad or } -4.37 \text{ rad.}$$

The other "root" (+4.37 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

11. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0\cos\left(3\pi(2.0)+\frac{\pi}{3}\right) = 3.0 \text{ m.}$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0)+\frac{\pi}{3}\right) = -49 \text{ m/s.}$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0)+\frac{\pi}{3}\right) = -2.7\times10^2 \text{ m/s}^2.$$

(d) In the second paragraph after Eq. 15-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \pi/3 \approx 20$ rad.

(e) Comparing with Eq. 15-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

12. We note (from the graph) that $v_m = \omega x_m = 5.00$ cm/s. Also the value at $t = 0$ is $v_o = 4.00$ cm/s. Then Eq. 15-6 leads to

$$\phi = \sin^{-1}(-4.00/5.00) = -0.927 \text{ rad or } +5.36 \text{ rad.}$$

The other "root" (+4.07 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

13. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position (x = 0). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -2kx.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{2k}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(7580 \text{ N/m})}{0.245 \text{ kg}}} = 39.6 \text{ Hz}.$$

14. The statement that "the spring does not affect the collision" justifies the use of elastic collision formulas in section 10-5. We are told the period of SHM so that we can find the mass of block 2:

$$T = 2\pi\sqrt{\frac{m_2}{k}} \quad \Rightarrow \quad m_2 = \frac{kT^2}{4\pi^2} = 0.600 \text{ kg}.$$

At this point, the rebound speed of block 1 can be found from Eq. 10-30:

$$|v_{1f}| = \left|\frac{0.200 \text{ kg} - 0.600 \text{ kg}}{0.200 \text{ kg} + 0.600 \text{ kg}}\right|(8.00 \text{ m/s}) = 4.00 \text{ m/s} .$$

This becomes the initial speed v_0 of the projectile motion of block 1. A variety of choices for the positive axis directions are possible, and we choose left as the $+x$ direction and down as the $+y$ direction, in this instance. With the "launch" angle being zero, Eq. 4-21 and Eq. 4-22 (with $-g$ replaced with $+g$) lead to

$$x - x_0 = v_0 t = v_0\sqrt{\frac{2h}{g}} = (4.00 \text{ m/s})\sqrt{\frac{2(4.90 \text{ m})}{9.8 \text{ m/s}^2}} .$$

Since $x - x_0 = d$, we arrive at $d = 4.00$ m.

15. (a) Eq. 15-8 leads to

$$a=-\omega^2 x \Rightarrow \omega=\sqrt{\frac{-a}{x}}=\sqrt{\frac{123\ \text{m/s}^2}{0.100\ \text{m}}}=35.07\ \text{rad/s}\,.$$

Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Eq. 15-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega=\sqrt{\frac{k}{m}} \quad \Rightarrow \quad m=\frac{400\ \text{N/m}}{(35.07\ \text{rad/s})^2}=0.325\text{kg}.$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2}kx_m^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2 \Rightarrow x_m=\frac{m}{k}v^2+x^2.$$

Consequently, $x_m=\sqrt{(0.325\ \text{kg}/400\ \text{N/m})(13.6\ \text{m/s})^2+(0.100\ \text{m})^2}=0.400\text{m}.$

16. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m = \frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$. With $x = x_m \cos(\omega t + \phi)$, we obtain

$$\frac{1}{4}d = \left(\frac{1}{2}d\right)\cos\left(0.503t + 0\right) \;\Rightarrow\; \frac{1}{2} = \cos(0.503t)$$

which has t = 2.08 h as the smallest positive root. The calculator is in radians mode during this calculation.

17. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8 \text{ m/s}^2)}{(2\pi \times 2.0 \text{ Hz})^2} = 0.031 \text{ m}.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

18. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2}x_m$ where

$$x_1 = x_m\cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m\cos(\omega t + \phi_2).$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m\omega\sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m\omega\sin(\omega t + \phi_2).$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the wt term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

19. (a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50$ s,

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{dx_2}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for $t = 0.50$ s and find they are both negative-valued, indicating that the particles are moving in the same direction.

20. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where $\omega = 1.20$ rad/s in this problem. Evaluating this at $t = 0$ and using the values from the graphs shown in the problem, we find

$$\phi = \tan^{-1}(-v_o/x_o\omega) = \tan^{-1}\big(+4.00/(2 \times 1.20)\big) = 1.03 \text{ rad (or } -5.25 \text{ rad)}.$$

One can check that the other "root" (4.17 rad) is unacceptable since it would give the wrong signs for the individual values of v_o and x_o.

21. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8$ m/s^2.

(a) Using Eq. 15-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \Rightarrow x_m = \frac{gT^2}{4\pi^2} = 0.25 \text{ m}.$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$(2\pi f)^2 x_m = g \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2 \text{ Hz}.$$

22. Eq. 15-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{2.00 \text{ kg}}} = 7.07 \text{rad/s}.$$

Energy methods (discussed in §15-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 15-3 and Eq. 15-6.

(a) Dividing Eq. 15-6 by Eq. 15-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase ($\omega t + \phi$) is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415 \text{ m/s}}{(7.07 \text{ rad/s})(0.129 \text{ m})}\right).$$

With the calculator in radians mode, this gives the phase equal to –1.31 rad. Plugging this back into Eq. 15-3 leads to $0.129 \text{m} = x_m \cos(-1.31) \Rightarrow x_m = 0.500 \text{m}$.

(b) Since $\omega t + \phi = -1.31$ rad at $t = 1.00$ s, we can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_o = x_m \cos \phi = -0.251$ m.

(c) And we obtain $v_o = -x_m \omega \sin\phi = 3.06$ m/s.

23. Let the spring constants be k_1 and k_2. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x + k_2\,x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/d^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -k_1x - k_2x.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 30\text{ Hz}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 45\text{ Hz},$$

respectively. Comparing these expressions, it is clear that

$$f = \sqrt{f_1^2 + f_2^2} = \sqrt{(30\text{ Hz})^2 + (45\text{ Hz})^2} = 54\text{ Hz}.$$

24. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M} x_m = \mu_s g$$

which leads to

$$x_m = \frac{\mu_s g(m+M)}{k} = \frac{(0.40)(9.8 \text{ m/s}^2)(1.8 \text{ kg} + 10 \text{ kg})}{200 \text{ N/m}} = 0.23 \text{ m} = 23 \text{ cm}.$$

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\Rightarrow\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\text{ N})\sin 40.0°}{120\text{ N/m}} = 0.0750\text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\text{ N}/9.80\text{ m/s}^2}{120\text{ N/m}}} = 0.686\text{ s.}$$

26. We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\Delta x_\ell$. Thus $k_{\text{eff}} = k\Delta x_\ell / 2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}.$$

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.

27. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2\ \text{N/m}\right)\left(0.024\ \text{m}\right)^2 = 3.7\times10^{-2}\ \text{J}.$$

28. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where E = 1.00 J and x_m = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200 \text{ N / m}.$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \Rightarrow m = \frac{2E}{v_m^2} = 1.39 \text{ kg}.$$

(c) Eq. 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91 \text{ Hz}.$$

29. The total energy is given by $E=\frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

(a) The fraction of the energy that is kinetic is

$$\frac{K}{E}=\frac{E-U}{E}=1-\frac{U}{E}=1-\frac{1}{4}=\frac{3}{4}=0.75$$

where the result from part (b) has been used.

(b) When $x=\frac{1}{2}x_m$ the potential energy is $U=\frac{1}{2}kx^2=\frac{1}{8}kx_m^2$. The ratio is

$$\frac{U}{E}=\frac{kx_m^2/8}{kx_m^2/2}=\frac{1}{4}=0.25.$$

(c) Since $E=\frac{1}{2}kx_m^2$ and $U=\frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve x^2/x_m^2 = 1/2 for x. We should get $x=x_m/\sqrt{2}$.

30. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position ($x = 0$):

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.0\ \text{kg})(0.85\ \text{m/s})^2 = 0.72\ \text{J}.$$

Looking at the graph in the problem, we see that $U(x=10)=0.5$ J. Since the potential function has the form $U(x)=bx^2$, the constant is $b=5.0\times10^{-3}\ \text{J/cm}^2$. Thus, $U(x) = 0.72$ J when $x = 12$ cm.

(a) Thus, the mass does turn back before reaching $x = 15$ cm.

(b) It turns back at $x = 12$ cm.

31. (a) Eq. 15-12 (divided by 2π) yields

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{1000\text{ N/m}}{5.00\text{ kg}}}=2.25\,\text{Hz}.$$

(b) With x_o = 0.500 m, we have $U_0=\frac{1}{2}kx_0^2=125\text{ J}$.

(c) With v_o = 10.0 m/s, the initial kinetic energy is $K_0=\frac{1}{2}mv_0^2=250\text{ J}$.

(d) Since the total energy $E = K_o + U_o$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E=\frac{1}{2}kx_m^2 \Rightarrow x_m=\sqrt{\frac{2E}{k}}=0.866\text{ m}.$$

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently x_m = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant k:

$$\frac{1}{2} k\, x_m^{\,2} = 6.0\ \text{J} \quad \Rightarrow \quad k = 8.3 \times 10^2\ \text{N/m}\ .$$

33. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000$ m/s^2 leads to $\omega = 2000$ rad/s. Using Newton's second law with $m = 0.010$ kg, we have

$$F = ma = m\left(-a_m \cos(\omega t + \phi)\right) = -(80 \text{ N})\cos\left(2000t - \frac{\pi}{3}\right)$$

where t is understood to be in seconds.

(a) Eq. 15-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

(b) The relation $v_m = \omega x_m$ can be used to solve for v_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 15-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s.}$$

(c) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080 \text{ J}$.

(d) At the maximum displacement, the force acting on the particle is

$$F = kx = (4.0\times10^4 \text{ N/m})(2.0\times10^{-3}\text{m}) = 80 \text{ N.}$$

(e) At half of the maximum displacement, $x = 1.0 \text{ mm}$, and the force is

$$F = kx = (4.0\times10^4 \text{ N/m})(1.0\times10^{-3}\text{m}) = 40 \text{ N.}$$

34. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where ω is given by Eq. 15-12. Since the kinetic energy is $\frac{1}{2}mv^2$ and the potential energy is $\frac{1}{2}kx^2$ (which may be conveniently written as $\frac{1}{2}m\omega^2x^2$) then the ratio of kinetic to potential energy is simply

$$(v/x)^2/\omega^2 = \tan^2(\omega t + \phi),$$

which at $t = 0$ is $\tan^2\phi$. Since $\phi = \pi/6$ in this problem, then the ratio of kinetic to potential energy at $t = 0$ is $\tan^2(\pi/6) = 1/3$.

35. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$. With $m = 9.5$ g, $M = 5.4$ kg and $v = 630$ m/s, we obtain $v' = 1.1$ m/s.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2}(m+M)(v')^2 = \frac{1}{2}kx_m^2 \quad\Rightarrow\quad \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}kx_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3}\text{kg})(630\text{ m/s})}{\sqrt{(6000\text{ N/m})(9.5\times10^{-3}\text{kg} + 5.4\text{kg})}} = 3.3\times10^{-2}\text{ m.}$$

36. We note that the spring constant is

$$k = 4\pi^2 m_1/T^2 = 1.97 \times 10^5 \text{ N/m}.$$

It is important to determine where in its simple harmonic motion (which "phase" of its motion) block 2 is when the impact occurs. Since $\omega = 2\pi/T$ and the given value of t (when the collision takes place) is one-fourth of T, then $\omega t = \pi/2$ and the location then of block 2 is $x = x_m\cos(\omega t + \phi)$ where $\phi = \pi/2$ which gives $x = x_m\cos(\pi/2 + \pi/2) = -x_m$. This means block 2 is at a turning point in its motion (and thus has zero speed right before the impact occurs); this means, too, that the spring is stretched an amount of 1 cm = 0.01 m at this moment. To calculate its after-collision speed (which will be the same as that of block 1 right after the impact, since they stick together in the process) we use momentum conservation and obtain v = (4.0 kg)(6.0 m/s)/(6.0 kg) = 4.0 m/s. Thus, at the end of the impact itself (while block 1 is still at the same position as before the impact) the system (consisting now of a total mass M = 6.0 kg) has kinetic energy

$$K = \tfrac{1}{2}(6.0 \text{ kg})(4.0 \text{ m/s})^2 = 48 \text{ J}$$

and potential energy

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(1.97 \times 10^5 \text{ N/m})(0.010 \text{ m})^2 \approx 10 \text{ J},$$

meaning the total mechanical energy in the system at this stage is approximately $E = K + U$ = 58 J. When the system reaches its new turning point (at the new amplitude X) then this amount must equal its (maximum) potential energy there: $E = \tfrac{1}{2}(1.97 \times 10^5 \text{ N/m})\, X^2$. Therefore, we find

$$X = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(58 \text{ J})}{1.97\times10^5 \text{ N/m}}} = 0.024 \text{ m}.$$

37. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and

$$f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}\,.$$

Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\ \text{m/s}^2}{0.050\ \text{m}}} = 2.2\ \text{Hz}.$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the y axis to be positive in the downward direction and let y = 0.080 m. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes $0 = \frac{1}{2}ky^2 - mgy + \frac{1}{2}mv^2$. We solve for the speed:

$$v = \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.080\,\text{m}\right) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)\left(0.080\,\text{m}\right)^2}$$
$$= 0.56\,\text{m/s}$$

(c) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m+\Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve $\sqrt{k/(m+\Delta m)} = \frac{1}{2}\sqrt{k/m}$ for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives

$$m = \Delta m/3 = (300\text{ g})/3 = 100\text{ g} = 0.100\text{ kg}.$$

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant k. Use $k = m\omega^2 = m(2\pi f)^2$. Then

$$y\frac{(m+\Delta m)g}{m(2\pi f)^2} = \frac{(0.100\ \text{kg} + 0.300\ \text{kg})(9.80\ \text{m/s}^2)}{(0.100\ \text{kg})(2\pi\times 2.24\ \text{Hz})^2} = 0.200\ \text{m}.$$

This is measured from the initial position.

38. From Eq. 15-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20\ \text{N}\cdot\text{m}}{0.85\ \text{rad}} = 0.235\ \ \text{N}\cdot\text{m/rad}\,.$$

With $I = 2mR^2/5$ (the rotational inertia for a solid sphere — from Chapter 11), Eq. 15–23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}mR^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}(95\ \text{kg})(0.15\ \text{m})^2}{0.235\ \ \text{N}\cdot\text{m/rad}}} = 12\ \text{s.}$$

39. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad / s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta .$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2\left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

40. (a) Comparing the given expression to Eq. 15-3 (after changing notation $x \rightarrow \theta$), we see that $\omega = 4.43$ rad/s. Since $\omega = \sqrt{g/L}$ then we can solve for the length: $L = 0.499$ m.

(b) Since $v_m = \omega x_m = \omega L \theta_m = (4.43 \text{ rad/s})(0.499 \text{ m})(0.0800 \text{ rad})$ and $m = 0.0600$ kg, then we can find the maximum kinetic energy: $\frac{1}{2} m v_m^2 = 9.40 \times 10^{-4}$ J.

41. (a) Referring to Sample Problem 15-5, we see that the distance between P and C is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 15–30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2.$$

Eq. 15-29 then gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) We note that this T is identical to that computed in Sample Problem 15-5. As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot which is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

42. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem 15-5, but treating in our case a more general possibility for I. Canceling 2π, squaring both sides, and canceling g leads directly to the result; $L_o = I/mh$.

43. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r+L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2+\frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$
\begin{aligned}
I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\
&= \frac{1}{2}(0.500\,\text{kg})(0.100\,\text{m})^2 + (0.500\,\text{kg})(0.500\,\text{m}+0.100\,\text{m})^2 + \frac{1}{3}(0.270\,\text{kg})(0.500\,\text{m})^2 \\
&= 0.205\,\text{kg}\cdot\text{m}^2.
\end{aligned}
$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\text{ m} + 0.100\text{ m} = 0.600\text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\text{ m})/2 = 0.250\text{ m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\text{ kg})(0.600\text{ m}) + (0.270\text{ kg})(0.250\text{ m})}{0.500\text{ kg} + 0.270\text{ kg}} = 0.477\text{ m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\text{ kg}\cdot\text{m}^2}{(0.500\text{ kg}+0.270\text{ kg})(9.80\text{ m/s}^2)(0.447\text{ m})}} = 1.50\text{ s}.$$

44. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

45. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} = 2\pi\sqrt{\frac{(2.35\text{ cm})^2 + 2(1.75\text{ cm})^2}{2(980\text{ cm/s}^2)(1.75\text{ cm})}} = 0.366\text{ s.}$$

46. To use Eq. 15-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass so the center of mass of the system is $h = \frac{1}{2}(0.50 \text{ m}) = 0.25\text{m}$ below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where L = 1.00 m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15–29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, T = 1.83 s.

47. From Eq. 15-28, we find the length of the pendulum when the period is $T = 8.85$ s:

$$L = \frac{gT^2}{4\pi^2}.$$

The new length is $L' = L - d$ where $d = 0.350$ m. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields $T' = 8.77$ s.

48. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = 1/3ML^2$. Therefore, Eq. 15-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\left(L/2\right)}} \quad \Rightarrow \quad \frac{3gT^2}{8\pi^2}$$

so that L = 0.84 m.

(b) By energy conservation

$$E_{\text{bottom of swing}} = E_{\text{end of swing}} \quad \Rightarrow \quad K_m = U_m$$

where $U = Mg\ell(1 - \cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = \left(0.5 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where θ_m = 0.17 rad. Thus, $K_m = U_m$ = 0.031 J. If we calculate $(1 - \cos\theta)$ straightforwardly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

49. This is similar to the situation treated in Sample Problem 15-5, except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Eq. 15-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{\left(L^2 + 12x^2\right)}{12gx}}.$$

(a) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12} = (1.85\text{ m})/\sqrt{12} = 0.53\text{ m}$ as the value of x which should produce the smallest possible value of T.

(b) With L = 1.85 m and x = 0.53 m, we obtain T = 2.1 s from the expression derived in part (a).

50. Consider that the length of the spring as shown in the figure (with one of the block's corners lying directly above the block's center) is some value L (its rest length). If the (constant) distance between the block's center and the point on the wall where the spring attaches is a distance r, then $r\cos\theta = d/\sqrt{2}$ and $r\cos\theta = L$ defines the angle θ measured from a line on the block drawn from the center to the top corner to the line of r (a straight line from the center of the block to the point of attachment of the spring on the wall). In terms of this angle, then, the problem asks us to consider the dynamics that results from increasing θ from its original value θ_o to $\theta_o + 3°$ and then releasing the system and letting it oscillate. If the new (stretched) length of spring is L' (when $\theta = \theta_o + 3°$), then it is a straightforward trigonometric exercise to show that

$$(L')^2 = r^2 + (d/\sqrt{2})^2 - 2r(d/\sqrt{2})\cos(\theta_o + 3°) = L^2 + d^2 - d^2\cos(3°) + \sqrt{2}\,Ld\sin(3°) .$$

since $\theta_o = 45°$. The difference between L' (as determined by this expression) and the original spring length L is the amount the spring has been stretched (denoted here as x_m). If one plots x_m versus L over a range that seems reasonable considering the figure shown in the problem (say, from $L = 0.03$ m to $L = 0.10$ m) one quickly sees that $x_m \approx 0.00222$ m is an excellent approximation (and is very close to what one would get by approximating x_m as the arc length of the path made by that upper block corner as the block is turned through 3°, even though this latter procedure should in principle overestimate x_m). Using this value of x_m with the given spring constant leads to a potential energy of $U = \frac{1}{2} k\, x_m^2 =$ 0.00296 J. Setting this equal to the kinetic energy the block has as it passes back through the initial position, we have

$$K = 0.00296 \text{ J} = \tfrac{1}{2}\, I\, \omega_m^2$$

where ω_m is the maximum angular speed of the block (and is not to be confused with the angular frequency ω of the oscillation, though they are related by $\omega_m = \theta_o\omega$ if θ_o is expressed in radians). The rotational inertia of the block is $I = \frac{1}{6} Md^2 = 0.0018 \text{ kg·m}^2$. Thus, we can solve the above relation for the maximum angular speed of the block:

$$\omega_m = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(0.00296 \text{ J})}{0.0018 \text{ kg}\cdot\text{m}^2}} = 1.81 \text{ rad/s} .$$

Therefore the angular frequency of the oscillation is $\omega = \omega_m/\theta_o = 34.6$ rad/s. Using Eq. 15-5, then, the period is $T =$ 0.18 s.

51. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = 2\pi/\omega = 2\pi\sqrt{I/C},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1 - \cos\theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}\ .$$

If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 10-2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With $m = 0.600$ kg and $k = 1850$ N/m, we obtain $T = 0.0653$ s.

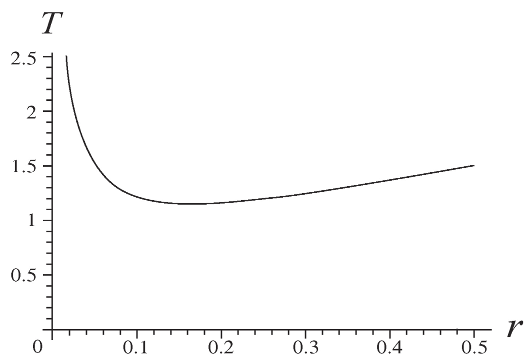
52. (a) For the "physical pendulum" we have

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{com} + mh^2}{mgh}} \ .$$

If we substitute r for h and use item (i) in Table 10-2, we have

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{a^2+b^2}{12r}+r}$$

In the figure below, we plot T as a function of r, for a = 0.35 m and b = 0.45 m.



(b) The minimum of T can be located by setting its derivative to zero, $dT/dr = 0$. This yields

$$r = \sqrt{\frac{a^2+b^2}{12}} = \sqrt{\frac{(0.35\text{ m})^2 + (0.45\text{ m})^2}{12}} = 0.16\text{ m.}$$

(c) The direction from the center does not matter, so the locus of points is a circle around the center, of radius $[(a^2 + b^2)/12]^{1/2}$.

53. Replacing x and v in Eq. 15-3 and Eq. 15-6 with θ and $d\theta/dt$, respectively, we identify 4.44 rad/s as the angular frequency ω. Then we evaluate the expressions at $t = 0$ and divide the second by the first:

$$\left(\frac{d\theta/dt}{\theta}\right)_{\text{at } t=0} = -\,\omega \tan\phi \ .$$

(a) The value of θ at $t = 0$ is 0.0400 rad, and the value of $d\theta/dt$ then is –0.200 rad/s, so we are able to solve for the phase constant:

$$\phi = \tan^{-1}[0.200/(0.0400 \times 4.44)] = 0.845 \text{ rad.}$$

(b) Once ϕ is determined we can plug back in to $\theta_o = \theta_m \cos\phi$ to solve for the angular amplitude. We find $\theta_m = 0.0602$ rad.

54. We note that the initial angle is $\theta_o = 7° = 0.122$ rad (though it turns out this value will cancel in later calculations). If we approximate the initial stretch of the spring as the arc-length that the corresponding point on the plate has moved through ($x = r\theta_o$ where $r =$ 0.025 m) then the initial potential energy is approximately $\frac{1}{2}kx^2 =$ 0.0093 J. This should equal to the kinetic energy of the plate ($\frac{1}{2}I\omega_m^2$ where this ω_m is the maximum angular speed of the plate, not the angular frequency ω). Noting that the maximum angular speed of the plate is $\omega_m = \omega\theta_o$ where $\omega = 2\pi/T$ with $T = 20$ ms = 0.02 s as determined from the graph, then we can find the rotational inertial from $\frac{1}{2}\, I\ \omega_m^2$ = 0.0093 J. Thus, $I = 1.3\times10^{-5}\ \text{kg}\cdot\text{m}^2$.

55. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I / mgd}$, where I is its rotational inertia, m = 22.1 g is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ with L = 2.20 m. According to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus,

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}.$$

Minimizing T with respect to d, $dT/d(d)$=0, we obtain $d = L/\sqrt{12}$. Therefore, the minimum period T is

$$T_{\min} = 2\pi\sqrt{\frac{L^2 + 12(L/\sqrt{12})^2}{12g(L/\sqrt{12})}} = 2\pi\sqrt{\frac{2L}{\sqrt{12}g}} = 2\pi\sqrt{\frac{2(2.20\ \text{m})}{\sqrt{12}(9.80\ \text{m/s}^2)}} = 2.26\ \text{s}.$$

(b) If d is chosen to minimize the period, then as L is increased the period will increase as well.

(c) The period does not depend on the mass of the pendulum, so T does not change when m increases.

56. The table of moments of inertia in Chapter 11, plus the parallel axis theorem found in that chapter, leads to

$$I_P = \tfrac{1}{2}MR^2 + Mh^2 = \tfrac{1}{2}(2.5\text{ kg})(0.21\text{ m})^2 + (2.5\text{ kg})(0.97\text{ m})^2 = 2.41\text{ kg}\cdot\text{m}^2$$

where P is the hinge pin shown in the figure (the point of support for the physical pendulum), which is a distance $h = 0.21\text{ m} + 0.76\text{ m}$ away from the center of the disk.

(a) Without the torsion spring connected, the period is

$$T = 2\pi\sqrt{\frac{I_P}{Mgh}} = 2.00\text{ s} .$$

(b) Now we have two "restoring torques" acting in tandem to pull the pendulum back to the vertical position when it is displaced. The magnitude of the torque-sum is $(Mgh + \kappa)\theta = I_P\,\alpha$, where the small angle approximation ($\sin\theta \approx \theta$ in radians) and Newton's second law (for rotational dynamics) have been used. Making the appropriate adjustment to the period formula, we have

$$T' = 2\pi\sqrt{\frac{I_P}{Mgh + \kappa}} .$$

The problem statement requires $T = T' + 0.50$ s. Thus, $T' = (2.00 - 0.50)\text{s} = 1.50$ s. Consequently,

$$\kappa = \frac{4\pi^2}{T'^2} I_P - Mgh = 18.5\ \text{N}\cdot\text{m/rad} .$$

57. (a) We want to solve $e^{-bt/2m} = 1/3$ for t. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore, $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$. Thus,

$$t = \frac{2(1.50 \text{ kg})}{0.230 \text{ kg/s}} \ln 3 = 14.3 \text{ s}.$$

(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00 \text{ N/m}}{1.50 \text{ kg}} - \frac{(0.230 \text{ kg/s})^2}{4(1.50 \text{ kg})^2}} = 2.31 \text{ rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31 \text{ rad/s}) = 2.72$ s and the number of oscillations is

$$t/T = (14.3 \text{ s})/(2.72 \text{ s}) = 5.27.$$

58. Referring to the numbers in Sample Problem 15-7, we have m = 0.25 kg, b = 0.070 kg/s and T = 0.34 s. Thus, when t = 20T, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39.$$

59. Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m}.$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6.0 %.

60. (a) From Hooke's law, we have

$$k = \frac{(500 \text{ kg})(9.8 \text{ m/s}^2)}{10 \text{cm}} = 4.9 \times 10^2 \text{ N/cm}.$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'}.$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000 \text{ N/m}}{500 \text{ kg}}} \approx 9.9 \text{ rad/s},$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T} \ln 2 \approx \frac{2(500 \text{ kg})}{0.63 \text{ s}} (0.69) = 1.1 \times 10^3 \text{ kg/s}.$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 15-43 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2 \ln 2 \sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086 \text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

61. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

62. With $\omega = 2\pi/T$ then Eq. 15-28 can be used to calculate the angular frequencies for the given pendulums. For the given range of $2.00 < \omega < 4.00$ (in rad/s), we find only two of the given pendulums have appropriate values of ω: pendulum (d) with length of 0.80 m (for which ω = 3.5 rad/s) and pendulum (e) with length of 1.2 m (for which ω = 2.86 rad/s).

63. With $M = 1000$ kg and $m = 82$ kg, we adapt Eq. 15-12 to this situation by writing

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{M+4m}} \; .$$

If $d = 4.0$ m is the distance traveled (at constant car speed v) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M+4m}} \;\Rightarrow\; k = (M+4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with $v = 16000/3600 = 4.44$ m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M+4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050 \text{ m}.$$

64. Its total mechanical energy is equal to its maximum potential energy $\frac{1}{2}kx_m^2$, and its potential energy at $t = 0$ is $\frac{1}{2}kx_o^2$ where $x_o = x_m\cos(\pi/5)$ in this problem. The ratio is therefore $\cos^2(\pi/5) = 0.655 = 65.5\%$.

65. (a) From the graph, we find x_m = 7.0 cm = 0.070 m, and T = 40 ms = 0.040 s. Thus, the angular frequency is $\omega = 2\pi/T$ = 157 rad/s. Using m = 0.020 kg, the maximum kinetic energy is then $\frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 x_m^2 = 1.2$ J.

(b) Using Eq. 15-5, we have $f = \omega/2\pi$ = 50 oscillations per second. Of course, Eq. 15-2 can also be used for this.

66. (a) From the graph we see that $x_m = 7.0$ cm = 0.070 m and $T = 40$ ms = 0.040 s. The maximum speed is $x_m\omega = x_m 2\pi/T = 11$ m/s.

(b) The maximum acceleration is $x_m\omega^2 = x_m(2\pi/T)^2 = 1.7 \times 10^3$ m/s^2.

67. Setting 15 mJ (0.015 J) equal to the maximum kinetic energy leads to v_{max} = 0.387 m/s. Then one can use either an "exact" approach using $v_{max} = \sqrt{2gL(1 - \cos(\theta_{max}))}$ or the "SHM" approach where

$$v_{max} = L\omega_{max} = L\omega\theta_{max} = L\sqrt{g/L}\,\theta_{max}$$

to find L. Both approaches lead to L = 1.53 m.

68. Since $\omega = 2\pi f$ where $f = 2.2$ Hz, we find that the angular frequency is $\omega = 13.8$ rad/s. Thus, with $x = 0.010$ m, the acceleration amplitude is $a_m = x_m\,\omega^2 = 1.91$ m/s^2. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g.$$

69. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m + M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050\text{ kg})(150\text{ m/s})}{0.050\text{ kg} + 4.0\text{ kg}} = 1.85\text{ m/s}.$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M + m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m.$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050\text{ kg}+4.0\text{ kg})(1.85\text{ m/s})^2}{500\text{ N/m}} - \frac{(0.050\text{ kg})(9.8\text{ m/s}^2)^2}{(500\text{ N/m})^2}\left[2(4.0\text{ kg})+0.050\text{ kg}\right]} \\ &= 0.166\text{ m}. \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\text{ kg})(150\text{ m/s})^2 = 563\text{ J}$. The kinetic energy of the bullet-block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\text{ kg}+4.0\text{ kg})(1.85\text{ m/s})^2 = 6.94\text{ J}.$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is

$$E/E_0 = (6.94\text{ J})/(563\text{ J}) = 0.0123 \text{ or } 1.23\%.$$

70. (a) We note that

$$\omega = \sqrt{k/m} = \sqrt{1500/0.055} = 165.1 \text{ rad/s}.$$

We consider the most direct path in each part of this problem. That is, we consider in part (a) the motion directly from $x_1 = +0.800x_m$ at time t_1 to $x_2 = +0.600x_m$ at time t_2 (as opposed to, say, the block moving from $x_1 = +0.800x_m$ through $x = +0.600x_m$, through $x = 0$, reaching $x = -x_m$ and after returning back through $x = 0$ then getting to $x_2 = +0.600x_m$). Eq. 15-3 leads to

$$\omega t_1 + \phi = \cos^{-1}(0.800) = 0.6435 \text{ rad}$$

$$\omega t_2 + \phi = \cos^{-1}(0.600) = 0.9272 \text{ rad} .$$

Subtracting the first of these equations from the second leads to

$$\omega(t_2 - t_1) = 0.9272 - 0.6435 = 0.2838 \text{ rad} .$$

Using the value for ω computed earlier, we find $t_2 - t_1 = 1.72 \times 10^{-3}$ s.

(b) Let t_3 be when the block reaches $x = -0.800x_m$ in the direct sense discussed above. Then the reasoning used in part (a) leads here to

$$\omega(t_3 - t_1) = (2.4981 - 0.6435) \text{ rad} = 1.8546 \text{ rad}$$

and thus to $t_3 - t_1 = 11.2 \times 10^{-3}$ s.

71. (a) The problem gives the frequency f = 440 Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency ω is similar to frequency except that ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2.8\times10^3$ rad/s.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With x_m = 0.00075 m and the above value for ω, this expression yields v_m = 2.1 m/s.

(c) In the discussion immediately after Eq. 15-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value ω = 2765 rad/s is used) yields a_m = 5.7 km/s.

72. (a) The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \left(2\pi(1000 \text{ Hz})\right)^2 (0.00040 \text{ m}) = 1.6\times10^4 \text{ m/s}^2.$$

(b) Similarly, in the discussion after Eq. 15-6, we find $v_m = \omega x_m$ so that

$$v_m = \left(2\pi(1000 \text{ Hz})\right)(0.00040 \text{ m}) = 2.5 \text{ m/s}.$$

(c) From Eq. 15-8, we have (in absolute value)

$$|a| = \left(2\pi(1000 \text{ Hz})\right)^2 (0.00020 \text{ m}) = 7.9\times10^3 \text{ m/s}^2.$$

(d) This can be approached with the energy methods of §15-4, but here we will use trigonometric relations along with Eq. 15-3 and Eq. 15-6. Thus, allowing for both roots stemming from the square root,

$$\sin(\omega t+\phi) = \pm\sqrt{1-\cos^2(\omega t+\phi)} \quad\Rightarrow\quad -\frac{v}{\omega x_m} = \pm\sqrt{1-\frac{x^2}{x_m^2}}.$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi(1000)\sqrt{0.00040^2 - 0.00020^2} = 2.2 \text{ m/s}.$$

73. (a) The rotational inertia is $I = \frac{1}{2} MR^2 = \frac{1}{2}(3.00 \text{ kg})(0.700 \text{ m})^2 = 0.735 \text{ kg} \cdot \text{m}^2$.

(b) Using Eq. 15-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600 \text{ N} \cdot \text{m}}{2.5 \text{ rad}} = 0.0240 \text{ N} \cdot \text{m/rad}.$$

(c) Using Eq. 15-5, Eq. 15-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024 \text{N} \cdot \text{m/rad}}{0.735 \text{kg} \cdot \text{m}^2}} = 0.181 \text{ rad/s}.$$

74. (a) We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = R = 0.126$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.873\,\text{s}.$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2}.$$

Thus, our result is $r = 0.126/2 = 0.0630$ m.

75. (a) The frequency for small amplitude oscillations is $f=(1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives

$$f=(1/2\pi)\sqrt{(9.80\ \mathrm{m/s^2})/(2.0\ \mathrm{m})}=0.35\ \mathrm{Hz}.$$

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T}+m\vec{g}=m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a}=\vec{a}_e+\vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g}-\vec{a}_e)+\vec{T}=\ m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g}-\vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f=(1/2\pi)\sqrt{g/L}$. Thus

$$f=\frac{1}{2\pi}\sqrt{\frac{g+a_e}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.8\,\mathrm{m/s^2}+2.0\,\mathrm{m/s^2}}{2.0\,\mathrm{m}}}=0.39\ \mathrm{Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g}-\vec{a}_e=0$. To find the frequency for small amplitude oscillations, replace g with zero in $f=(1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

76. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_o = x_m = 0.37$ cm. It is straightforward to infer from this that the phase constant ϕ in Eq. 15-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable t is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \pi/2$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Eq. 15-3 becomes $x = (0.37 \text{ cm}) \cos(\pi t/2)$.

(e) The derivative of x is $v = -(0.37 \text{ cm/s})(\pi/2) \sin(\pi t/2) \approx (-0.58 \text{ cm/s}) \sin(\pi t/2)$.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91 \text{ cm/s}^2$.

(h) Making sure our calculator is in radians mode, we find $x = (0.37) \cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of π in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58 \text{ cm/s})\sin(\pi(3.0)/2) = 0.58$ cm/s.

77. Since $T = 0.500$ s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so m = 0.0500 kg and $v_m = 0.150$ m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = (4\pi \text{ rad/s})^2 (0.0500 \text{ kg}) = 7.90 \text{ N/m}.$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m}.$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

78. (a) Hooke's law readily yields (0.300 kg)(9.8 m/s^2)/(0.0200 m) = 147 N/m.

(b) With m = 2.00 kg, the period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.733\ \text{s}.$$

79. Using $\Delta m = 2.0$ kg, $T_1 = 2.0$ s and $T_2 = 3.0$ s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}}\,.$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m+\Delta m}{m}}$$

which (after squaring both sides) simplifies to $m = \dfrac{\Delta m}{(T_2/T_1)^2 - 1} = 1.6\,\text{kg}.$

80. (a) Comparing with Eq. 15-3, we see $\omega = 10$ rad/s in this problem. Thus, $f = \omega/2\pi = 1.6$ Hz.

(b) Since $v_m = \omega x_m$ and $x_m = 10$ cm (see Eq. 15-3), then $v_m = (10 \text{ rad/s})(10 \text{ cm}) = 100$ cm/s or 1.0 m/s.

(c) The maximum occurs at $t = 0$.

(d) Since $a_m = \omega^2 x_m$ then $v_m = (10 \text{ rad/s})^2(10 \text{ cm}) = 1000 \text{ cm/s}^2$ or 10 m/s^2.

(e) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or $x = \pm 10$ cm.

(f) Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (0.10 \text{ kg})(10 \text{ rad / s})^2 = 10 \text{ N / m}.$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

81. (a) We require $U = \frac{1}{2}E$ at some value of x. Using Eq. 15-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \Rightarrow x = \frac{x_m}{\sqrt{2}}.$$

We compare the given expression x as a function of t with Eq. 15-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/\sqrt{2} \approx 3.5\text{ m}$.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54\text{ s}.$$

Since we are asked for the interval $t_{eq} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{eq} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29\text{ s}.$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

82. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law:

$$\Delta x = F/k = (0.20 \text{ kg})(9.8 \text{ m/s}^2)/(19 \text{ N/m}) = 0.103 \text{ m}.$$

(a) The body, once released, will not only fall through the Δx distance but continue through the equilibrium position to a "turning point" equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 15-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6 \text{ Hz}.$$

(c) The maximum distance from the equilibrium position is the amplitude: $x_m = \Delta x = 0.10$ m.

83. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is 180/(60 s) = 3.0 Hz and the amplitude is half the stroke, or x_m = 0.38 m. Thus,

$$v_m = 2\pi(3.0 \text{ Hz})(0.38 \text{ m}) = 7.2 \text{ m/s}.$$

84. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2} I\omega^2 + \frac{1}{2} kx^2$$

and θ is in radians. We note that $r\omega = v$ (where $v = dx/dt$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{mR^2}{r^2}\right)v^2 + \frac{1}{2} kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in §15-4 *if* we identify the mass m in that section with the term mR^2/r^2 appearing in this problem. Making this identification, Eq. 15-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}.$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back towards its equilibrium position).

85. (a) Hooke's law readily yields

$$k = (15\text{ kg})(9.8\text{ m/s}^2)/(0.12\text{ m}) = 1225\text{ N/m}.$$

Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

(b) We are told f = 2.00 Hz = 2.00 cycles/sec. Since a cycle is equivalent to 2π radians, we have ω = 2π(2.00) = 4π rad/s (understood to be valid to three significant figures). Using Eq. 15-12, we find

$$\omega=\sqrt{\frac{k}{m}} \quad\Rightarrow\quad m=\frac{1225\text{ N/m}}{(4\pi\text{ rad/s})^2}=7.76\text{kg}.$$

Consequently, the weight of the package is mg = 76.0 N.

86. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is

$$\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n + 1),$$

where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n + 1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n + 1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. With $k = 8600$ N/m and $n = L_1/L_2 = 0.70$, we obtain

$$k_1 = \left(\frac{n+1}{n}\right)k = \left(\frac{0.70+1.0}{0.70}\right)(8600\text{ N/m}) = 20886\text{ N/m} \approx 2.1\times10^4\,\text{N/m}$$

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n + 1)$, then solve for k_2. The result is $k_2 = k(n + 1)$.

$$k_2 = (n+1)k = (0.70+1.0)(8600\text{ N/m}) = 14620\text{ N/m} \approx 1.5\times10^4\,\text{N/m}$$

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n + 1)/n$. With $f = (1/2\pi)\sqrt{k/m}$, we obtain, for $f = 200$ Hz and $n = 0.70$

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}\,f = \sqrt{\frac{0.70+1.0}{0.70}}(200\text{ Hz}) = 3.1\times10^2\text{ Hz}.$$

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n + 1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}\,f = \sqrt{0.70+1.0}(200\text{ Hz}) = 2.6\times10^2\,\text{Hz}.$$

87. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x = (10000 \text{ kg})(9.8 \text{ m/s}^2)\sin\theta$$

where θ = 30° (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3\omega_x$ causes the cable to stretch x = 0.15 m. Since the cable is expected to obey Hooke's law, its spring constant is

$$k = \frac{3w_x}{x} = 9.8\times10^5 \text{ N/m}.$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 15-12 (divided by 2π).

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\times10^5 \text{ N/m}}{20000 \text{ kg}}} = 1.1 \text{ Hz}.$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x = \frac{3w_x - 2w_x}{k} = 0.050 \text{ m}.$$

88. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{eff} = \sqrt{g^2 + (v^2 / R)^2}.$$

Then, since frequency is the reciprocal of the period, Eq. 15-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}}.$$

With v = 70 m/s, R = 50m, and L = 0.20 m, we have $f \approx 3.5\ \text{s}^{-1} = 3.5$ Hz.

89. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where $y = 0.096$ m is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3$ kg is the mass of the block. Thus

$$k = mg/y = (1.3 \text{ kg})(9.8 \text{ m/s}^2)/(0.096 \text{ m}) = 1.33 \times 10^2 \text{ N/m}.$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3 \text{ kg}}{133 \text{ N/m}}} = 0.62 \text{ s}.$$

(c) The frequency is $f = 1/T = 1/0.62$ s $= 1.6$ Hz.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3 \text{ kg})(9.8 \text{ m/s}^2)(0.146 \text{ m}) + \frac{1}{2}(133 \text{ N/m})(0.146 \text{ m})^2 = -0.44 \text{ J}.$$

When the block is at the equilibrium point, the elongation of the spring is $y = 9.6$ cm and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3 \text{ kg})(9.8 \text{ m/s}^2)(0.096 \text{ m}) + \frac{1}{2}(133 \text{ N/m})(0.096 \text{ m})^2 = -0.61 \text{ J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44 \text{ J} + 0.61 \text{ J})}{1.3 \text{kg}}} = 0.51 \text{ m/s}.$$

90. (a) The Hooke's law force (of magnitude (100)(0.30) = 30 N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v = 0$) at $x = -x_m$ where the amplitude is $x_m = 0.30 - 0.20 = 0.10$ m.

(c) Using Eq. 15-13 with $m = W/g \approx 2.0$ kg, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.90 \text{ s}.$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}(100 \text{ N / m})(0.10 \text{ m})^2 = 0.50 \text{ J}.$$

91. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given v = 5.2 m/s at x = 0 is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20 \text{ rad/s}$).

(a) Since $\omega = 2\pi f$, we find f = 3.2 Hz.

(b) We have v_m = 5.2 m/s = (20 rad/s)x_m, which leads to x_m = 0.26 m.

(c) With meters, seconds and radians understood,

$$\begin{aligned} x &= (0.26 \text{ m})\cos(20t + \phi) \\ v &= -(5.2 \text{ m/s})\sin(20t + \phi). \end{aligned}$$

The requirement that x = 0 at t = 0 implies (from the first equation above) that either ϕ = $+\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that v > 0 when t = 0; that choice is $\phi = -\pi/2$. Therefore,

$$x = (0.26 \text{ m})\cos\left(20t - \frac{\pi}{2}\right) = (0.26 \text{ m})\sin(20t).$$

92. (a) Eq. 15-21 leads to

$$E = \frac{1}{2}kx_m^2 \quad \Rightarrow \quad x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0\text{ J})}{200\text{ N/m}}} = 0.20\text{ m}.$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80\text{ kg}/200\text{ N/m}} \approx 0.4\text{ s}$, then the block completes 10/0.4 = 25 cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \Rightarrow 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

Therefore, when $x = 0.15$ m, we find $v = 2.1$ m/s.

93. The time for one cycle is T = (50 s)/20 = 2.5 s. Thus, from Eq. 15-23, we find

$$I = \kappa \left(\frac{T}{2\pi} \right)^2 = (0.50)\left(\frac{2.5}{2\pi} \right)^2 = 0.079 \text{ kg} \cdot \text{m}^2 .$$

94. The period formula, Eq. 15-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From the figure, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M\left(L+R\right)^2,$$

where $M = 1.0\text{ kg}$. Thus, Eq. 15-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M\left(L+R\right)^2}{Mg\left(L+R\right)}}$$

which leads to $L = 0.8315$ m.

95. (a) By Eq. 15-13, the mass of the block is

$$m_b = \frac{kT_0^2}{4\pi^2} = 2.43 \text{ kg}.$$

Therefore, with $m_p = 0.50$ kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44 \text{ s}.$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, $v_0 = 3.14$ m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision.

$$V = v_0 \frac{m_b}{m_p + m_b} = 2.61 \text{ m/s}.$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

96. (a) Hooke's law provides the spring constant: $k = (20\text{ N})/(0.20\text{ m}) = 1.0\times10^2$ N/m.

(b) The attached mass is $m = (5.0\text{ N})/(9.8\text{ m/s}^2) = 0.51$ kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51\text{ kg}}{100\text{ N/m}}} = 0.45\text{ s}.$$

97. (a) Hooke's law provides the spring constant:

$$k = (4.00 \text{ kg})(9.8 \text{ m/s}^2)/(0.160 \text{ m}) = 245 \text{ N/m}.$$

(b) The attached mass is $m = 0.500$ kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284 \text{ s}.$$

98. (a) We are told that when $t = 4T$, with $T = 2\pi / \omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 15-43),

$$e^{-bt/2m} = \frac{3}{4}.$$

Thus,

$$T \approx 2\pi\sqrt{(2.00\text{kg})/(10.0\,\text{N/m})} = 2.81\,\text{s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \quad \Rightarrow \quad b = \frac{2(2.00\text{ kg})(0.288)}{4(2.81\,\text{s})} = 0.102\text{ kg/s}.$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{m\,o}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313\text{ J}$. At $t = 4T$,

$$E = \tfrac{1}{2}k(\tfrac{3}{4}x_{m\,o})^2 = 0.176\text{ J}.$$

Therefore, $E_o - E = 0.137$ J.

99. Since d_m is the amplitude of oscillation, then the maximum acceleration being set to $0.2g$ provides the condition: $\omega^2 d_m = 0.2g$. Since d_s is the amount the spring stretched in order to achieve vertical equilibrium of forces, then we have the condition $kd_s = mg$. Since we can write this latter condition as $m\omega^2 d_s = mg$, then $\omega^2 = g/d_s$. Plugging this into our first condition, we obtain

$$d_s = d_m/0.2 = (10\text{ cm})/0.2 = 50\text{ cm}.$$

100. We note (from the graph) that $a_{\mathrm{m}} = \omega^2 x_m = 4.00$ cm/s^2. Also the value at $t = 0$ is $a_{\mathrm{o}} = 1.00$ cm/s^2. Then Eq. 15-7 leads to

$$\phi = \cos^{-1}(-1.00/4.00) = +1.82 \text{ rad or } -4.46 \text{ rad.}$$

The other "root" (+4.46 rad) can be rejected on the grounds that it would lead to a negative slope at $t = 0$.

101. (a) The graphs suggest that T = 0.40 s and κ = 4/0.2 = 0.02 N·m/rad. With these values, Eq. 15-23 can be used to determine the rotational inertia:

$$I = \kappa T^2/4\pi^2 = 8.11 \times 10^{-5}\ \text{kg·m}^2.$$

(b) We note (from the graph) that θ_{max} = 0.20 rad. Setting the maximum kinetic energy ($\frac{1}{2}I\omega_{max}^2$) equal to the maximum potential energy (see the hint in the problem) leads to $\omega_{max} = \theta_{max}\sqrt{\kappa/I}$ = 3.14 rad/s.

102. The angular frequency of the simple harmonic oscillation is given by Eq. 15-13:

$$\omega = \sqrt{\frac{k}{m}}\ .$$

Thus, for two different masses m_1 and m_2, with the same spring constant *k*, the ratio of the frequencies would be

$$\frac{\omega_1}{\omega_2} = \frac{\sqrt{k/m_1}}{\sqrt{k/m_2}} = \sqrt{\frac{m_2}{m_1}}\ .$$

In our case, with $m_1 = m$ and $m_2 = 2.5m$, the ratio is $\dfrac{\omega_1}{\omega_2} = \sqrt{\dfrac{m_2}{m_1}} = \sqrt{2.5} = 1.58$.

103. For simple harmonic motion, Eq. 15-24 must reduce to

$$\tau = -L\left(F_g \sin\theta\right) \rightarrow -L\left(F_g \theta\right)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{\left(-LF_g \sin\theta\right) - \left(-LF_g\theta\right)}{-LF_g \sin\theta}\right| = \left|1 - \frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1 - \frac{\theta}{\theta - \theta^3/6}\right| \approx 0.010 \Rightarrow \frac{1}{1 - \theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24 \text{ rad} = 14.0°$. A more accurate value (found numerically) for the θ value which results in a 1.0% deviation is 13.986°.

104. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) The period of the simple harmonic oscillator is given by Eq. 15-13:

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of x is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -2.0 \times 10^2$ m/s^2.

(e) The graph shows $v_m = 6.28$ m/s, so

$$K_m = \frac{1}{2} m v_m^2 = 4.0 \text{ J}.$$

105. (a) From the graph, it is clear that $x_m = 0.30$ m.

(b) With $F = -kx$, we see k is the (negative) slope of the graph — which is 75/0.30 = 250 N/m. Plugging this into Eq. 15-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.28 \text{ s.}$$

(c) As discussed in §15-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \frac{k}{m} x_m = 1.5\times10^2 \text{ m/s}^2.$$

Alternatively, we could arrive at this result using $a_m = (2\pi/T)^2\, x_m$.

(d) Also in §15-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2$$

which yields 11.3 ≈ 11 J. We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

106. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\frac{1}{2}kx_m^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}^2\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$$
$$= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3$ = 0.125 J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625\ \text{J}$.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125\text{J} \approx 3.13\times10^{-2}\ \text{J}$.

(c) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{3}{4}Mv_{\mathbf{cm}}^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right) = \frac{3}{2}Mv_{\mathbf{cm}}a_{\mathbf{cm}} + kxv_{\mathbf{cm}} = 0$$

which leads to

$$a_{\text{cm}} = -\left(\frac{2k}{3M}\right)x.$$

Comparing with Eq. 15-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

107. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45\text{ s}$.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass *m* attached) is *mg*/*k*, where in this problem *mg* = 10 N and *k* = 200 N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish *x* = 0 at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; i.e.,

$$E = K + U\ ,$$

where $U = kx^2/2$. Thus, as the block passes through the unstretched position, the energy is $E = 2.0 + \frac{1}{2}k(0.05)^2 = 2.25\text{ J}$. At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \Rightarrow x_m = \pm 0.15\text{ m}.$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

108. Using Eq. 15-12, we find $\omega = \sqrt{k/m} = 10 \text{ rad/s}$. We also use $v_m = x_m\omega$ and $a_m = x_m\omega^2$.

(a) The amplitude (meaning "displacement amplitude") is $x_m = v_m/\omega = 3/10 = 0.30$ m.

(b) The acceleration-amplitude is $a_m = (0.30 \text{ m})(10 \text{ rad/s})^2 = 30 \text{ m/s}^2$.

(c) One interpretation of this question is "what is the most negative value of the acceleration?" in which case the answer is $-a_m = -30 \text{ m/s}^2$. Another interpretation is "what is the smallest value of the absolute-value of the acceleration?" in which case the answer is zero.

(d) Since the period is $T = 2\pi/\omega = 0.628$ s. Therefore, seven cycles of the motion requires $t = 7T = 4.4$ s.

109. The mass is $m = \dfrac{0.108 \text{ kg}}{6.02 \times 10^{23}} = 1.8 \times 10^{-25}$ kg. Using Eq. 15-12 and the fact that $f = \omega/2\pi$, we have

$$1\times10^{13} \text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi\times10^{13}\right)^2\left(1.8\times10^{-25}\right) \approx 7\times10^2 \text{ N/m}.$$

110. (a) Eq. 15-28 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17m}{9.8\,\mathrm{m/s^2}}} = 8.3\text{ s.}$$

(b) Plugging $I = mL^2$ into Eq. 15-25, we see that the mass m cancels out. Thus, the characteristics (such as the period) of the periodic motion do not depend on the mass.

111. (a) The net horizontal force is F since the batter is assumed to exert no horizontal force on the bat. Thus, the horizontal acceleration (which applies as long as F acts on the bat) is $a = F/m$.

(b) The only torque on the system is that due to F, which is exerted at P, at a distance $L_o - \frac{1}{2}L$ from C. Since $L_o = 2L/3$ (see Sample Problem 15-5), then the distance from C to P is $\frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. Since the net torque is equal to the rotational inertia ($I = 1/12mL^2$ about the center of mass) multiplied by the angular acceleration, we obtain

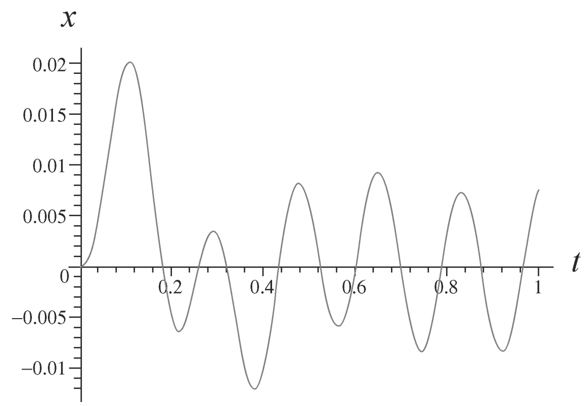
$$\alpha = \frac{\tau}{I} = \frac{F\left(\frac{1}{6}L\right)}{\frac{1}{12}mL^2} = \frac{2F}{mL}.$$

(c) The distance from C to O is $r = L/2$, so the contribution to the acceleration at O stemming from the angular acceleration (in the counterclockwise direction of Fig. 15-11) is $\alpha r = \frac{1}{2}\alpha L$ (leftward in that figure). Also, the contribution to the acceleration at O due to the result of part (a) is F/m (rightward in that figure). Thus, if we choose rightward as positive, then the net acceleration of O is

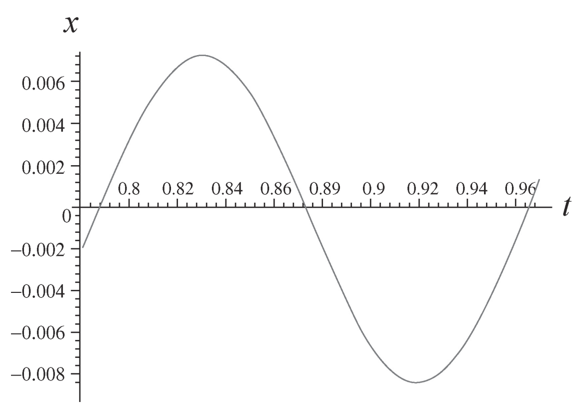
$$a_O = \frac{F}{m} - \frac{1}{2}\alpha L = \frac{F}{m} - \frac{1}{2}\left(\frac{2F}{mL}\right)L = 0.$$

(d) Point O stays relatively stationary in the batting process, and that might be possible due to a force exerted by the batter or due to a finely tuned cancellation such as we have shown here. We assumed that the batter exerted no force, and our first expectation is that the impulse delivered by the impact would make all points on the bat go into motion, but for this particular choice of impact point, we have seen that the point being held by the batter is naturally stationary and exerts no force on the batter's hands which would otherwise have to "fight" to keep a good hold of it.

112. (a) A plot of x versus t (in SI units) is shown below:

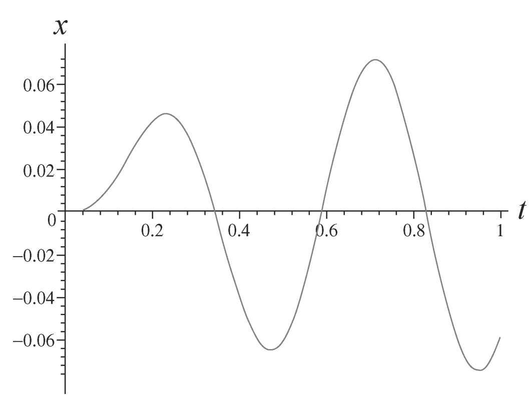


If we expand the plot near the end of that time interval we have



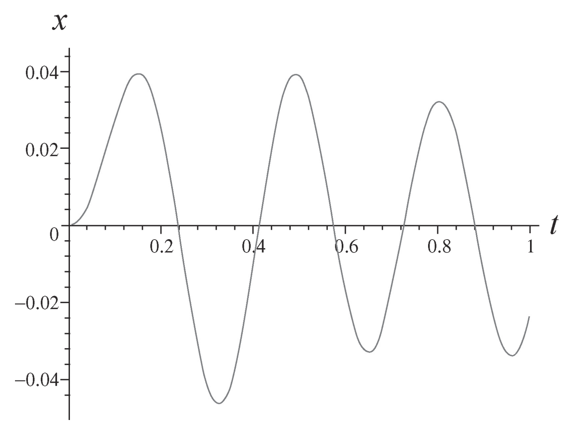
This is close enough to a regular sine wave cycle that we can estimate its period ($T = 0.18$ s, so $\omega = 35$ rad/s) and its amplitude ($y_m = 0.008$ m).

(b) Now, with the new driving frequency ($\omega_d = 13.2$ rad/s), the x versus t graph (for the first one second of motion) is as shown below:



It is a little more difficult in this case to estimate a regular sine-curve-like amplitude and period (for the part of the above graph near the end of that time interval), but we arrive at roughly $y_m = 0.07$ m, $T = 0.48$ s, and $\omega = 13$ rad/s.

(c) Now, with $\omega_d = 20$ rad/s, we obtain (for the behavior of the graph, below, near the end of the interval) the estimates: $y_m = 0.03$ m, $T = 0.31$ s, and $\omega = 20$ rad/s.



Chapter 16

1. (a) The angular wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \dfrac{\lambda\omega}{2\pi} = \dfrac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

2. The distance d between the beetle and the scorpion is related to the transverse speed v_t and longitudinal speed v_ℓ as

$$d = v_t t_t = v_\ell t_\ell$$

where t_t and t_ℓ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150 \text{ m/s}}{50 \text{ m/s}} = 3.0 .$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0 t_\ell - t_\ell = 2.0 t_\ell = 4.0\times10^{-3} \text{ s} \quad \Rightarrow \quad t_\ell = 2.0\times10^{-3} \text{ s},$$

then $d = v_\ell t_\ell = (150 \text{ m/s})(2.0\times10^{-3} \text{ s}) = 0.30 \text{ m} = 30 \text{ cm}.$

3. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T = 4(0.170\text{ s}) = 0.680\text{ s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

4. (a) The speed of the wave is the distance divided by the required time. Thus,

$$v = \frac{853 \text{ seats}}{39 \text{ s}} = 21.87 \text{ seats/s} \approx 22 \text{ seats/s}.$$

(b) The width w is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit. Thus,

$$w = vt = (21.87 \text{ seats/s})(1.8 \text{ s}) \approx 39 \text{ seats}.$$

5. Let $y_1 = 2.0$ mm (corresponding to time t_1) and $y_2 = -2.0$ mm (corresponding to time t_2). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0) .$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2 = 0.011$ s (or 1.1 ms).

6. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives

$$u = -\omega y_m \cos(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{du}{dt} = \frac{d(-\omega y_m \cos(-\omega t + \phi))}{dt} = -y_m \omega^2 \sin(-\omega t + \phi) \quad > 0 \text{ at } t = 0.$$

This implies that $-\sin\phi > 0$ and consequently that ϕ is in either the third or fourth quadrant. The graph shows (at t = 0) $u = -4$ m/s, and (at some later t) $u_{max} = 5$ m/s. We note that $u_{max} = y_m \omega$. Therefore,

$$u = -u_{max} \cos(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

7. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700 \text{ m} = 700 \text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

8. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100\text{ cm})\,(2.0\text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\max} = 2\pi\, f y_m = \left(4.0\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) $y(3.5\text{ cm}, 0.26\text{ s}) = (6.0\text{ cm})\sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

9. (a) Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16}{0.040} = 400\,\text{rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = 80/64 = 1.26$ m ≈ 1.3 m.

(c) The amplitude of the transverse displacement is $y_m = 4.0\ \text{cm} = 4.0\times10^{-2}\,\text{m}$.

(d) The wave number is $k = 2\pi/\lambda = 5.0$ rad/m.

(e) The angular frequency, as obtained in part (a), is $\omega = 16/0.040 = 4.0\times10^{2}$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin\left(5x - 400t + \phi\right)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.040$ at $x = t = 0$. Therefore, $\sin\phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

10. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t) .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2} .$$

Therefore, where $y = 12$, u must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

11. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, i.e., $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = k\,v = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times 10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right [in the $+x$ direction] – see section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times 10^2\, t) .$$

12. The slope that they are plotting is the physical slope of sinusoidal waveshape (not to be confused with the more abstract "slope" of its time development; the physical slope is an x-derivative whereas the more abstract "slope" would be the t-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d\, y_m \sin(kx - \omega t)}{dx} = y_m\, k \cos(kx - \omega t) .$$

The problem additionally gives $t = 0$, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40 \text{ m}} = 15.7 \text{ rad/m}.$$

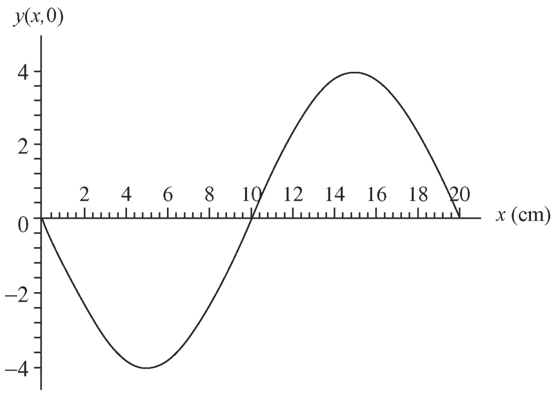
Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude y_m. We find

$$y_m = \frac{0.20}{15.7 \text{ rad/m}} = 0.0127 \text{ m} \approx 1.3 \text{ cm}.$$

13. From Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, i.e., $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At t =0, we then have



$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a negative sine function. A plot of y(x,0) is depicted on the right.

(b) From the figure we see that the amplitude is y_m = 4.0 cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of y with respect to t, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0,5.0) = -2.5$ cm/s.

14. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

15. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope:

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m}.$$

Thus,

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

16. The volume of a cylinder of height ℓ is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho \pi d^2 \, \ell/4}{\ell} = \frac{\pi \rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

17. (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

18. We use $v=\sqrt{\tau/\mu}\propto\sqrt{\tau}$ to obtain

$$\tau_2=\tau_1\left(\frac{v_2}{v_1}\right)^2=(120\,\mathrm{N})\left(\frac{180\,\mathrm{m/s}}{170\,\mathrm{m/s}}\right)^2=135\mathrm{N}.$$

19. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\ \text{m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\ \text{rad/s})/(2.0\ \text{m}^{-1}) = 15\ \text{m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\ \text{kg/m}\right)\left(15\ \text{m/s}\right)^2 = 0.036\ \text{N}.$$

20. (a) Comparing with Eq. 16-2, we see that k = 20/m and ω = 600/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017\,\text{kg/m} = 17\,\text{g/m}.$$

21. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})\,(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9\times10^2$ rad/s

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin\phi$. We wish to select ϕ so that

$$5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}.$$

The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x,t) = \left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

22. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at $x = 10$ cm, becomes $y(x = 10 \text{ cm}, t) = y_m \sin[k(10 \text{ cm} - \omega t)]$. Comparing this with the expression given, we find $\omega = 4.0$ rad/s, or $f = \omega/2\pi = 0.64$ Hz.

(b) Since $k(10 \text{ cm}) = 1.0$, the wave number is $k = 0.10$/cm. Consequently, the wavelength is $\lambda = 2\pi/k = 63$ cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is $k = 0.10$/cm.

(e) The angular frequency is $\omega = 4.0$ rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of k and ω into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t) = 5.0 \sin(0.10x - 4.0t).$$

(g) Since $v = \omega/k = \sqrt{\tau/\mu}$, the tension is

$$\tau = \frac{\omega^2 \mu}{k^2} = \frac{(4.0 \text{ g/cm})(4.0 \text{ s}^{-1})^2}{(0.10 \text{ cm}^{-1})^2} = 6400 \text{ g} \cdot \text{cm/s}^2 = 0.064 \text{ N}.$$

23. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30$ ms. If this time is denoted by t_0 then the coordinate of this wave at time t is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v = \sqrt{\frac{\tau L}{m}} = \sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}} = 158\,\text{m/s}.$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x = \frac{10.0\,\text{m} + (158\,\text{m/s})(30.0\times10^{-3}\,\text{s})}{2} = 7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L - x = (10.0\text{ m} - 7.37\text{ m}) = 2.63$ m.

24. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1\,\text{m/s}.$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1 = \frac{M}{1+\mu_2/\mu_1} = \frac{500\,\text{g}}{1+5.00/3.00} = 187.5\,\text{g} \approx 188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313\text{ g}$.

25. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by μgy, so the tension is $\tau = \mu gy$. The wave speed is $v = \sqrt{\mu gy/\mu} = \sqrt{gy}$.

(b) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $\mathrm{d}t = \mathrm{d}y/v = \mathrm{d}y/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{\mathrm{d}y}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\,\Bigg|_0^L = 2\sqrt{\frac{L}{g}}\,.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

27. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}} = 500$ Hz, and that the wavelength $\lambda = 0.20$ m. We also note from the graph that the maximum value of dK/dt is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\tfrac{1}{2}\mu\, v\, \omega^2\, y_m{}^2 = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2 \mu\lambda\, f^3}} = 0.0032 \text{ m} .$$

28. Comparing $y(x,t) = (3.00 \text{ mm})\sin[(4.00 \text{ m}^{-1})x - (7.00 \text{ s}^{-1})t]$ to the general expression $y(x,t) = y_m \sin(kx - \omega t)$, we see that $k = 4.00 \text{ m}^{-1}$ and $\omega = 7.00 \text{ rad/s}$. The speed of the wave is

$$v = \omega / k = (7.00 \text{ rad/s})/(4.00 \text{ m}^{-1}) = 1.75 \text{ m/s}.$$

29. The wave $y(x,t) = (2.00 \text{ mm})[(20 \text{ m}^{-1})x - (4.0 \text{ s}^{-1})t]^{1/2}$ is of the form $h(kx - \omega t)$ with angular wave number $k = 20 \text{ m}^{-1}$ and angular frequency $\omega = 4.0 \text{ rad/s}$. Thus, the speed of the wave is

$$v = \omega / k = (4.0 \text{ rad/s})/(20 \text{ m}^{-1}) = 0.20 \text{ m/s}.$$

30. The wave $y(x,t) = (4.00 \text{ mm})\, h[(30 \text{ m}^{-1})x + (6.0 \text{ s}^{-1})t]$ is of the form $h(kx - \omega t)$ with angular wave number $k = 30 \text{ m}^{-1}$ and angular frequency $\omega = 6.0 \text{ rad/s}$. Thus, the speed of the wave is

$$v = \omega / k = (6.0 \text{ rad/s})/(30 \text{ m}^{-1}) = 0.20 \text{ m/s}.$$

31. The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \ = 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41 y_m .$$

32. (a) Let the phase difference be ϕ. Then from Eq. 16–52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to $1.45 \text{ rad}/2\pi = 0.230$ wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that λ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude y_m. The amplitude of their resultant wave, as shown in the figure, is y'_m = 4.00 mm. Eq. 16-52 applies:

$$y'_m = 2\,y_m \cos(\tfrac{1}{2}\phi_2) \quad\Rightarrow\quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, ϕ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100t) \quad \text{and} \quad y_2 = (0.00900)\sin(16x + 1100t + 2.7)\ .$$

34. (a) We use Eq. 16-26 and Eq. 16-33 with μ = 0.00200 kg/m and y_m = 0.00300 m. These give $v = \sqrt{\tau / \mu}$ = 775 m/s and

$$P_{\text{avg}} = \tfrac{1}{2}\,\mu v\,\omega^2 y_m{}^2 = 10\text{ W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); P = 20 W.

(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-16(a)) then the amplitude y_m is doubled which means its square $y_m{}^2$ increases by a factor of 4. Thus, the answer now is four times that of part (a); P = 40 W.

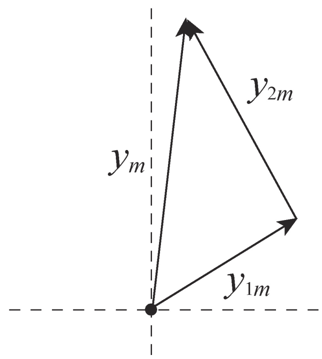
(d) Eq. 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi)$ = 1.618 times the original amplitude y_m. Squared, this results in an increase in the power by a factor of 2.618. Thus, P = 26 W in this case.

(e) Now the situation depicted in Fig. 16-16(b) applies, so P = 0.

35. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus y_m = 5.0 cm.



36. (a) As shown in Figure 16-16(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

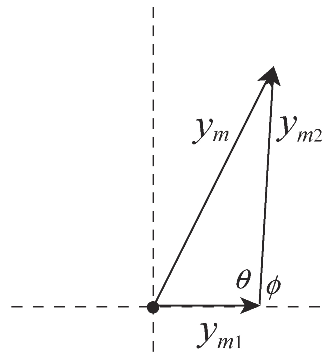
(c) As shown in Figure 16-16(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

(e) Using phasor terminology, the angle "between them" in this case is π/2 rad (90º), so the Pythagorean theorem applies:

$$\sqrt{(8.0\text{ mm})^2 + (5.0\text{ mm})^2} = 9.4\text{ mm}.$$

37. The phasor diagram is shown on the right. We use the cosine theorem:



$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$

We solve for $\cos\phi$:

$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.

38. We see that y_1 and y_3 cancel (they are 180º) out of phase, and y_2 cancels with y_4 because their phase difference is also equal to π rad (180º). There is no resultant wave in this case.

39. (a) Using the phasor technique, we think of these as two “vectors” (the first of “length” 4.6 mm and the second of “length” 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad).

(c) Clearly, it should in “in phase” with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

40. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\text{N}}{2.00 \times 10^{-3}\,\text{kg}/1.25\,\text{m}}} = 66.1\,\text{m/s}.$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where $L = 125$ cm. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\ \text{m/s}}{2(1.25\ \text{m})} = 26.4\ \text{Hz}.$$

41. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\text{ N}}{(10.0\text{ m})\,(0.100\text{ kg})}} = n\ (7.91\text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\text{ Hz}) = 23.7$ Hz.

42. The *n*th resonant frequency of string *A* is

$$f_{n,A} = \frac{v_A}{2l_A} n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string *B* it is

$$f_{n,B} = \frac{v_B}{2l_B} n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4} f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of *B* matches the frequency of *A*'s first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of *B* would match $f_{3,A} = \dfrac{3v_A}{2l_A} = \dfrac{3}{2L}\sqrt{\dfrac{\tau}{\mu}},$

43. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\text{ N})\,(8.40\text{ m})}{0.120\text{ kg}}} = 82.0\text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\text{ m}) = 16.8\text{ m}$.

(c) The frequency is $f = v/\lambda = (82.0\text{ m/s})/(16.8\text{ m}) = 4.88\text{ Hz}$.

44. The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T = 2(0.50 \text{ s}) = 1.0$ s. Thus, $f = 1/T = 1.0$ Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10 \text{ cm.}$$

45. (a) Eq. 16–26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\text{N}}{7.20\times 10^{-3}\ \text{kg/m}}} = 144.34\ \text{m/s} \approx 1.44\times 10^{2}\ \text{m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0\ \text{cm}) = 60.0\ \text{cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times 10^{2}\ \text{m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

46. Use Eq. 16–66 (for the resonant frequencies) and Eq. 16–26 $(v = \sqrt{\tau/\mu})$ to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3.$$

(b) And we get the new wavelength $\lambda_3' = \dfrac{v'}{f_3'} = \dfrac{2L}{3} = \lambda_3$.

47. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315 \text{ Hz}}{420 \text{ Hz} - 315 \text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315 \text{ Hz})/3 = 105$ Hz.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then

$$v = \lambda f = 2Lf = 2(0.75 \text{ m})(105 \text{ Hz}) = 158 \text{ m/s}.$$

48. Using Eq. 16-26, we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{65.2\times10^{6}\ \text{N}}{3.35\,\text{kg/m}}} = 4412\,\text{m/s}.$$

The corresponding resonant frequencies are

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The wavelength of the wave with the lowest (fundamental) resonant frequency f_1 is $\lambda_1 = 2L$, where $L = 347$ m. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{4412\ \text{m/s}}{2(347\ \text{m})} = 6.36\ \text{Hz}.$$

(b) The frequency difference between successive modes is

$$\Delta f = f_n - f_{n-1} = \frac{v}{2L} = \frac{4412\ \text{m/s}}{2(347\ \text{m})} = 6.36\ \text{Hz}.$$

49. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus, f_1 = (390 Hz – 325 Hz) = 65 Hz. This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

50. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16–23(b), where (see Eq. 16–65)

$$\lambda = L \quad \text{and} \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$ then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \Rightarrow \lambda = 4.0\,\text{m} \Rightarrow L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$ then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \Rightarrow v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16–26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\ \text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

The period is $T = 1/f = 0.11$ s.

51. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3$ cm^{-1}. The wave speed is

$$v = \omega/k = (40\pi \text{ rad/s})/(\pi/3 \text{ cm}^{-1}) = 1.2\times10^{2} \text{ cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3 \text{ cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by $u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi \text{ rad/s})\,(0.50\,\text{cm})\,\sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\left[\left(40\pi \text{ s}^{-1}\right)\left(\frac{9}{8}\text{ s}\right)\right] = 0.$$

52. The nodes are located from vanishing of the spatial factor sin $5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of x which corresponds to a node is $x = 0$.

(b) The second smallest value of x which corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of x which corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16–58 through Eq. 16–60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s $= 0.025$ s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s $= 0.050$ s.

53. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The amplitude y_m is half the maximum displacement of the standing wave, or 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L = 3.0$m, $\lambda = 2.0$ m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}.$$

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100 \text{ m/s})}{2(3.0 \text{ m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}.$$

(d) The two waves are

$$y_1 = \left(5.0\times10^{-3} \text{ m}\right)\sin\left[\left(3.14 \text{ m}^{-1}\right)x - \left(314 \text{ s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3} \text{ m}\right)\sin\left[\left(3.14 \text{ m}^{-1}\right)x + \left(314 \text{ s}^{-1}\right)t\right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

54. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form $y(x,t) = -(0.04)\cos(kx)\sin(\omega t)$, where $\omega = 2\pi / T = \pi$ rad/s, with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20\text{ m, }0.50\text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040\text{ m}.$$

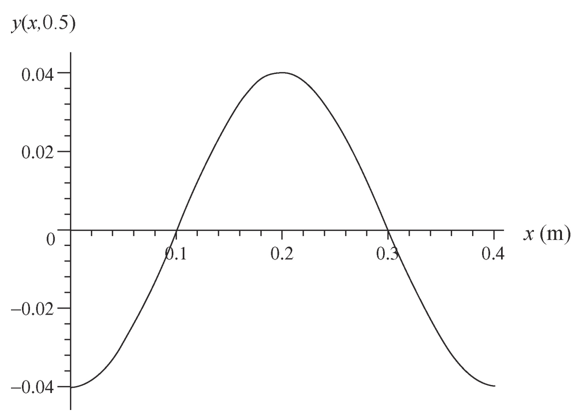
(b) The above equation yields $y(0.30\text{ m, }0.50\text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$.

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega\cos(kx)\cos(\omega t) = 0.$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown below:



55. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is

$$f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00 \text{ Hz}.$$

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi \text{ m}^{-1}$, so the wavelength is

$$\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00 \text{ m}.$$

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using $\cos\alpha = \sin(\alpha + \pi/2)$, then apply

$$\begin{aligned}\cos\alpha + \cos\beta &= \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) \\ &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\end{aligned}$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m\cos(kx+\omega t) + y_m\cos(kx-\omega t) = 2y_m\cos(kx)\cos(\omega t).$$

Nodes occur where $\cos(kx) = 0$ or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi \text{ m}^{-1}$, this means $x = \left(n + \tfrac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of x which corresponds to a node is $x = 0.500$ m (n=0).

(e) The second smallest value of x which corresponds to a node is $x = 1.50$ m (n=1).

(f) The third smallest value of x which corresponds to a node is $x = 2.50$ m (n=2).

(g) The displacement is a maximum where $\cos(kx) = \pm 1$. This means $kx = n\pi$, where n is an integer. Thus, $x = n(1.00 \text{ m})$. The smallest value of x which corresponds to an anti-node (maximum) is $x = 0$ (n=0).

(h) The second smallest value of x which corresponds to an anti-node (maximum) is $x = 1.00$ m (n=1).

(i) The third smallest value of x which corresponds to an anti-node (maximum) is $x = 2.00$ m (n=2).

56. Reference to point A as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Using Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90$ cm $= 9.0$ mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so $T = 12$ ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t]$$

and

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in y_1, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in y_2. Therefore, the sign in front of ω is minus.

57. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(k\,x \pm \omega\,t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point – which occurs during a half-period. Since the period T is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\,\pi} = 0.500 \text{ s} .$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \frac{\omega}{k} = 1.00$ m/s. Therefore, $\Delta x = (1.00 \text{ m/s})(0.250 \text{ s}) = 0.250$ m.

58. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The mass that allows the oscillator to set up the 4th harmonic ($n = 4$) on the string is

$$m = \left.\frac{4L^2 f^2 \mu}{n^2 g}\right|_{n=4} = \frac{4(1.20\ \text{m})^2(120\ \text{Hz})^2(0.00160\ \text{kg/m})}{(4)^2(9.80\ \text{m/s}^2)} = 0.846\ \text{kg}$$

(b) If the mass of the block is $m = 1.00\ \text{kg}$, the corresponding n is

$$n = \sqrt{\frac{4L^2 f^2 \mu}{g}} = \sqrt{\frac{4(1.20\ \text{m})^2(120\ \text{Hz})^2(0.00160\ \text{kg/m})}{9.80\ \text{m/s}^2}} = 3.68$$

which is not an integer. Therefore, the mass cannot set up a standing wave on the string.

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1 v_1/2f,$$

where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1 v_1/2L_1$. A similar expression holds for the steel section: $f = n_2 v_2/2L_2$. Since the frequency is the same for the two sections, $n_1 v_1/L_1 = n_2 v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$$

and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times10^3\ \text{kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\ \text{kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is

$$f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}.$$

The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus,

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\text{kg/m}^3)(1.00\times10^{-6}\text{m}^2)}} = 324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

60. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

The mass that allows the oscillator to set up the nth harmonic on the string is

$$m = \frac{4L^2 f^2 \mu}{n^2 g}.$$

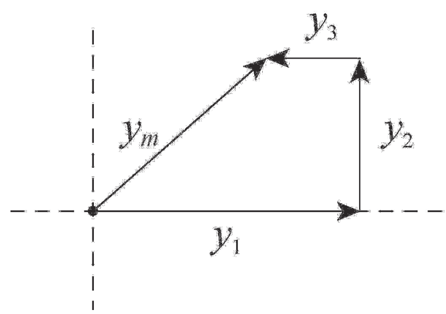
Thus, we see that the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number n then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n+1}{n^2}.$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer ($2n + 1$) divided by a squared integer (n^2). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging $m = 0.447$ kg, $n = 4$, and the other values given in the problem, we find

$$\mu = 0.000845 \text{ kg/m} = 0.845 \text{ g/m}.$$

61. (a) The phasor diagram is shown here: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave.



The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

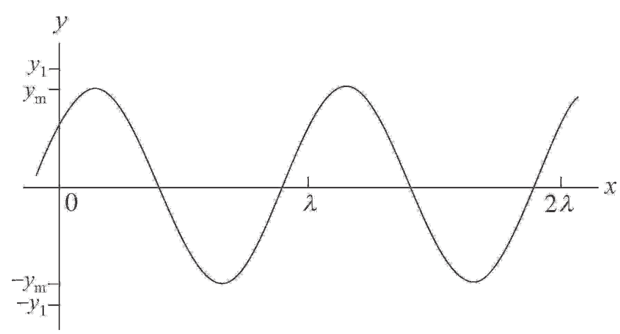
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



62. Setting $x = 0$ in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{dy}{dt} = \frac{d\, y_m \sin(-\omega t + \phi)}{dt} = -y_m \omega \cos(-\omega t + \phi) \quad > 0 \text{ at } t = 0.$$

This implies that $-\cos(\phi) > 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $y = 2.00$ mm, and (at some later t) $y_m = 6.00$ mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{3}\right) = 0.34 \text{ rad} \quad \text{or} \quad 2.8 \text{ rad}$$

(bear in mind that $\sin(\theta) = \sin(\pi - \theta)$), and we must choose $\phi = 2.8$ rad because this is about 161° and is in second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.8 - 2\pi = -3.48$ rad is also an acceptable result.

63. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m \cos\left(\frac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\frac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

64. Setting $x = 0$ in $a_y = -\omega^2 y$ (see the solution to part (b) of Sample Problem 16-2) where $y = y_m \sin(kx - \omega t + \phi)$ gives $a_y = -\omega^2 y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a negative "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{d\,a_y}{d\,t} = \frac{d\,(-\omega^2 y_m \sin(-\omega t + \phi))}{d\,t} = y_m \omega^3 \cos(-\omega t + \phi) \quad < 0 \text{ at } t = 0.$$

This implies that $\cos\phi < 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at t = 0) $a_y = -100$ m/s², and (at another t) $a_{max} = 400$ m/s². Therefore,

$$a_y = -a_{max} \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}(\tfrac{1}{4}) = 0.25 \text{ rad} \quad \text{or} \quad 2.9 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

65. We note that

$$dy/dt = -\omega\cos(kx - \omega t + \phi),$$

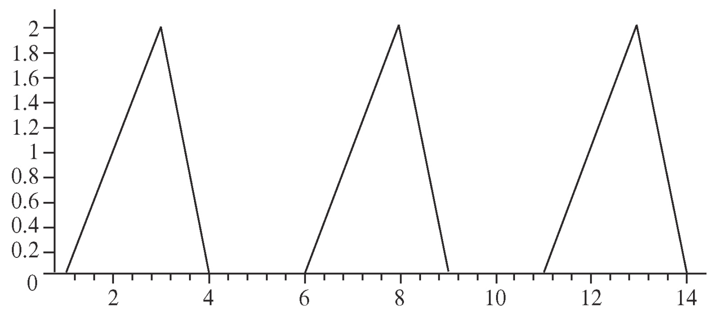
which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega\, y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2 \text{ rad}.$$

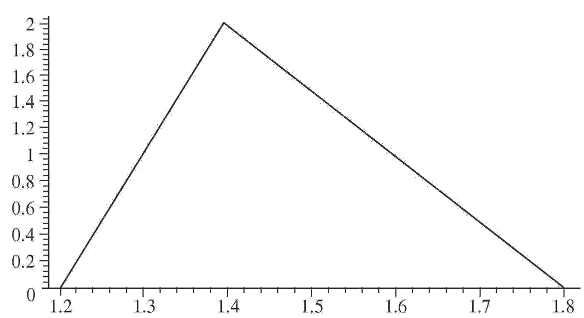
66. (a) Recalling the discussion in §16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where x is in centimeters and t is in seconds) must be 5.0 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16–48 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0 \text{ s}$. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.



(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4.0)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3.0)/5 = 1.4$ s. Finally, the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1.0)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



67. (a) The displacement of the string is assumed to have the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f = 120$ Hz and the angular frequency is $\omega = 2\pi f = 2\pi\,(120 \text{ Hz}) = 754$ rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754 \text{ rad/s})\,(5.00 \times 10^{-3} \text{ m}) = 3.77 \text{ m/s}.$$

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = k y_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = k y_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus,

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\,\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is

$$\tau_{\text{trans}} = (90.0 \text{ N}) \sin 7.83° = 12.3 \text{ N}.$$

We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is

$$u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\, \Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}} \frac{\Delta y}{\Delta t} = \tau_{\text{trans}} u.$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

68. We use Eq. 16-52 in interpreting the figure.

(a) Since $y' = 6.0$ mm when $\phi = 0$, then Eq. 16-52 can be used to determine $y_m = 3.0$ mm.

(b) We note that $y' = 0$ when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, $\lambda = 20$ cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since $f = 120$ Hz, $\omega = 2\pi f = 754$ rad/s $\approx 7.5\times10^2$ rad/s.

(d) The sign in front of ω is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm}) \sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]]$ (this applies to each wave when they are in phase).

69. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t),$$

and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

70. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is y_m = 2.0 cm = 0.020 m, as given in the problem.

(b) The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10 \text{ m}) = 63 \text{ m}^{-1}$

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400 \text{ Hz}) = 2510 \text{ rad/s} = 2.5\times10^3 \text{ rad/s}$.

(d) A minus sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left((62.8\,\text{m}^{-1})x - (2510\,\text{s}^{-1})t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of y:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m$ = (2510 rad/s)(0.020 m) = 50 m/s.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

71. (a) The amplitude is y_m = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is f = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the $-x$ direction, the sign in front of ω is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$
\begin{aligned}
y(x,t) = y_{\text{m}} \sin(kx+\omega t) &= y_{\text{m}} \sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] \\
&= (0.010\,\text{m})\sin\left[2\pi(550\,\text{Hz})\left(\frac{x}{330\,\text{m/s}}+t\right)\right] \\
&= (0.010\,\text{m})\ \sin[(10.5 \text{ rad/s})\, x + (3.46\times10^3 \text{ rad/s})t].
\end{aligned}
$$

72. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$(3.0\ \text{mm}) + (5.0\ \text{mm})\cos(70°) = 4.71\,\text{mm}\ \ \text{along}\ x\ \text{axis}$$
$$(5.0\ \text{mm})\sin(70°) = 4.70\ \text{mm}\ \ \text{along}\ y\ \text{axis.}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71\ \text{mm})^2 + (4.70\ \text{mm})^2} = 6.7\,\text{mm}$.

(b) And the angle (phase constant) is $\tan^{-1}(4.70/4.71) = 45°$.

73. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75\ \text{Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

74. By Eq. 16–66, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880$ Hz.

(b) The frequency of the third harmonic is and $f_3 = 3(440) = 1320$ Hz.

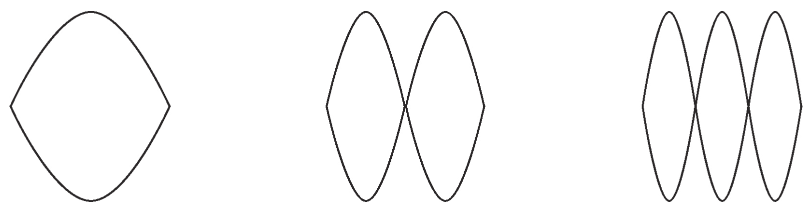
75. We make use of Eq. 16–65 with $L = 120$ cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown below:



76. (a) At $x = 2.3$ m and $t = 0.16$ s the displacement is

$$y(x,t) = 0.15\sin\left[(0.79)(2.3) - 13(0.16)\right]\text{m} = -0.039\,\text{m}.$$

(b) We choose $y_m = 0.15$ m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79\ \text{m}^{-1}$,

(d) and $\omega = 13$ rad/s .

(e) The wave must be traveling in $-x$ direction, implying a plus sign in front of ω.

Thus, its general form is $y'(x,t) = (0.15\text{ m})\sin(0.79x + 13t)$.

(f) The displacement of the standing wave at $x = 2.3$ m and $t = 0.16$ s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

77. (a) The wave speed is

$$v=\sqrt{\frac{\tau}{\mu}}=\sqrt{\frac{120\,\text{N}}{8.70\times10^{-3}\,\text{kg}/1.50\,\text{m}}}=144\ \text{m/s}.$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\text{ m}) = 3.00\text{ m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\text{ m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\text{ m/s})/(3.00\text{ m}) = 48.0\text{ Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\text{ m/s})/(1.50\text{ m}) = 96.0\text{ Hz}$.

78. We use $P = \frac{1}{2}\mu v \omega^2 y_m^2 \propto v f^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1$.

79. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed u.

(a) The amplitude is y_m = 2.0 mm.

(b) Since ω = 600 rad/s, the frequency is found to be f = 600/2π ≈ 95 Hz.

(c) Since k = 20 rad/m, the velocity of the wave is $v = \omega/k$ = 600/20 = 30 m/s in the $+x$ direction.

(d) The wavelength is λ = 2π/k ≈ 0.31 m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is u_m = (600)(2.0) = 1200 mm/s, or 1.2 m/s.

80. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx)\cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is

$$k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$$

and the angular frequency is

$$\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800 \text{ rad/s}.$$

With $y_m = 2.0$ mm, the displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos[(3800\,\text{s}^{-1})t].$$

81. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049 \text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27 \text{ m/s})^2(0.049 \text{ kg/m}) = 36 \text{ N}.$$

82. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = (2000\,\text{m/s})(40\times10^{-6}\,\text{s}) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve a = 300/40 × 10^{-6}) we find the stopping distance d:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2(40\times10^{-6})}{2(300)}$$

which gives d = 6.0×10^{-3} m. This and the radius r form the legs of a right triangle (where r is opposite from θ = 60°). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d\tan 60° = 1.0\times10^{-2}\,\text{m}.$$

83. (a) Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^{8}\ \mathrm{N/m^2}}{7800\,\mathrm{kg/m^3}}} = 3.00\times10^{2}\ \mathrm{m/s}$$

(b) The result does not depend on the diameter of the wire.

84. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then

$$z(y,t) = z_m \sin(ky - \omega t),$$

where $z_m = 3.0$ mm, $k = 60\text{ cm}^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20\text{ s} = 10\pi\text{ s}^{-1}$. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

85. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin(-\pi/4)$ which yields $a = 16.7$ m/s^2.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s^2.

86. Repeating the steps of Eq. 16-47 $\rightarrow$ Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative x, the smallest value to produce $\cos \pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x](-4\pi\sin 4\pi t)$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at x=0 has zero velocity is $t = 0$.

(c) Using the result obtained in (b), the second time where the velocity at x =0 vanishes would be $t = 0.25$ s,

(d) and the third time is $t = 0.50$ s.

87. (a) From the frequency information, we find $\omega = 2\pi f = 10\pi$ rad/s. A point on the rope undergoing simple harmonic motion (discussed in Chapter 15) has maximum speed as it passes through its "middle" point, which is equal to $y_m\omega$. Thus,

$$5.0 \text{ m/s} = y_m\omega \quad \Rightarrow \quad y_m = 0.16 \text{ m} \quad .$$

(b) Because of the oscillation being in the *fundamental* mode (as illustrated in Fig. 16-23(a) in the textbook), we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2 \text{ N}.$$

(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$, and we observe that the anti-node having zero displacement at $t = 0$ suggests the use of sine instead of cosine for the simple harmonic motion factor. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m}) \sin[(1.57 \text{ m}^{-1})x] \sin[(31.4 \text{ rad/s})t]$$

88. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Recalling from Ch. 11 the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad\Rightarrow\quad y_m = 3.2 \text{ cm.}$$

(c) As already noted, $f = 0.25$ Hz.

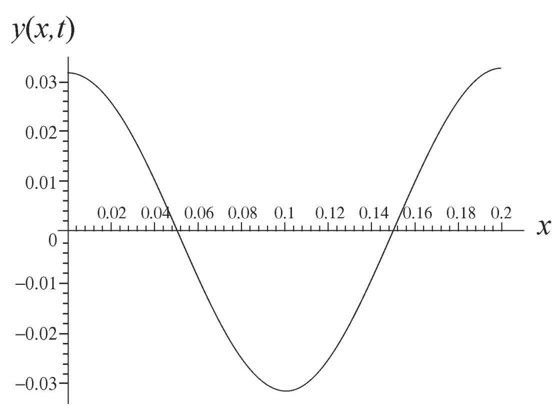
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the t and x terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is 0.050 $\sin \pi t/2$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050\sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at $t = 2.0$ s for $0 \le x \le 0.20$ m is shown below.



89. (a) The wave speed is

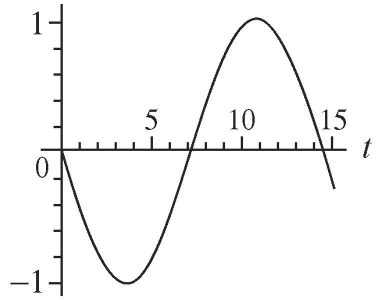
$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}}=\sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}.$$

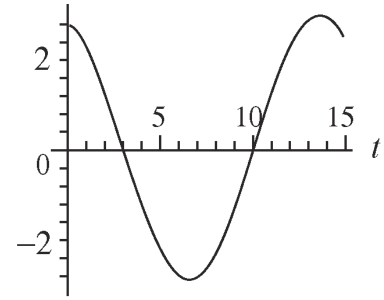
(b) The time required is

$$t=\frac{2\pi(\ell+\Delta\ell)}{v}=\frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}}=2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

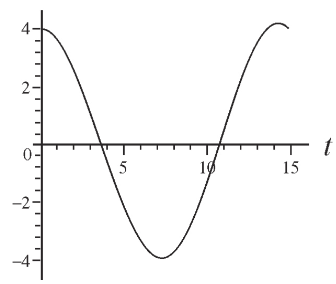
Thus if $\ell/\Delta\ell \gg 1$, then $t\propto\sqrt{\ell/\Delta\ell}\propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t\simeq 2\pi\sqrt{m/k}=$ const.

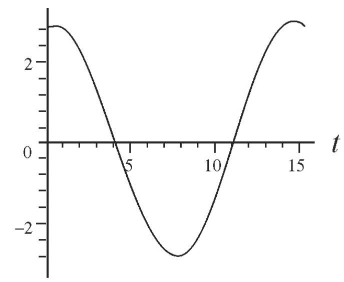
90. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250.3$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.

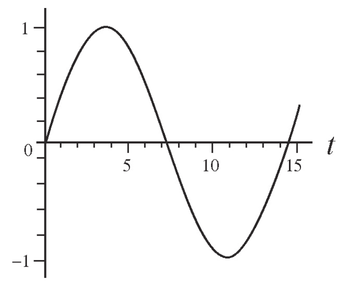




The following three graphs show the oscillation at $x = \lambda/4 = 62.6$ mm ≈ 63 mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.







(b) We can think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in §16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{max} = 4.0$ mm and occurs at $x = \lambda/4 = 62.6$ mm.

(e) The smallest amplitude of oscillation is $y_{min} = 1.0$ mm and occurs at $x = 0$ and at $x = \lambda/2 = 125$ mm.

(f) The largest amplitude can be related to the amplitudes of y_1 and y_2 in a simple way: $y_{max} = y_{1m} + y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is $y_{min} = y_{1m} - y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

91. Using Eq. 16-50, we have

$$y' = \left[0.60 \cos \frac{\pi}{6}\right] \sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be

$$0.60 \cos \frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}.$$

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12) $v = \dfrac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

92. (a) For visible light

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz}.$$

(b) For radio waves

$$\lambda_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\,\text{m}.$$

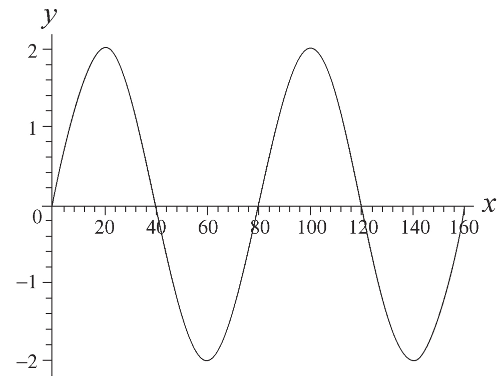
(c) For X rays

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{5.0\times10^{-9}\,\text{m}} = 6.0\times10^{16}\,\text{Hz}$$

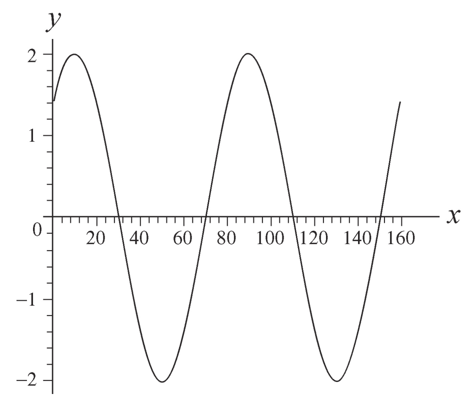
and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-11}\,\text{m}} = 3.0\times10^{19}\ \text{Hz}.$$

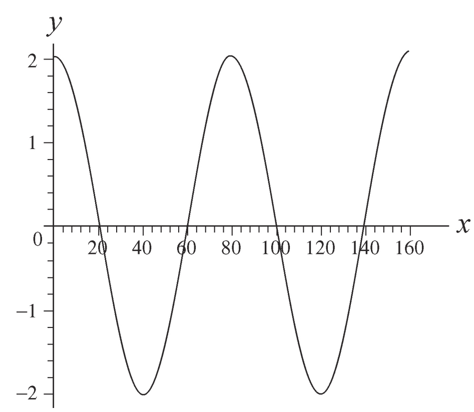
93. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at $t = 0$, and this next one is at $t = 0.050$ s.



And the final one, shown below, is at $t = 0.010$ s.



(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0 \times 10^2$ cm/s $= 2.0$ m/s

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the $-x$ direction.

Chapter 17

1. (a) When the speed is constant, we have $v = d/t$ where v = 343 m/s is assumed. Therefore, with t = 15/2 s being the time for sound to travel to the far wall we obtain d = (343 m/s) × (15/2 s) which yields a distance of 2.6 km.

(b) Just as the $\frac{1}{2}$ factor in part (a) was $1/(n + 1)$ for n = 1 reflection, so also can we write

$$d = (343\,\text{m/s})\left(\frac{15\,\text{s}}{n+1}\right) \quad \Rightarrow \quad n = \frac{(343)(15)}{d} - 1$$

for multiple reflections (with d in meters). For d = 25.7 m, we find n = 199 $\approx 2.0\times10^{2}$.

2. The time it takes for a soldier in the rear end of the column to switch from the left to the right foot to stride forward is t = 1 min/120 = 1/120 min = 0.50 s. This is also the time for the sound of the music to reach from the musicians (who are in the front) to the rear end of the column. Thus the length of the column is

$$l = vt = (343\,\text{m/s})(0.50\,\text{s}) = 1.7 \times 10^2\,\text{m}.$$

3. (a) The time for the sound to travel from the kicker to a spectator is given by d/v, where d is the distance and v is the speed of sound. The time for light to travel the same distance is given by d/c, where c is the speed of light. The delay between seeing and hearing the kick is $\Delta t = (d/v) - (d/c)$. The speed of light is so much greater than the speed of sound that the delay can be approximated by $\Delta t = d/v$. This means $d = v\,\Delta t$. The distance from the kicker to spectator A is

$$d_A = v\,\Delta t_A = (343\ \text{m/s})(0.23\ \text{s}) = 79\ \text{m}.$$

(b) The distance from the kicker to spectator B is $d_B = v\,\Delta t_B = (343\ \text{m/s})(0.12\ \text{s}) = 41\ \text{m}$.

(c) Lines from the kicker to each spectator and from one spectator to the other form a right triangle with the line joining the spectators as the hypotenuse, so the distance between the spectators is

$$D = \sqrt{d_A^2 + d_B^2} = \sqrt{(79\,\text{m})^2 + (41\,\text{m})^2} = 89\,\text{m}\,.$$

4. The density of oxygen gas is

$$\rho = \frac{0.0320\,\text{kg}}{0.0224\,\text{m}^3} = 1.43\,\text{kg/m}^3.$$

From $v = \sqrt{B/\rho}$ we find

$$B = v^2\rho = (317\,\text{m/s})^2\left(1.43\,\text{kg/m}^3\right) = 1.44\times10^5\ \text{Pa}.$$

5. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives

$$d = \frac{1}{2}gt_f^2 \quad \Rightarrow \quad t_f = \sqrt{2d/g}.$$

The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d. Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

$$gd^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for d. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.

6. Using Eqs. 16-13 and 17-3, the speed of sound can be expressed as

$$v = \lambda f = \sqrt{\frac{B}{\rho}},$$

where $B = -(dp/dV)/V$. Since V, λ and ρ are not changed appreciably, the frequency ratio becomes

$$\frac{f_s}{f_i} = \frac{v_s}{v_i} = \sqrt{\frac{B_s}{B_i}} = \sqrt{\frac{(dp/dV)_s}{(dp/dV)_i}}.$$

Thus, we have

$$\frac{(dV/dp)_s}{(dV/dp)_i} = \frac{B_i}{B_s} = \left(\frac{f_i}{f_s}\right)^2 = \left(\frac{1}{0.333}\right)^2 = 9.00.$$

7. If d is the distance from the location of the earthquake to the seismograph and v_s is the speed of the S waves then the time for these waves to reach the seismograph is $t_s. = d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

so

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(3.0\,\text{min})(60\,\text{s}/\text{min})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9\times10^3\ \text{km}.$$

We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

8. Let ℓ be the length of the rod. Then the time of travel for sound in air (speed v_s) will be $t_s = \ell / v_s$. And the time of travel for compressional waves in the rod (speed v_r) will be $t_r = \ell / v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12\,\text{s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right).$$

Thus, with v_s = 343 m/s and $v_r = 15v_s$ = 5145 m/s, we find $\ell = 44\,\text{m}$.

9. (a) Using $\lambda = v/f$, where v is the speed of sound in air and f is the frequency, we find

$$\lambda = \frac{343\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 7.62\times10^{-5}\ \text{m}.$$

(b) Now, $\lambda = v/f$, where v is the speed of sound in tissue. The frequency is the same for air and tissue. Thus

$$\lambda = (1500\ \text{m/s})/(4.50 \times 10^{6}\ \text{Hz}) = 3.33 \times 10^{-4}\ \text{m}.$$

10. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: $p_m = 1.50$ Pa.

(b) We identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi = 158$ Hz.

(c) We also obtain $\lambda = 2\pi/k = 2.22$ m.

(d) The speed of the wave is $v = \omega/k = 350$ m/s.

11. Without loss of generality we take $x = 0$, and let $t = 0$ be when $s = 0$. This means the phase is $\phi = -\pi/2$ and the function is $s = (6.0 \text{ nm})\sin(\omega t)$ at $x = 0$. Noting that $\omega = 3000$ rad/s, we note that at $t = \sin^{-1}(1/3)/\omega = 0.1133$ ms the displacement is $s = +2.0$ nm. Doubling that time (so that we consider the excursion from –2.0 nm to +2.0 nm) we conclude that the time required is 2(0.1133 ms) = 0.23 ms.

12. The key idea here is that the time delay Δt is due to the distance d that each wavefront must travel to reach your left ear (L) after it reaches your right ear (R).

(a) From the figure, we find $\Delta t = \frac{d}{v} = \frac{D \sin\theta}{v}$.

(b) Since the speed of sound in water is now v_w, with $\theta = 90°$, we have

$$\Delta t_w = \frac{D \sin 90°}{v_w} = \frac{D}{v_w}.$$

(c) The apparent angle can be found by substituting D / v_w for Δt:

$$\Delta t = \frac{D \sin\theta}{v} = \frac{D}{v_w}.$$

Solving for θ with $v_w = 1482 \text{ m/s}$ (see Table 17-1), we obtain

$$\theta = \sin^{-1}\left(\frac{v}{v_w}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1482 \text{ m/s}}\right) = \sin^{-1}(0.231) = 13°$$

13. (a) Consider a string of pulses returning to the stage. A pulse which came back just before the previous one has traveled an extra distance of $2w$, taking an extra amount of time $\Delta t = 2w/v$. The frequency of the pulse is therefore

$$f = \frac{1}{\Delta t} = \frac{v}{2w} = \frac{343\,\text{m/s}}{2(0.75\,\text{m})} = 2.3\times10^{2}\ \text{Hz}.$$

(b) Since $f \propto 1/w$, the frequency would be higher if w were smaller.

14. (a) The period is T = 2.0 ms (or 0.0020 s) and the amplitude is Δp_m = 8.0 mPa (which is equivalent to 0.0080 N/m^2). From Eq. 17-15 we get

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi/T)} = 6.1 \times 10^{-9}\ \text{m}\ .$$

where ρ = 1.21 kg/m^3 and v = 343 m/s.

(b) The angular wave number is $k = \omega/v = 2\pi/vT$ = 9.2 rad/m.

(c) The angular frequency is $\omega = 2\pi/T$ = 3142 rad/s $\approx 3.1 \times 10^3$ rad/s.

The results may be summarized as $s(x, t)$ = (6.1 nm) cos[(9.2 m^{-1})x – (3.1 × 10^3 s^{-1})t].

(d) Using similar reasoning, but with the new values for density (ρ' = 1.35 kg/m^3) and speed (v' = 320 m/s), we obtain

$$s_m = \frac{\Delta p_m}{v'\rho'\omega} = \frac{\Delta p_m}{v'\rho'(2\pi/T)} = 5.9 \times 10^{-9}\ \text{m}.$$

(e) The angular wave number is $k = \omega/v' = 2\pi/v'T$ = 9.8 rad/m.

(f) The angular frequency is $\omega = 2\pi/T$ = 3142 rad/s $\approx 3.1 \times 10^3$ rad/s.

The new displacement function is $s(x, t)$ = (5.9 nm) cos[(9.8 m^{-1})x – (3.1 × 10^3 s^{-1})t].

15. The problem says "At one instant.." and we choose that instant (without loss of generality) to be $t = 0$. Thus, the displacement of "air molecule A" at that instant is

$$s_A = +s_m = s_m\cos(kx_A - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_A + \phi),$$

where $x_A = 2.00$ m. Regarding "air molecule B" we have

$$s_B = +\tfrac{1}{3}s_m = s_m\cos(\,kx_B - \omega t + \phi\,)\big|_{t=0} = s_m\cos(\,kx_B + \phi\,).$$

These statements lead to the following conditions:

$$kx_A + \phi = 0$$
$$kx_B + \phi = \cos^{-1}(1/3) = 1.231$$

where $x_B = 2.07$ m. Subtracting these equations leads to

$$k(x_B - x_A) = 1.231 \quad \Rightarrow \quad k = 17.6 \text{ rad/m}.$$

Using the fact that $k = 2\pi/\lambda$ we find $\lambda = 0.357$ m, which means

$$f = v/\lambda = 343/0.357 = 960 \text{ Hz}.$$

Another way to complete this problem (once k is found) is to use $kv = \omega$ and then the fact that $\omega = 2\pi f$.

16. Let the separation between the point and the two sources (labeled 1 and 2) be x_1 and x_2, respectively. Then the phase difference is

$$\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{x_1}{\lambda} + ft\right) - 2\pi\left(\frac{x_2}{\lambda} + ft\right) = \frac{2\pi(x_1 - x_2)}{\lambda} = \frac{2\pi(4.40\,\text{m} - 4.00\,\text{m})}{(330\,\text{m/s})/540\,\text{Hz}} = 4.12\,\text{rad}.$$

17. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We note that at all points (at large distance from the origin) along the *x* axis there will be quiet ones; one way to see this is to note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value 3.5, implying a half-wavelength (180º) phase difference (destructive interference) between the waves. To distinguish the destructive interference along the +*x* axis from the destructive interference along the –*x* axis, we label one with +3.5 and the other –3.5. This labeling is useful in that it suggests that the complete enumeration of the quiet directions in the upper-half plane (including the *x* axis) is: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5. Similarly, the complete enumeration of the loud directions in the upper-half plane is: –3, –2, –1, 0, +1, +2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The discussion about the "quiet" directions was started in part (a). The number of values in the list: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5 along with –2.5, –1.5, –0.5, +0.5, +1.5, +2.5 (for the lower-half plane) is 14. There are 14 "quiet" directions.

18. At the location of the detector, the phase difference between the wave which traveled straight down the tube and the other one which took the semi-circular detour is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r).$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus,

$$r_{\min} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0\,\text{cm}}{2(\pi - 2)} = 17.5\,\text{cm}.$$

19. Let L_1 be the distance from the closer speaker to the listener. The distance from the other speaker to the listener is $L_2 = \sqrt{L_1^2 + d^2}$, where d is the distance between the speakers. The phase difference at the listener is $\phi = 2\pi(L_2 - L_1)/\lambda$, where λ is the wavelength.

For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where n is an integer. Thus,

$$\lambda = 2(L_2 - L_1)/(2n + 1).$$

The frequency is

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{2\left(\sqrt{L_1^2 + d^2} - L_1\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range. This means n ranges from 0 to 28.

(a) The lowest frequency that gives minimum signal is ($n = 0$) $f_{\min,1} = 343$ Hz.

(b) The second lowest frequency is ($n = 1$) $f_{\min,2} = [2(1)+1]343\text{ Hz} = 1029\text{ Hz} = 3f_{\min,1}$. Thus, the factor is 3.

(c) The third lowest frequency is (n=2) $f_{\min,3} = [2(2)+1]343\text{ Hz} = 1715\text{ Hz} = 5f_{\min,1}$. Thus, the factor is 5.

For a maximum in intensity at the listener, $\phi = 2n\pi$, where n is any positive integer. Thus $\lambda = (1/n)\left(\sqrt{L_1^2 + d^2} - L_1\right)$ and

$$f = \frac{v}{\lambda} = \frac{nv}{\sqrt{L_1^2 + d^2} - L_1} = \frac{n(343\,\text{m/s})}{\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, n must be in the range from 1 to 29 for the frequency to be audible.

(d) The lowest frequency that gives maximum signal is (n =1) $f_{\max,1} = 686$ Hz.

(e) The second lowest frequency is (n = 2) $f_{\max,2} = 2(686\text{ Hz}) = 1372\text{ Hz} = 2f_{\max,1}$. Thus, the factor is 2.

(f) The third lowest frequency is (n = 3) $f_{\max,3} = 3(686\text{ Hz}) = 2058\text{ Hz} = 3f_{\max,1}$. Thus, the factor is 3.

20. (a) To be out of phase (and thus result in destructive interference if they superpose) means their path difference must be $\lambda/2$ (or $3\lambda/2$ or $5\lambda/2$ or …). Here we see their path difference is L, so we must have (in the least possibility) $L = \lambda/2$, or $q = L/\lambda = 0.5$.

(b) As noted above, the next possibility is $L = 3\lambda/2$, or $q = L/\lambda = 1.5$.

21. Building on the theory developed in §17 – 5, we set $\Delta L/\lambda = n-1/2,\ n=1,2,...$ in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f_{\min,n} = \frac{(2n-1)v}{2\Delta L} = (n-1/2)(286\text{ Hz})$$

where we have used v = 343 m/s (note the remarks made in the textbook at the beginning of the exercises and problems section) and ΔL = (19.5 – 18.3) m = 1.2 m.

(a) The lowest frequency that gives destructive interference is (n = 1)

$$f_{\min,1} = (1-1/2)(286\text{ Hz}) = 143\text{ Hz}.$$

(b) The second lowest frequency that gives destructive interference is (n = 2)

$$f_{\min,2} = (2-1/2)(286\text{ Hz}) = 429\text{ Hz} = 3(143\text{ Hz}) = 3f_{\min,1}.$$

So the factor is 3.

(c) The third lowest frequency that gives destructive interference is (n = 3)

$$f_{\min,3} = (3-1/2)(286\text{ Hz}) = 715\text{ Hz} = 5(143\text{ Hz}) = 5f_{\min,1}.$$

So the factor is 5.

Now we set $\Delta L/\lambda = \frac{1}{2}$ (even numbers) — which can be written more simply as "(all integers n = 1, 2,…)" — in order to establish constructive interference. Thus,

$$f_{\max,n} = \frac{nv}{\Delta L} = n(286\text{ Hz}).$$

(d) The lowest frequency that gives constructive interference is (n =1) $f_{\max,1} = (286\text{ Hz})$.

(e) The second lowest frequency that gives constructive interference is (n = 2)

$$f_{\max,2} = 2(286\text{ Hz}) = 572\text{ Hz} = 2f_{\max,1}.$$

Thus, the factor is 2.

(f) The third lowest frequency that gives constructive interference is (n = 3)

$$f_{\max,3} = 3(286\text{ Hz}) = 858\text{ Hz} = 3f_{\max,1}.$$

Thus, the factor is 3.

22. (a) The problem indicates that we should ignore the decrease in sound amplitude which means that all waves passing through point P have equal amplitude. Their superposition at P if $d = \lambda/4$ results in a net effect of zero there since there are four sources (so the first and third are $\lambda/2$ apart and thus interfere destructively; similarly for the second and fourth sources).

(b) Their superposition at P if $d = \lambda/2$ also results in a net effect of zero there since there are an even number of sources (so the first and second being $\lambda/2$ apart will interfere destructively; similarly for the waves from the third and fourth sources).

(c) If $d = \lambda$ then the waves from the first and second sources will arrive at P in phase; similar observations apply to the second and third, and to the third and fourth sources. Thus, four waves interfere constructively there with net amplitude equal to $4s_m$.

23. (a) If point P is infinitely far away, then the small distance d between the two sources is of no consequence (they seem effectively to be the same distance away from P). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces fully constructive interference.

(c) For finite values of x, the difference in source positions becomes significant. The path lengths for waves to travel from S_1 and S_2 become now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

The path length difference for waves traveling from S_1 and S_2 is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \qquad \text{for} \quad x > 0.$$

The phase difference in "cycles" (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2 + x^2} - x}{\lambda}.$$

Thus, in terms of λ, the phase difference is identical to the path length difference: $|\Delta\phi| = \Delta\ell > 0$. Consider $\Delta\ell = \lambda/2$. Then $\sqrt{d^2 + x^2} = x + \lambda/2$. Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4}.$$

In general, if $\Delta\ell = \xi\lambda$ for some multiplier $\xi > 0$, we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda = \frac{64.0}{\xi} - \xi$$

where we have used $d = 16.0$ m and $\lambda = 2.00$ m.

(d) For $\Delta\ell = 0.50\lambda$, or $\xi = 0.50$, we have $x = (64.0/0.50 - 0.50)\ \text{m} = 127.5\ \text{m} \approx 128\ \text{m}$.

(e) For $\Delta\ell = 1.00\lambda$, or $\xi = 1.00$, we have $x = (64.0/1.00 - 1.00)\ \text{m} = 63.0\ \text{m}$.

(f) For $\Delta\ell = 1.50\lambda$, or $\xi = 1.50$, we have $x = (64.0/1.50 - 1.50)\ \text{m} = 41.2\ \text{m}$.

Note that since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the $\xi = 1, 2$ cases give constructive interference. A shift of a half-cycle brings "troughs" of one wave in superposition with "crests" of the other, thereby canceling the waves; therefore, the $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ cases produce destructive interference.

24. (a) Since intensity is power divided by area, and for an isotropic source the area may be written $A = 4\pi r^2$ (the area of a sphere), then we have

$$I = \frac{P}{A} = \frac{1.0\,\text{W}}{4\pi(1.0\,\text{m})^2} = 0.080\,\text{W/m}^2.$$

(b) This calculation may be done exactly as shown in part (a) (but with $r = 2.5$ m instead of $r = 1.0$ m), or it may be done by setting up a ratio. We illustrate the latter approach. Thus,

$$\frac{I'}{I} = \frac{P/4\pi(r')^2}{P/4\pi r^2} = \left(\frac{r}{r'}\right)^2$$

leads to $I' = (0.080\ \text{W/m}^2)(1.0/2.5)^2 = 0.013\ \text{W/m}^2$.

25. The intensity is the rate of energy flow per unit area perpendicular to the flow. The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where A ($= 4\pi r^2$) is the surface area of a sphere of radius r. Thus

$$P = 4\pi(2.50 \text{ m})^2 (1.91 \times 10^{-4} \text{ W/m}^2) = 1.50 \times 10^{-2} \text{ W}.$$

26. Sample Problem 17-5 shows that a decibel difference $\Delta\beta$ is directly related to an intensity ratio (which we write as $\mathcal{R} = I'/I$). Thus,

$$\Delta\beta = 10\log(\mathcal{R}) \;\Rightarrow\; \mathcal{R} = 10^{\Delta\beta/10} = 10^{0.1} = 1.26.$$

27. The intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where ρ is the density of air, v is the speed of sound in air, ω is the angular frequency, and s_m is the displacement amplitude for the sound wave. Replace ω with $2\pi f$ and solve for s_m:

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00\times10^{-6}\ \mathrm{W/m^2}}{2\pi^2 (1.21\,\mathrm{kg/m^3})(343\,\mathrm{m/s})(300\,\mathrm{Hz})^2}} = 3.68\times10^{-8}\ \mathrm{m}.$$

28. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r = 3.00$ m,

$$I = \frac{1.00\times10^{-6}\ \text{W}}{4\pi(3.00\,\text{m})^2} = 8.84\times10^{-9}\ \text{W/m}^2.$$

(b) The sound level there is

$$\beta = 10\ \log\left(\frac{8.84\times10^{-9}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 39.5\,\text{dB}.$$

29. (a) Let I_1 be the original intensity and I_2 be the final intensity. The original sound level is $\beta_1 = (10 \text{ dB}) \log(I_1/I_0)$ and the final sound level is $\beta_2 = (10 \text{ dB}) \log(I_2/I_0)$, where I_0 is the reference intensity. Since $\beta_2 = \beta_1 + 30$ dB which yields

$$(10 \text{ dB}) \log(I_2/I_0) = (10 \text{ dB}) \log(I_1/I_0) + 30 \text{ dB},$$

or

$$(10 \text{ dB}) \log(I_2/I_0) - (10 \text{ dB}) \log(I_1/I_0) = 30 \text{ dB}.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 3$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2 / I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of 1.0×10^3.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} \approx 32$.

30. (a) Eq. 17-29 gives the relation between sound level β and intensity I, namely

$$I = I_0 10^{(\beta/10\text{dB})} = (10^{-12}\ \text{W/m}^2)10^{(\beta/10\text{dB})} = 10^{-12+(\beta/10\text{dB})}\ \text{W/m}^2$$

Thus we find that for a β = 70 dB level we have a high intensity value of I_{high} = 10 μW/m^2.

(b) Similarly, for β = 50 dB level we have a low intensity value of I_{low} = 0.10 μW/m^2.

(c) Eq. 17-27 gives the relation between the displacement amplitude and I. Using the values for density and wave speed, we find s_m = 70 nm for the high intensity case.

(d) Similarly, for the low intensity case we have s_m = 7.0 nm.

We note that although the intensities differed by a factor of 100, the amplitudes differed by only a factor of 10.

31. We use $\beta = 10 \log(I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m^2 and Eq. 17–27 with $\omega = 2\pi f = 2\pi(260\text{ Hz})$, $v = 343$ m/s and $\rho = 1.21$ kg/m^3.

$$I = I_o\left(10^{8.5}\right) = \frac{1}{2}\rho v\left(2\pi f\right)^2 s_m^2 \quad \Rightarrow \quad s_m = 7.6\times10^{-7}\text{ m} = 0.76\ \mu\text{m}.$$

32. (a) Since $\omega = 2\pi f$, Eq. 17-15 leads to

$$\Delta p_m = v\rho\left(2\pi f\right)s_m \quad \Rightarrow \quad s_m = \frac{1.13\times10^{-3}\ \text{Pa}}{2\pi\left(1665\,\text{Hz}\right)\left(343\,\text{m/s}\right)\left(1.21\ \text{kg/m}^3\right)}$$

which yields s_m = 0.26 nm. The nano prefix represents 10^{-9}. We use the speed of sound and air density values given at the beginning of the exercises and problems section in the textbook.

(b) We can plug into Eq. 17–27 or into its equivalent form, rewritten in terms of the pressure amplitude:

$$I = \frac{1}{2}\frac{\left(\Delta p_m\right)^2}{\rho v} = \frac{1}{2}\frac{\left(1.13\times10^{-3}\ \text{Pa}\right)^2}{\left(1.21\,\text{kg/m}^3\right)\left(343\,\text{m/s}\right)} = 1.5\ \ \text{nW/m}^2.$$

33. We use $\beta = 10 \log (I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m^2 and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\,\text{m})^2\left(1\times10^{-12}\ \text{W/m}^2\right)10^{6.2} = 2\times10^{-6}\ \text{W} = 2\ \mu\text{W}.$$

34. The difference in sound level is given by Eq. 17-37:

$$\Delta\beta = \beta_f - \beta_i = (10\ \text{db})\log\left(\frac{I_f}{I_i}\right).$$

Thus, if $\Delta\beta = 5.0\ \text{db}$, then $\log(I_f / I_i) = 1/2$, which implies that $I_f = \sqrt{10} I_i$. On the other hand, the intensity at a distance r from the source is $I = \dfrac{P}{4\pi r^2}$, where P is the power of the source. A fixed P implies that $I_i r_i^2 = I_f r_f^2$. Thus, with $r_i = 1.2\ \text{m},$ we obtain

$$r_f = \left(\frac{I_i}{I_f}\right)^{1/2} r_i = \left(\frac{1}{10}\right)^{1/4} (1.2\ \text{m}) = 0.67\ \text{m}.$$

35. (a) The intensity is

$$I = \frac{P}{4\pi r^2} = \frac{30.0\,\text{W}}{(4\pi)(200\,\text{m})^2} = 5.97\times10^{-5}\ \text{W/m}^2.$$

(b) Let A (= 0.750 cm^2) be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P' = IA = 0 = (6.0\times10^{-5}\ \text{W/m}^2)(0.750\,\text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2) = 4.48\times10^{-9}\ \text{W}.$$

36. Combining Eqs.17-28 and 17-29 we have $\beta = 10 \log\left(\frac{P}{I_o 4\pi r^2}\right)$. Taking differences (for sounds A and B) we find

$$\Delta\beta = 10 \log\left(\frac{P_A}{I_o 4\pi r^2}\right) - 10 \log\left(\frac{P_B}{I_o 4\pi r^2}\right) = 10 \log\left(\frac{P_A}{P_B}\right)$$

using well-known properties of logarithms. Thus, we see that $\Delta\beta$ is independent of r and can be evaluated anywhere.

(a) We can solve the above relation (once we know $\Delta\beta = 5.0$) for the ratio of powers; we find $P_A/P_B \approx 3.2$.

(b) At $r = 1000$ m it is easily seen (in the graph) that $\Delta\beta = 5.0$ dB. This is the same $\Delta\beta$ we expect to find, then, at $r = 10$ m.

37. (a) As discussed on page 408, the average potential energy transport rate is the same as that of the kinetic energy. This implies that the (average) rate for the total energy is

$$\left(\frac{dE}{dt}\right)_{\text{avg}} = 2\left(\frac{dK}{dt}\right)_{\text{avg}} = 2\left(\tfrac{1}{4}\rho A v \omega^2 s_m^2\right)$$

using Eq. 17-44. In this equation, we substitute $\rho = 1.21$ kg/m^3, $A = \pi r^2 = \pi(0.020 \text{ m})^2$, v = 343 m/s, $\omega = 3000$ rad/s, $s_m = 12 \times 10^{-9}$ m, and obtain the answer 3.4×10^{-10} W.

(b) The second string is in a separate tube, so there is no question about the waves superposing. The total rate of energy, then, is just the addition of the two: $2(3.4 \times 10^{-10} \text{ W}) = 6.8 \times 10^{-10}$ W.

(c) Now we *do* have superposition, with $\phi = 0$, so the resultant amplitude is twice that of the individual wave which leads to the energy transport rate being four times that of part (a). We obtain $4(3.4 \times 10^{-10} \text{ W}) = 1.4 \times 10^{-9}$ W.

(d) In this case $\phi = 0.4\pi$, which means (using Eq. 17-39)

$$s_m{}' = 2\, s_m \cos(\phi/2) = 1.618 s_m.$$

This means the energy transport rate is $(1.618)^2 = 2.618$ times that of part (a). We obtain $2.618(3.4 \times 10^{-10} \text{ W}) = 8.8 \times 10^{-10}$ W.

(e) The situation is as shown in Fig. 17-14(b). The answer is zero.

38. (a) Using Eq. 17–39 with v = 343 m/s and n = 1, we find $f = nv/2L$ = 86 Hz for the fundamental frequency in a nasal passage of length L = 2.0 m (subject to various assumptions about the nature of the passage as a "bent tube open at both ends").

(b) The sound would be perceptible as *sound* (as opposed to just a general vibration) of very low frequency.

(c) Smaller L implies larger f by the formula cited above. Thus, the female's sound is of higher pitch (frequency).

39. (a) From Eq. 17–53, we have

$$f = \frac{nv}{2L} = \frac{(1)(250\,\mathrm{m/s})}{2(0.150\,\mathrm{m})} = 833\,\mathrm{Hz}.$$

(b) The frequency of the wave on the string is the same as the frequency of the sound wave it produces during its vibration. Consequently, the wavelength in air is

$$\lambda = \frac{v_{\mathrm{sound}}}{f} = \frac{348\,\mathrm{m/s}}{833\,\mathrm{Hz}} = 0.418\,\mathrm{m}.$$

40. The distance between nodes referred to in the problem means that $\lambda/2 = 3.8$ cm, or $\lambda = 0.076$ m. Therefore, the frequency is

$$f = v/\lambda = (1500 \text{ m/s})/(0.076 \text{ m}) \approx 20 \times 10^3 \text{ Hz}.$$

41. (a) We note that 1.2 = 6/5. This suggests that both even and odd harmonics are present, which means the pipe is open at both ends (see Eq. 17-39).

(b) Here we observe 1.4 = 7/5. This suggests that only odd harmonics are present, which means the pipe is open at only one end (see Eq. 17-41).

42. At the beginning of the exercises and problems section in the textbook, we are told to assume v_{sound} = 343 m/s unless told otherwise. The second harmonic of pipe A is found from Eq. 17–39 with $n = 2$ and $L = L_A$, and the third harmonic of pipe B is found from Eq. 17–41 with $n = 3$ and $L = L_B$. Since these frequencies are equal, we have

$$\frac{2v_{\text{sound}}}{2L_A} = \frac{3v_{\text{sound}}}{4L_B} \Rightarrow L_B = \frac{3}{4}L_A.$$

(a) Since the fundamental frequency for pipe A is 300 Hz, we immediately know that the second harmonic has $f = 2(300\text{ Hz}) = 600$ Hz. Using this, Eq. 17–39 gives

$$L_A = (2)(343\text{ m/s})/2(600\text{ s}^{-1}) = 0.572\text{ m}.$$

(b) The length of pipe B is $L_B = \frac{3}{4}L_A = 0.429\,\text{m}$.

43. (a) When the string (fixed at both ends) is vibrating at its lowest resonant frequency, exactly one-half of a wavelength fits between the ends. Thus, $\lambda = 2L$. We obtain

$$v = f\lambda = 2Lf = 2(0.220 \text{ m})(920 \text{ Hz}) = 405 \text{ m/s}.$$

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. If M is the mass of the (uniform) string, then $\mu = M/L$. Thus,

$$\tau = \mu v^2 = (M/L)v^2 = [(800 \times 10^{-6} \text{ kg})/(0.220 \text{ m})]\,(405 \text{ m/s})^2 = 596 \text{ N}.$$

(c) The wavelength is $\lambda = 2L = 2(0.220 \text{ m}) = 0.440 \text{ m}$.

(d) The frequency of the sound wave in air is the same as the frequency of oscillation of the string. The wavelength is different because the wave speed is different. If v_a is the speed of sound in air the wavelength in air is

$$\lambda_a = v_a/f = (343 \text{ m/s})/(920 \text{ Hz}) = 0.373 \text{ m}.$$

44. The frequency is f = 686 Hz and the speed of sound is v_{sound} = 343 m/s. If L is the length of the air-column, then using Eq. 17–41, the water height is (in unit of meters)

$$h = 1.00 - L = 1.00 - \frac{nv}{4f} = 1.00 - \frac{n(343)}{4(686)} = (1.00 - 0.125n) \text{ m}$$

where n = 1, 3, 5,… with only one end closed.

(a) There are 4 values of n (n = 1,3,5,7) which satisfies $h > 0$.

(b) The smallest water height for resonance to occur corresponds to n = 7 with $h = 0.125$ m.

(c) The second smallest water height corresponds to n = 5 with h = 0.375 m.

45. (a) Since the pipe is open at both ends there are displacement antinodes at both ends and an integer number of half-wavelengths fit into the length of the pipe. If L is the pipe length and λ is the wavelength then $\lambda = 2L/n$, where n is an integer. If v is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now $L = 0.457$ m, so

$$f = n(344 \text{ m/s})/2(0.457 \text{ m}) = 376.4n \text{ Hz}.$$

To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set $f = 1000$ Hz and solve for n, then set $f = 2000$ Hz and again solve for n. The results are 2.66 and 5.32, which imply that $n = 3$, 4, and 5 are the appropriate values of n. Thus, there are 3 frequencies.

(b) The lowest frequency at which resonance occurs is ($n = 3$) $f = 3(376.4 \text{ Hz}) = 1129$ Hz.

(c) The second lowest frequency at which resonance occurs is ($n = 4$)

$$f = 4(376.4 \text{ Hz}) = 1506 \text{ Hz}.$$

46. (a) Since the difference between consecutive harmonics is equal to the fundamental frequency (see section 17-6) then $f_1 = (390 - 325)$ Hz = 65 Hz. The next harmonic after 195 Hz is therefore (195 + 65) Hz = 260 Hz.

(b) Since $f_n = nf_1$ then $n = 260/65 = 4$.

(c) Only *odd* harmonics are present in tube *B* so the difference between consecutive harmonics is equal to *twice* the fundamental frequency in this case (consider taking differences of Eq. 17-41 for various values of *n*). Therefore,

$$f_1 = \tfrac{1}{2}(1320 - 1080)\text{ Hz} = 120\text{ Hz}.$$

The next harmonic after 600 Hz is consequently [600 + 2(120)] Hz = 840 Hz.

(d) Since $f_n = nf_1$ (for n odd) then $n = 840/120 = 7$.

47. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f = (n/2L)\sqrt{\tau/\mu}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_2 = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_2 - f_1 = (1/2L)\sqrt{\tau/\mu}$ and

$$\tau = 4L^2\mu(f_2 - f_1)^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\,\text{kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 = 45.3\,\text{N}.$$

48. (a) Using Eq. 17–39 with n = 1 (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{(1)v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343\,\text{m/s}}{4(1.20\,\text{m})} = 71.5\,\text{Hz}.$$

(b) For the wire (using Eq. 17–53) we have

$$f' = \frac{nv_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{\tau}{\mu}}$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$ (both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension τ.

$$\tau = (2L_{\text{wire}}\,f)^2\left(\frac{m_{\text{wire}}}{L_{\text{wire}}}\right) = 4f^2 m_{\text{wire}}L_{\text{wire}} = 4(71.5\,\text{Hz})^2(9.60\times10^{-3}\,\text{kg})(0.330\,\text{m}) = 64.8\,\text{N}.$$

49. The top of the water is a displacement node and the top of the well is a displacement anti-node. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If d is the depth and λ is the wavelength then $\lambda = 4d$. The frequency is $f = v/\lambda = v/4d$, where v is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of air in the well. Thus $f = (1/4d)\sqrt{B/\rho}$ and

$$d = \frac{1}{4f}\sqrt{\frac{B}{\rho}} = \frac{1}{4(7.00\,\text{Hz})}\sqrt{\frac{1.33\times10^5\ \text{Pa}}{1.10\,\text{kg/m}^3}} = 12.4\,\text{m}.$$

50. We observe that “third lowest … frequency” corresponds to harmonic number $n_A = 3$ for pipe *A* which is open at both ends. Also, “second lowest … frequency” corresponds to harmonic number $n_B = 3$ for pipe *B* which is closed at one end.

(a) Since the frequency of *B* matches the frequency of A, using Eqs. 17-39 and 17-41, we have

$$f_A = f_B \quad \Rightarrow \frac{3v}{2L_A} = \frac{3v}{4L_B}$$

which implies $L_B = L_A/2 = (1.20\text{ m})/2 = 0.60\text{ m}$. Using Eq. 17-40, the corresponding wavelength is

$$\lambda = \frac{4L_B}{3} = \frac{4(0.60\text{ m})}{3} = 0.80\text{ m}.$$

The change from node to anti-node requires a distance of $\lambda/4$ so that every increment of 0.20 m along the *x* axis involves a switch between node and anti-node. Since the closed end is a node, the next node appears at $x = 0.40$ m So there are 2 nodes. The situation corresponds to that illustrated in Fig. 17-15(b) with $n = 3$.

(b) The smallest value of *x* where a node is present is $x = 0$.

(c) The second smallest value of *x* where a node is present is $x = 0.40$m.

(d) Using $v = 343$ m/s, we find $f_3 = v/\lambda = 429$ Hz. Now, we find the fundamental resonant frequency by dividing by the harmonic number, $f_1 = f_3/3 = 143$ Hz.

51. Let the period be T. Then the beat frequency is $1/T - 440\,\text{Hz} = 4.00\,\text{beats/s}$. Therefore, $T = 2.25 \times 10^{-3}$ s. The string that is “too tightly stretched” has the higher tension and thus the higher (fundamental) frequency.

52. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass-spring oscillator is proportional to $1/\sqrt{m}$). Since the beat frequency also decreases the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

53. Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is

$$f = v/\lambda = (1/2L)\sqrt{\tau/\mu},$$

where $v = \sqrt{\tau/\mu}$ is the wave speed for the wire, τ is the tension in the wire, and μ is the linear mass density of the wire. Suppose the tension in one wire is τ and the oscillation frequency of that wire is f_1. The tension in the other wire is $\tau + \Delta\tau$ and its frequency is f_2. You want to calculate $\Delta\tau/\tau$ for f_1 = 600 Hz and f_2 = 606 Hz. Now, $f_1 = (1/2L)\sqrt{\tau/\mu}$ and $f_2 = (1/2L)\sqrt{(\tau + \Delta\tau/\mu}$, so

$$f_2 / f_1 = \sqrt{(\tau + \Delta\tau)/\tau} = \sqrt{1 + (\Delta\tau/\tau)}.$$

This leads to $\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [(606\,\text{Hz})/(600\,\text{Hz})]^2 - 1 = 0.020.$

54. (a) The number of different ways of picking up a pair of tuning forks out of a set of five is $5!/(2!3!) = 10$. For each of the pairs selected, there will be one beat frequency. If these frequencies are all different from each other, we get the maximum possible number of 10.

(b) First, we note that the minimum number occurs when the frequencies of these forks, labeled 1 through 5, increase in equal increments: $f_n = f_1 + n\Delta f$, where $n = 2, 3, 4, 5$. Now, there are only 4 different beat frequencies: $f_{\text{beat}} = n\Delta f$, where $n = 1, 2, 3, 4$.

55. In the general Doppler shift equation, the trooper's speed is the source speed and the speeder's speed is the detector's speed. The Doppler effect formula, Eq. 17–47, and its accompanying rule for choosing ± signs, are discussed in §17-10. Using that notation, we have $v = 343$ m/s,

$$v_D = v_S = 160 \text{ km/h} = (160000 \text{ m})/(3600 \text{ s}) = 44.4 \text{ m/s},$$

and $f = 500$ Hz. Thus,

$$f' = (500 \text{ Hz})\left(\frac{343 \text{ m/s} - 44.4 \text{ m/s}}{343 \text{ m/s} - 44.4 \text{ m/s}}\right) = 500\,\text{Hz} \;\Rightarrow\; \Delta f = 0.$$

56. The Doppler effect formula, Eq. 17–47, and its accompanying rule for choosing ± signs, are discussed in §17-10. Using that notation, we have v = 343 m/s, v_D = 2.44 m/s, f' = 1590 Hz and f = 1600 Hz. Thus,

$$f' = f\left(\frac{v+v_D}{v+v_S}\right) \Rightarrow v_S = \frac{f}{f'}(v+v_D) - v = 4.61\,\text{m/s}.$$

57. We use $v_S = r\omega$ (with r = 0.600 m and ω = 15.0 rad/s) for the linear speed during circular motion, and Eq. 17–47 for the Doppler effect (where f = 540 Hz, and v = 343 m/s for the speed of sound).

(a) The lowest frequency is

$$f' = f\left(\frac{v+0}{v+v_S}\right) = 526 \text{ Hz}.$$

(b) The highest frequency is

$$f' = f\left(\frac{v+0}{v-v_S}\right) = 555 \text{ Hz}.$$

58. We are combining two effects: the reception of a moving object (the truck of speed u = 45.0 m/s) of waves emitted by a stationary object (the motion detector), and the subsequent emission of those waves by the moving object (the truck) which are picked up by the stationary detector. This could be figured in two steps, but is more compactly computed in one step as shown here:

$$f_{\text{final}} = f_{\text{initial}}\left(\frac{v+u}{v-u}\right) = (0.150\,\text{MHz})\left(\frac{343\,\text{m/s} + 45\,\text{m/s}}{343\,\text{m/s} - 45\,\text{m/s}}\right) = 0.195\,\text{MHz}.$$

59. In this case, the intruder is moving *away* from the source with a speed u satisfying $u/v \ll 1$. The Doppler shift (with $u = -0.950$ m/s) leads to

$$f_{\text{beat}} = \left|f_r - f_s\right| \approx \frac{2\,|u|}{v} f_s = \frac{2(0.95\,\text{m/s})(28.0\,\text{kHz})}{343\,\text{m/s}}) = 155\,\text{Hz}\,.$$

60. We use Eq. 17–47 with f = 1200 Hz and v = 329 m/s.

(a) In this case, v_D = 65.8 m/s and v_S = 29.9 m/s, and we choose signs so that f' is larger than f:

$$f' = f\left(\frac{329 \text{ m/s} + 65.8 \text{ m/s}}{329 \text{ m/s} - 29.9 \text{ m/s}}\right) = 1.58\times10^3 \text{ Hz}.$$

(b) The wavelength is $\lambda = v/f' = 0.208$ m.

(c) The wave (of frequency f') "emitted" by the moving reflector (now treated as a "source," so v_S = 65.8 m/s) is returned to the detector (now treated as a detector, so v_D = 29.9 m/s) and registered as a new frequency f'':

$$f'' = f'\left(\frac{329 \text{ m/s} + 29.9 \text{ m/s}}{329 \text{ m/s} - 65.8 \text{ m/s}}\right) = 2.16\times10^3 \text{ Hz}.$$

(d) This has wavelength $v/f'' = 0.152$ m.

61. We denote the speed of the French submarine by u_1 and that of the U.S. sub by u_2.

(a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v+u_2}{v-u_1}\right) = (1.000\times 10^3\text{ Hz})\left(\frac{5470\text{ km/h} + 70.00\text{ km/h}}{5470\text{ km/h} - 50.00\text{ km/h}}\right) = 1.022\times 10^3\text{ Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be $f_r = f_1(v+u_2)/(v-u_2)$. Since the French sub is moving towards the reflected signal with speed u_1, then

$$f_r' = f_r\left(\frac{v+u_1}{v}\right) = f_1\frac{(v+u_1)(v+u_2)}{v(v-u_2)} = \frac{(1.000\times 10^3\text{ Hz})(5470+50.00)(5470+70.00)}{(5470)(5470-70.00)}$$

$$= 1.045\times 10^3\text{ Hz}.$$

62. When the detector is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = \frac{f}{1 - v_s / v}$$

where v_s is the speed of the source (assumed to be approaching the detector in the way we've written it, above). The difference between the approach and the recession is

$$f' - f'' = f\left(\frac{1}{1 - v_s/v} - \frac{1}{1 + v_s/v}\right) = f\left(\frac{2\, v_s/v}{1 - (v_s/v)^2}\right)$$

which, after setting $(f' - f'')/f = 1/2$, leads to an equation which can be solved for the ratio v_s/v. The result is $\sqrt{5} - 2 = 0.236$. Thus, $v_s/v = 0.236$.

63. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f'\left(\frac{v+u_{\text{bat}}}{v-u_{\text{bat}}}\right) = (3.9\times10^4\ \text{Hz})\left(\frac{v+v/40}{v-v/40}\right) = 4.1\times10^4\ \text{Hz}.$$

64. The “third harmonic” refers to a resonant frequency $f_3 = 3 f_1$, where f_1 is the fundamental lowest resonant frequency. When the source is stationary, with respect to the air, then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right)$$

where v_d is the speed of the detector (assumed to be moving away from the source, in the way we’ve written it, above). The problem, then, wants us to find v_d such that $f' = f_1$ when the emitted frequency is $f = f_3$. That is, we require $1 - v_d/v = 1/3$. Clearly, the solution to this is $v_d/v = 2/3$ (independent of length and whether one or both ends are open [the latter point being due to the fact that the odd harmonics occur in both systems]). Thus,

(a) For tube 1, $v_d = 2v/3$.

(b) For tube 2, $v_d = 2v/3$.

(c) For tube 3, $v_d = 2v/3$.

(d) For tube 4, $v_d = 2v/3$.

65. (a) The expression for the Doppler shifted frequency is

$$f' = f\frac{v \pm v_D}{v \mp v_S},$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector (the uncle), and v_S is the speed of the source (the locomotive). All speeds are relative to the air. The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is $v_S = 10$ m/s. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\frac{v}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} + 10.00\,\text{m/s}}\right) = 485.8\,\text{Hz}.$$

(b) The girl is now the detector. Relative to the air she is moving with speed $v_D = 10.00$ m/s toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at $v_S = 10.00$ m/s away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0$ Hz.

(c) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at $v_D = 10.00$ m/s toward the locomotive. Use the plus sign in the numerator. Thus

$$f' = f\frac{v + v_D}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}.$$

(d) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the girl and the girl is moving at $v_D = 20.00$ m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0$ Hz.

66. We use Eq. 17–47 with f = 500 Hz and v = 343 m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 30.5\text{ m/s}}{343\text{ m/s} - 30.5\text{ m/s}}\right) = 598\,\text{Hz}.$$

(b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 0}{343\text{ m/s} - 2(30.5\text{ m/s})}\right) = 608\,\text{Hz}.$$

(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 2(30.5\text{ m/s})}{343\text{ m/s} - 0}\right) = 589\,\text{Hz}.$$

67. The Doppler shift formula, Eq. 17–47, is valid only when both u_S and u_D are measured with respect to a stationary medium (i.e., no wind). To modify this formula in the presence of a wind, we switch to a new reference frame in which there is no wind.

(a) When the wind is blowing from the source to the observer with a speed w, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f\left(\frac{v+u'_D}{v+u'_S}\right) = f\left(\frac{v+w}{v+w}\right) = 2.0\times10^3\ \text{Hz}.$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both u'_D and u'_S. The result is that there is still no Doppler shift:

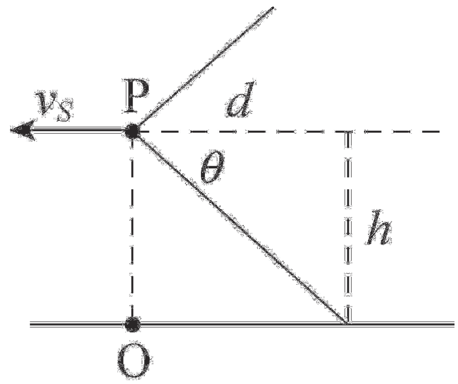
$$f' = f\left(\frac{v-u'_D}{v-u'_S}\right) = f\left(\frac{v-w}{v-w}\right) = 2.0\times10^3\ \text{Hz}.$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

68. We note that 1350 km/h is v_S = 375 m/s. Then, with $\theta = 60°$, Eq. 17-57 gives v = 3.3×10^2 m/s.

69. (a) The half angle θ of the Mach cone is given by sin $\theta = v/v_S$, where v is the speed of sound and v_S is the speed of the plane. Since $v_S = 1.5v$, sin $\theta = v/1.5v = 1/1.5$. This means $\theta = 42°$.

(b) Let h be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance d behind the plane. The situation is shown on the diagram below, with P indicating the plane and O indicating the observer. The cone angle is related to h and d by tan $\theta = h/d$, so $d = h/\tan\theta$. The shock wave reaches O in the time the plane takes to fly the distance d:



$$t = \frac{d}{v} = \frac{h}{v\tan\theta} = \frac{5000\text{ m}}{1.5(331\text{ m/s})\tan 42°} = 11\text{ s}.$$

70. The altitude H and the horizontal distance x for the legs of a right triangle, so we have

$$H = x\tan\theta = v_p t\tan\theta = 1.25vt\sin\theta$$

where v is the speed of sound, v_p is the speed of the plane and

$$\theta = \sin^{-1}\left(\frac{v}{v_p}\right) = \sin^{-1}\left(\frac{v}{1.25v}\right) = 53.1°.$$

Thus the altitude is

$$H = x\tan\theta = (1.25)(330\,\text{m/s})(60\,\text{s})(\tan 53.1°) = 3.30\times10^4\ \text{m}.$$

71. (a) Incorporating a term ($\lambda/2$) to account for the phase shift upon reflection, then the path difference for the waves (when they come back together) is

$$\sqrt{L^2 + (2d)^2} - L + \lambda/2 = \Delta(\text{path}) .$$

Setting this equal to the condition needed to destructive interference ($\lambda/2$, $3\lambda/2$, $5\lambda/2$ …) leads to d = 0, 2.10 m, … Since the problem explicitly excludes the d = 0 possibility, then our answer is d = 2.10 m.

(b) Setting this equal to the condition needed to constructive interference (λ, 2λ, 3λ …) leads to d = 1.47 m, … Our answer is d = 1.47 m.

72. When the source is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right),$$

where v_d is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The difference between the approach and the recession is

$$f'' - f' = f\left[\left(1 + \frac{v_d}{v}\right) - \left(1 - \frac{v_d}{v}\right)\right] = f\left(2\frac{v_d}{v}\right)$$

which, after setting $(f'' - f')/f = 1/2$, leads to an equation which can be solved for the ratio v_d/v. The result is 1/4. Thus, $v_d/v = 0.250$.

73. (a) Adapting Eq. 17-39 to the notation of this chapter, we have

$$s_m' = 2\, s_m \cos(\phi/2) = 2(12 \text{ nm}) \cos(\pi/6) = 20.78 \text{ nm}.$$

Thus, the amplitude of the resultant wave is roughly 21 nm.

(b) The wavelength (λ = 35 cm) does not change as a result of the superposition.

(c) Recalling Eq. 17-47 (and the accompanying discussion) from the previous chapter, we conclude that the standing wave amplitude is 2(12 nm) = 24 nm when they are traveling in opposite directions.

(d) Again, the wavelength (λ = 35 cm) does not change as a result of the superposition.

74. (a) The separation distance between points *A* and *B* is one-quarter of a wavelength; therefore, $\lambda = 4(0.15 \text{ m}) = 0.60 \text{ m}$. The frequency, then, is

$$f = v/\lambda = (343 \text{ m/s})/(0.60 \text{ m}) = 572 \text{ Hz}.$$

(b) The separation distance between points *C* and *D* is one-half of a wavelength; therefore, $\lambda = 2(0.15 \text{ m}) = 0.30 \text{ m}$. The frequency, then, is

$$f = v/\lambda = (343 \text{ m/s})/(0.30 \text{ m}) = 1144 \text{ Hz (or approximately 1.14 kHz).}$$

75. Any phase changes associated with the reflections themselves are rendered inconsequential by the fact that there are an even number of reflections. The additional path length traveled by wave A consists of the vertical legs in the zig-zag path: $2L$. To be (minimally) out of phase means, therefore, that $2L = \lambda/2$ (corresponding to a half-cycle, or 180°, phase difference). Thus, $L = \lambda/4$, or $L/\lambda = 1/4 = 0.25$.

76. Since they are approaching each other, the sound produced (of emitted frequency f) by the flatcar-trumpet received by an observer on the ground will be of higher pitch f'. In these terms, we are told $f' - f = 4.0$ Hz, and consequently that $f'/f = 444/440 = 1.0091$. With v_S designating the speed of the flatcar and $v = 343$ m/s being the speed of sound, the Doppler equation leads to

$$\frac{f'}{f} = \frac{v+0}{v-v_S} \quad \Rightarrow \quad v_S = (343 \text{ m/s})\frac{1.0091-1}{1.0091} = 3.1 \text{ m/s}.$$

77. The siren is between you and the cliff, moving away from you and towards the cliff. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17–47 (and see the discussion in the textbook immediately after that equation regarding the selection of ± signs). The source is the siren with $v_S = 10$ m/s. The problem asks us to use $v = 330$ m/s for the speed of sound.

(a) With $f = 1000$ Hz, the frequency f_y you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = 970.6\,\text{Hz} \approx 9.7\times10^2\ \text{Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = 1031.3\,\text{Hz} \approx 1.0\times10^3\ \text{Hz}.$$

(c) The beat frequency is $f_c - f_y = 60$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

78. Let r stand for the ratio of the source speed to the speed of sound. Then, Eq. 17-55 (plus the fact that frequency is inversely proportional to wavelength) leads to

$$2\left(\frac{1}{1+r}\right) = \frac{1}{1-r} .$$

Solving, we find $r = 1/3$. Thus, $v_s/v = 0.33$.

79. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

(a) With $I_1 = 9.60 \times 10^{-4}$ W/m^2, $r_1 = 6.10$ m, and $r_2 = 30.0$ m, we find

$$I_2 = (9.60 \times 10^{-4}\ \text{W/m}^2)(6.10/30.0)^2 = 3.97 \times 10^{-5}\ \text{W/m}^2.$$

(b) Using Eq. 17–27 with $I_1 = 9.60 \times 10^{-4}$ W/m^2, $\omega = 2\pi(2000$ Hz), $v = 343$ m/s and $\rho = 1.21$ kg/m^3, we obtain

$$s_m = \sqrt{\frac{2I}{\rho v \omega^2}} = 1.71 \times 10^{-7}\ \text{m}.$$

(c) Eq. 17-15 gives the pressure amplitude:

$$\Delta p_m = \rho v \omega s_m = 0.893\ \ \text{Pa}.$$

80. When $\phi = 0$ it is clear that the superposition wave has amplitude $2\Delta p_m$. For the other cases, it is useful to write

$$\Delta p_1 + \Delta p_2 = \Delta p_m \left(\sin(\omega t) + \sin(\omega t - \phi)\right) = \left(2\Delta p_m \cos\frac{\phi}{2}\right)\sin\left(\omega t - \frac{\phi}{2}\right).$$

The factor in front of the sine function gives the amplitude Δp_r. Thus, $\Delta p_r / \Delta p_m = 2\cos(\phi/2)$.

(a) When $\phi = 0$, $\Delta p_r / \Delta p_m = 2\cos(0) = 2.00$.

(b) When $\phi = \pi/2$, $\Delta p_r / \Delta p_m = 2\cos(\pi/4) = \sqrt{2} = 1.41$.

(c) When $\phi = \pi/3$, $\Delta p_r / \Delta p_m = 2\cos(\pi/6) = \sqrt{3} = 1.73$.

(d) When $\phi = \pi/4$, $\Delta p_r / \Delta p_m = 2\cos(\pi/8) = 1.85$.

81. (a) With r = 10 m in Eq. 17–28, we have

$$I = \frac{P}{4\pi r^2} \quad \Rightarrow \quad P = 10\,\text{W}.$$

(b) Using that value of P in Eq. 17–28 with a new value for r, we obtain

$$I = \frac{P}{4\pi\left(5.0\right)^2} = 0.032\,\frac{\text{W}}{\text{m}^2}.$$

Alternatively, a ratio $I'/I = (r/r')^2$ could have been used.

(c) Using Eq. 17–29 with I = 0.0080 W/m^2, we have

$$\beta = 10\log\frac{I}{I_0} = 99\,\text{dB}$$

where $I_0 = 1.0 \times 10^{-12}$ W/m^2.

82. We use $v = \sqrt{B/\rho}$ to find the bulk modulus B:

$$B = v^2\rho = \left(5.4\times10^3\ \text{m/s}\right)^2\left(2.7\times10^3\ \text{kg/m}^3\right) = 7.9\times10^{10}\ \text{Pa}.$$

83. Let the frequencies of sound heard by the person from the left and right forks be f_l and f_r, respectively.

(a) If the speeds of both forks are u, then $f_{l,r} = fv/(v \pm u)$ and

$$f_{\text{beat}} = |f_r - f_l| = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right) = \frac{2\,fuv}{v^2 - u^2} = \frac{2(440\,\text{Hz})(3.00\,\text{m/s})(343\,\text{m/s})}{(343\,\text{m/s})^2 - (3.00\,\text{m/s})^2}$$

$$= 7.70\,\text{Hz}.$$

(b) If the speed of the listener is u, then $f_{l,r} = f(v \pm u)/v$ and

$$f_{\text{beat}} = |f_l - f_r| = 2f\left(\frac{u}{v}\right) = 2(440\,\text{Hz})\left(\frac{3.00\,\text{m/s}}{343\,\text{m/s}}\right) = 7.70\,\text{Hz}.$$

84. The rule: if you divide the time (in seconds) by 3, then you get (approximately) the straight-line distance d. We note that the speed of sound we are to use is given at the beginning of the problem section in the textbook, and that the speed of light is very much larger than the speed of sound. The proof of our rule is as follows:

$$t = t_{\text{sound}} - t_{\text{light}} \approx t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{343\,\text{m/s}} = \frac{d}{0.343\,\text{km/s}}.$$

Cross-multiplying yields (approximately) (0.3 km/s)t = d which (since 1/3 ≈ 0.3) demonstrates why the rule works fairly well.

85. (a) The intensity is given by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where ρ is the density of the medium, v is the speed of sound, ω is the angular frequency, and s_m is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v \omega s_m$, so $s_m = \Delta p_m/\rho v \omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w},$$

where the subscript a denotes air and the subscript w denotes water. Since $I_a = I_w$,

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998\times10^3\ \mathrm{kg/m^3})(1482\ \mathrm{m/s})}{(1.21\ \mathrm{kg/m^3})(343\ \mathrm{m/s})}} = 59.7.$$

The speeds of sound are given in Table 17-1 and the densities are given in Table 15-1.

(b) Now, $\Delta p_{mw} = \Delta p_{ma}$, so

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\ \mathrm{kg/m^3})(343\ \mathrm{m/s})}{(0.998\times10^3\ \mathrm{kg/m^3})(1482\ \mathrm{m/s})} = 2.81\times10^{-4}.$$

86. We use $\Delta\beta_{12} = \beta_1 - \beta_2 = (10 \text{ dB}) \log(I_1/I_2)$.

(a) Since $\Delta\beta_{12} = (10 \text{ dB}) \log(I_1/I_2) = 37$ dB, we get

$$I_1/I_2 = 10^{37 \text{ dB}/10 \text{ dB}} = 10^{3.7} = 5.0 \times 10^3.$$

(b) Since $\Delta p_m \propto s_m \propto \sqrt{I}$, we have

$$\Delta p_{m1} / \Delta p_{m2} = \sqrt{I_1 / I_2} = \sqrt{5.0 \times 10^3} = 71.$$

(c) The displacement amplitude ratio is $s_{m1} / s_{m2} = \sqrt{I_1 / I_2} = 71$.

87. (a) When the right side of the instrument is pulled out a distance d the path length for sound waves increases by $2d$. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where λ is the wavelength. Thus $\lambda = 4d$ and, if v is the speed of sound, the frequency is

$$f = v/\lambda = v/4d = (343 \text{ m/s})/4(0.0165 \text{ m}) = 5.2 \times 10^3 \text{ Hz}.$$

(b) The displacement amplitude is proportional to the square root of the intensity (see Eq. 17–27). Write $\sqrt{I} = Cs_m$, where I is the intensity, s_m is the displacement amplitude, and C is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$. Solve

$$\sqrt{100} = C(s_{SAD} - s_{SBD}) \quad \text{and} \quad \sqrt{900} = C(s_{SAD} - s_{SBD})$$

for s_{SAD} and s_{SBD}. Adding the equations give

$$s_{SAD} = (\sqrt{100} + \sqrt{900} / 2C = 20/C,$$

while subtracting them yields

$$s_{SBD} = (\sqrt{900} - \sqrt{100})/2C = 10/C.$$

Thus, the ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

88. The angle is $\sin^{-1}(v/v_s) = \sin^{-1}(343/685) = 30°$.

89. The round-trip time is $t = 2L/v$ where we estimate from the chart that the time between clicks is 3 ms. Thus, with $v = 1372$ m/s, we find $L = \frac{1}{2}vt = 2.1 \text{ m}$.

90. The wave is written as $s(x,t) = s_m \cos(kx \pm \omega t)$.

(a) The amplitude s_m is equal to the maximum displacement: $s_m = 0.30 \text{ cm}$.

(b) Since λ = 24 cm, the angular wave number is $k = 2\pi / \lambda = 0.26 \text{ cm}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(25 \text{ Hz}) = 1.6\times10^2 \text{ rad/s}$.

(d) The speed of the wave is $v = \lambda f$ = (24 cm)(25 Hz) = 6.0×10^2 cm/s.

(e) Since the direction of propagation is $-x$, the sign is plus, i.e., $s(x,t) = s_m \cos(kx + \omega t)$.

91. The source being a "point source" means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

From the discussion in §17-5, we know that the intensity ratio between "barely audible" and the "painful threshold" is $10^{-12} = I_2/I_1$. Thus, with $r_2 = 10000$ m, we find

$$r_1 = r_2\sqrt{10^{-12}} = 0.01\,\text{m} = 1\text{ cm}.$$

92. (a) The time it takes for sound to travel in air is $t_a = L/v$, while it takes $t_m = L/v_m$ for the sound to travel in the metal. Thus,

$$\Delta t = t_a - t_m = \frac{L}{v} - \frac{L}{v_m} = \frac{L(v_m - v)}{v_m v}.$$

(b) Using the values indicated (see Table 17-1), we obtain

$$L = \frac{\Delta t}{1/v - 1/v_m} = \frac{1.00\,\text{s}}{1/(343\,\text{m/s}) \; - \; 1/(5941\,\text{m/s})} = 364\,\text{m}.$$

93. (a) We observe that "third lowest … frequency" corresponds to harmonic number n = 5 for such a system. Using Eq. 17–41, we have

$$f = \frac{nv}{4L} \quad \Rightarrow \quad 750\,\text{Hz} = \frac{5v}{4(0.60\text{ m})}$$

so that $v = 3.6\times10^2$ m/s.

(b) As noted, $n = 5$; therefore, $f_1 = 750/5 = 150$ Hz.

94. We note that waves 1 and 3 differ in phase by π radians (so they cancel upon superposition). Waves 2 and 4 also differ in phase by π radians (and also cancel upon superposition). Consequently, there is no resultant wave.

95. Since they oscillate out of phase, then their waves will cancel (producing a node) at a point exactly midway between them (the midpoint of the system, where we choose $x = 0$). We note that Figure 17-14, and the $n = 3$ case of Figure 17-15(a) have this property (of a node at the midpoint). The distance Δx between nodes is $\lambda/2$, where $\lambda = v/f$ and $f = 300$ Hz and $v = 343$ m/s. Thus, $\Delta x = v/2f = 0.572$ m.

Therefore, nodes are found at the following positions:

$$x = n\Delta x = n(0.572\,\text{m}), \;\; n = 0, \pm 1, \pm 2, \ldots$$

(a) The shortest distance from the midpoint where nodes are found is $\Delta x = 0$.

(b) The second shortest distance from the midpoint where nodes are found is $\Delta x = 0.572$ m.

(c) The third shortest distance from the midpoint where nodes are found is $2\Delta x = 1.14$ m.

96. (a) With f = 686 Hz and v = 343 m/s, then the "separation between adjacent wavefronts" is $\lambda = v/f = 0.50$ m.

(b) This is one of the effects which are part of the Doppler phenomena. Here, the wavelength shift (relative to its "true" value in part (a)) equals the source speed v_s (with appropriate ± sign) relative to the speed of sound v:

$$\frac{\Delta\lambda}{\lambda} = \pm\frac{v_s}{v}.$$

In front of the source, the shift in wavelength is –(0.50 m)(110 m/s)/(343 m/s) = –0.16 m, and the wavefront separation is 0.50 m – 0.16 m = 0.34 m.

(c) Behind the source, the shift in wavelength is +(0.50 m)(110 m/s)/(343 m/s) = +0.16 m, and the wavefront separation is 0.50 m + 0.16 m = 0.66 m.

97. We use $I \propto r^{-2}$ appropriate for an isotropic source. We have

$$\frac{I_{r=d}}{I_{r=D-d}} = \frac{(D-d)^2}{D^2} = \frac{1}{2},$$

where d = 50.0 m. We solve for

$$D : D = \sqrt{2}d / \left(\sqrt{2}-1\right) = \sqrt{2}\,(50.0\,\text{m}) / \left(\sqrt{2}-1\right) = 171\,\text{m}.$$

98. (a) Using m = 7.3 × 10^7 kg, the initial gravitational potential energy is $U = mgy = 3.9\times10^{11}$ J, where h = 550 m. Assuming this converts primarily into kinetic energy during the fall, then K = 3.9 × 10^{11} J just before impact with the ground. Using instead the mass estimate m = 1.7 × 10^8 kg, we arrive at K = 9.2 × 10^{11} J.

(b) The process of converting this kinetic energy into other forms of energy (during the impact with the ground) is assumed to take Δt = 0.50 s (and in the average sense, we take the "power" P to be wave-energy/Δt). With 20% of the energy going into creating a seismic wave, the intensity of the body wave is estimated to be

$$I = \frac{P}{A_{\text{hemisphere}}} = \frac{(0.20)K/\Delta t}{\frac{1}{2}\left(4\pi r^2\right)} = 0.63\,\text{W/m}^2$$

using r = 200 × 10^3 m and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain I = 1.5 W/m^2.

(c) The surface area of a cylinder of "height" d is $2\pi rd$, so the intensity of the surface wave is

$$I = \frac{P}{A_{\text{cylinder}}} = \frac{(0.20)K/\Delta t}{(2\pi rd)} = 25\times10^3\ \text{W/m}^2$$

using d = 5.0 m, r = 200 × 10^3 m and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain I = 58 kW/m^2.

(d) Although several factors are involved in determining which seismic waves are most likely to be detected, we observe that on the basis of the above findings we should expect the more intense waves (the surface waves) to be more readily detected.

99. (a) The period is the reciprocal of the frequency:

$$T = 1/f = 1/(90\ \text{Hz}) = 1.1 \times 10^{-2}\ \text{s}.$$

(b) Using $v = 343$ m/s, we find $\lambda = v/f = 3.8$ m.

100. (a) The problem asks for the source frequency f. We use Eq. 17–47 with great care (regarding its ± sign conventions).

$$f' = f\left(\frac{340\text{ m/s} - 16\text{ m/s}}{340\text{ m/s} - 40\text{ m/s}}\right)$$

Therefore, with f' = 950 Hz, we obtain f = 880 Hz.

(b) We now have

$$f' = f\left(\frac{340\text{ m/s} + 16\text{ m/s}}{340\text{ m/s} + 40\text{ m/s}}\right)$$

so that with f = 880 Hz, we find f' = 824 Hz.

101. (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two-step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right)$$

where $v_x = v_{\text{blood}} \cos\theta$. If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90\,\text{m/s}$$

where $v = 1540$ m/s, $\theta = 20°$, and $R = 1 + 5495/5 \times 10^6$.

(c) We interpret the question as asking how Δf (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle θ changes. Since larger θ means smaller horizontal component of velocity v_x then we expect Δf to decrease towards zero as θ is increased towards 90°.

102. Pipe A (which can only support odd harmonics – see Eq. 17-41) has length L_A. Pipe B (which supports both odd and even harmonics [any value of n] – see Eq. 17-39) has length $L_B = 4L_A$. Taking ratios of these equations leads to the condition:

$$\left(\frac{n}{2}\right)_B = (n_{\rm odd})_A \quad .$$

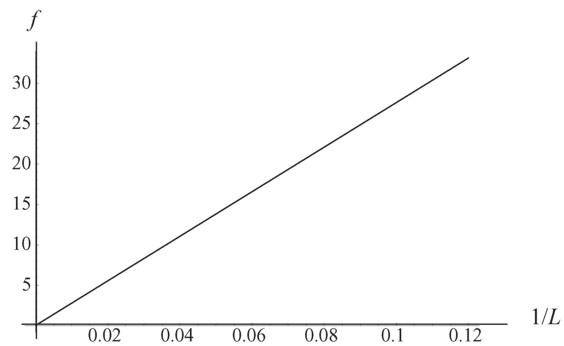
Solving for n_B we have $n_B = 2n_{\rm odd}$.

(a) Thus, the smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(1)=2$.

(b) The second smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(3)=6$.

(c) The third smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(5)=10$.

103. The points and the least-squares fit is shown in the graph that follows.



The graph has frequency in Hertz along the vertical axis and $1/L$ in inverse meters along the horizontal axis. The function found by the least squares fit procedure is $f = 276(1/L) + 0.037$. We shall assume that this fits either the model of an open organ pipe (mathematically similar to a string fixed at both ends) or that of a pipe closed at one end.

(a) In a tube with two open ends, $f = v/2L$. If the least-squares slope of 276 fits the first model, then a value of

$$v = 2(276\text{ m/s}) = 553\text{ m/s} \approx 5.5\times10^2\text{ m/s}$$

is implied.

(b) In a tube with only one open end, $f = v/4L$, and we find $v = 4(276\text{ m/s}) = 1106\text{ m/s}$ $\approx 1.1\times10^3$ m/s which is more "in the ballpark" of the 1400 m/s value cited in the problem.

(c) This suggests that the acoustic resonance involved in this situation is more closely related to the $n = 1$ case of Figure 17-15(b) than to Figure 17-14.

104. (a) Since the source is moving toward the wall, the frequency of the sound as received at the wall is

$$f' = f\left(\frac{v}{v - v_S}\right) = (440\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} - 20.0\,\text{m/s}}\right) = 467\,\text{Hz}.$$

(b) Since the person is moving with a speed *u* toward the reflected sound with frequency f', the frequency registered at the source is

$$f_r = f'\left(\frac{v + u}{v}\right) = (467\,\text{Hz})\left(\frac{343\,\text{m/s} + 20.0\,\text{m/s}}{343\,\text{m/s}}\right) = 494\,\text{Hz}.$$

105. Using Eq. 17-47 with great care (regarding its ± sign conventions), we have

$$f' = (440\text{ Hz})\left(\frac{340\text{ m/s} - 80.0\text{ m/s}}{340\text{ m/s} - 54.0\text{ m/s}}\right) = 400\text{ Hz}.$$

106. (a) Let P be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity I at the sphere surface and the area of the sphere. For a sphere of radius r, $P = 4\pi r^2 I$ and $I = P/4\pi r^2$. The intensity is proportional to the square of the displacement amplitude s_m. If we write $I = Cs_m^2$, where C is a constant of proportionality, then $Cs_m^2 = P/4\pi r^2$. Thus,

$$s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r).$$

The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius r in phase. If ω is the angular frequency and k is the angular wave number then the time dependence is $\sin(kr - \omega t)$. Letting $b = \sqrt{P/4\pi C}$, the displacement wave is then given by

$$s(r,t) = \sqrt{\frac{P}{4\pi C}}\frac{1}{r}\sin(kr - \omega t) = \frac{b}{r}\sin(kr - \omega t).$$

(b) Since s and r both have dimensions of length and the trigonometric function is dimensionless, the dimensions of b must be length squared.

107. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We consider the resultant wave (at large distance from the origin) along the $+x$ axis; we note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value $n = 3.2$, implying a sort of intermediate condition between constructive interference (which would follow if, say, $n = 3$) and destructive interference (such as the $n = 3.5$ situation found in the solution to the previous problem) between the waves. To distinguish this resultant along the $+x$ axis from the similar one along the $-x$ axis, we label one with $n = +3.2$ and the other $n = -3.2$. This labeling facilitates the complete enumeration of the loud directions in the upper-half plane: $n = -3, -2, -1, 0, +1, +2, +3$. Counting also the "other" $-3, -2, -1, 0, +1, +2, +3$ values for the *lower*-half plane, then we conclude there are a total of $7 + 7 = 14$ "loud" directions.

(b) The labeling also helps us enumerate the quiet directions. In the upper-half plane we find: $n = -2.5, -1.5, -0.5, +0.5, +1.5, +2.5$. This is duplicated in the lower half plane, so the total number of quiet directions is $6 + 6 = 12$.

108. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$. Since intensity is proportional to the square of the amplitude (see Eq. 17–27), this further implies

$$\frac{I_2}{I_1} = \left(\frac{s_{m2}}{s_{m1}}\right)^2 = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

or $s_{m2}/s_{m1} = r_1/r_2$.

(a) $I = P/4\pi r^2 = (10\ \text{W})/4\pi(3.0\ \text{m})^2 = 0.088\ \text{W/m}^2$.

(b) Using the notation A instead of s_m for the amplitude, we find

$$\frac{A_4}{A_3} = \frac{3.0\,\text{m}}{4.0\,\text{m}} = 0.75\,.$$

109. (a) In regions where the speed is constant, it is equal to distance divided by time. Thus, we conclude that the time difference is

$$\Delta t = \left(\frac{L - d}{V} + \frac{d}{V - \Delta V} \right) - \frac{L}{V}$$

where the first term is the travel time through bone and rock and the last term is the expected travel time purely through rock. Solving for d and simplifying, we obtain

$$d = \Delta t \, \frac{V(V - \Delta V)}{\Delta V} \approx \Delta t \frac{V^2}{\Delta V}.$$

(b) If we estimate $d \approx 10$ cm (as the lower limit of a range that goes up to a diameter of 20 cm), then the above expression (with the numerical values given in the problem) leads to $\Delta t = 0.8$ μs (as the lower limit of a range that goes up to a time difference of 1.6 μs).

110. (a) We expect the center of the star to be a displacement node. The star has spherical symmetry and the waves are spherical. If matter at the center moved it would move equally in all directions and this is not possible.

(b) We assume the oscillation is at the lowest resonance frequency. Then, exactly one-fourth of a wavelength fits the star radius. If λ is the wavelength and R is the star radius then $\lambda = 4R$. The frequency is $f = v/\lambda = v/4R$, where v is the speed of sound in the star. The period is $T = 1/f = 4R/v$.

(c) The speed of sound is $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of stellar material. The radius is $R = 9.0 \times 10^{-3} R_s$, where R_s is the radius of the Sun (6.96×10^8 m). Thus

$$T = 4R\sqrt{\frac{\rho}{B}} = 4(9.0\times10^{-3})(6.96\times10^{8}\text{ m})\sqrt{\frac{1.0\times10^{10}\text{ kg/m}^3}{1.33\times10^{22}\text{ Pa}}} = 22\text{ s}.$$

111. We find the difference in the two applications of the Doppler formula:

$$f_2 - f_1 = 37 \text{ Hz} = f\left(\frac{340 \text{ m/s} + 25 \text{ m/s}}{340 \text{ m/s} - 15 \text{ m/s}} - \frac{340 \text{ m/s}}{340 \text{ m/s} - 15 \text{ m/s}}\right) = f\left(\frac{25 \text{ m/s}}{340 \text{ m/s} - 15 \text{ m/s}}\right)$$

which leads to $f = 4.8 \times 10^2 \text{ Hz}$.

112. (a) We proceed by dividing the (velocity) equation involving the new (fundamental) frequency f' by the equation when the frequency f is 440 Hz to obtain

$$\frac{f'\lambda}{f\lambda}=\sqrt{\frac{\tau'/\mu}{\tau/\mu}} \quad\Rightarrow\quad \frac{f'}{f}=\sqrt{\frac{\tau'}{\tau}}$$

where we are making an assumption that the mass-per-unit-length of the string does not change significantly. Thus, with $\tau' = 1.2\tau$, we have $f'/440=\sqrt{1.2}$, which gives $f'=482\text{ Hz}$.

(b) In this case, neither tension nor mass-per-unit-length change, so the wave speed v is unchanged. Hence, using Eq. 17–38 with $n=1$,

$$f'\lambda' = f\lambda \;\Rightarrow\; f'(2L') = f(2L)$$

Since $L' = \frac{2}{3}L$, we obtain $f'=\frac{3}{2}(440)=660\text{ Hz}$.

Chapter 18

1. Let T_L be the temperature and p_L be the pressure in the left-hand thermometer. Similarly, let T_R be the temperature and p_R be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be p_3. Writing Eq. 18-5 for each thermometer,

$$T_L = (273.16\,\mathrm{K})\left(\frac{p_L}{p_3}\right) \quad \text{and} \quad T_R = (273.16\,\mathrm{K})\left(\frac{p_R}{p_3}\right),$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16\,\mathrm{K})\left(\frac{p_L - p_R}{p_3}\right).$$

First, we take T_L = 373.125 K (the boiling point of water) and T_R = 273.16 K (the triple point of water). Then, $p_L - p_R$ = 120 torr. We solve

$$373.125\,\mathrm{K} - 273.16\,\mathrm{K} = (273.16\,\mathrm{K})\left(\frac{120\,\mathrm{torr}}{p_3}\right)$$

for p_3. The result is p_3 = 328 torr. Now, we let T_L = 273.16 K (the triple point of water) and T_R be the unknown temperature. The pressure difference is $p_L - p_R$ = 90.0 torr. Solving the equation

$$273.16\,\mathrm{K} - T_R = (273.16\,\mathrm{K})\left(\frac{90.0\,\mathrm{torr}}{328\,\mathrm{torr}}\right)$$

for the unknown temperature, we obtain T_R = 348 K.

2. We take p_3 to be 80 kPa for both thermometers. According to Fig. 18-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 18-5 to compute the pressure:

$$p_N = \frac{T}{273.16\,\text{K}} p_3 = \left(\frac{373.35\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.343\,\text{kPa}.$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_H = \left(\frac{373.16\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.287\,\text{kPa}.$$

(a) The difference is $p_N - p_H = 0.056$ kPa ≈ 0.06 kPa .

(b) The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer.

3. From Eq. 18-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio: (373.15 K)/(273.16 K) = 1.366.

4. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. For $x = -71$°C, this gives $y = -96$°F.

(b) The relationship between y and x may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for $y = 134$ we find $x \approx 56.7$ on the Celsius scale.

5. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. If we require $y = 2x$, then we have

$$2x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = (5)(32) = 160°\text{C}$$

which yields $y = 2x = 320°\text{F}$.

(b) In this case, we require $y = \frac{1}{2}x$ and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = -\frac{(10)(32)}{13} \approx -24.6°\text{C}$$

which yields $y = x/2 = -12.3°\text{F}$.

6. We assume scales X and Y are linearly related in the sense that reading x is related to reading y by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$-70.00 = m(-125.0) + b$$
$$-30.00 = m(375.0) + b$$

which yield the solutions $m = 40.00/500.0 = 8.000 \times 10^{-2}$ and $b = -60.00$. With these values, we find x for $y = 50.00$:

$$x = \frac{y - b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375°X.$$

7. We assume scale X is a linear scale in the sense that if its reading is x then it is related to a reading y on the Kelvin scale by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$\begin{aligned} 373.15 &= m(-53.5) + b \\ 273.15 &= m(-170) + b \end{aligned}$$

which yield the solutions $m = 100/(170 - 53.5) = 0.858$ and $b = 419$. With these values, we find x for $y = 340$:

$$x = \frac{y-b}{m} = \frac{340-419}{0.858} = -92.1°X.$$

8. The change in length for the aluminum pole is

$$\Delta \ell = \ell_0 \alpha_{Al} \Delta T = (33\,\mathrm{m})(23\times 10^{-6}\,/\,\mathrm{C}^\circ)(15\ ^\circ\mathrm{C}) = 0.011\,\mathrm{m}.$$

9. Since a volume is the product of three lengths, the change in volume due to a temperature change ΔT is given by $\Delta V = 3\alpha V\,\Delta T$, where V is the original volume and α is the coefficient of linear expansion. See Eq. 18-11. Since $V = (4\pi/3)R^3$, where R is the original radius of the sphere, then

$$\Delta V = 3\alpha\left(\frac{4\pi}{3}R^3\right)\Delta T = (23\times10^{-6}/\mathrm{C}^\circ)(4\pi)(10\,\mathrm{cm})^3(100\,^\circ\mathrm{C}) = 29\,\mathrm{cm}^3.$$

The value for the coefficient of linear expansion is found in Table 18-2.

10. (a) The coefficient of linear expansion α for the alloy is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{10.015\,\text{cm} - 10.000\,\text{cm}}{(10.01\,\text{cm})(100°\text{C} - 20.000°\text{C})} = 1.88\times10^{-5}\,/\,\text{C}°.$$

Thus, from 100°C to 0°C we have

$$\Delta L = L\alpha\Delta T = (10.015\,\text{cm})(1.88\times10^{-5}\,/\,\text{C}°)(0°\text{C} - 100°\text{C}) = -1.88 \times 10^{-2}\,\text{cm}.$$

The length at 0°C is therefore $L' = L + \Delta L$ = (10.015 cm – 0.0188 cm) = 9.996 cm.

(b) Let the temperature be T_x. Then from 20°C to T_x we have

$$\Delta L = 10.009\,\text{cm} - 10.000\,\text{cm} = \alpha L\Delta T = (1.88\times10^{-5}\,/\,\text{C}°)(10.000\,\text{cm})\,\Delta T,$$

giving ΔT = 48 °C. Thus, T_x = (20°C + 48 °C)= 68°C.

11. The new diameter is

$$D = D_0(1+\alpha_{Al}\Delta T) = (2.725\,\text{cm})[1+(23\times10^{-6}\,/\,\text{C}°)(100.0°\text{C}-0.000°\text{C})] = 2.731\,\text{cm}.$$

12. The increase in the surface area of the brass cube (which has six faces), which had side length is L at 20°, is

$$\Delta A = 6(L+\Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_b L^2 \Delta T = 12\ (19\times10^{-6}\,/\,\mathrm{C}°)\ (30\,\mathrm{cm})^2 (75°\mathrm{C} - 20°\mathrm{C})$$
$$= 11\,\mathrm{cm}^2.$$

13. The volume at 30°C is given by

$$V' = V(1+\beta\Delta T) = V(1+3\alpha\Delta T) = (50.00\,\text{cm}^3)[1+3(29.00\times10^{-6}\,/\,\text{C°})\,(30.00\text{°C}-60.00\text{°C})]$$
$$= 49.87\,\text{cm}^3$$

where we have used $\beta = 3\alpha$.

14. (a) We use $\rho = m/V$ and

$$\Delta\rho = \Delta(m/V) = m\Delta(1/V) \simeq -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L).$$

The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%.$$

(b) Since $\alpha = \Delta L/(L\Delta T\,) = (0.23 \times 10^{-2}) / (100°\text{C} - 0.0°\text{C}) = 23 \times 10^{-6}\ /\text{C}°$, the metal is aluminum (using Table 18-2).

15. If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c \Delta T$. See Eq. 18-11. If β is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is $\Delta V_g = \beta V_c \Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\Delta V_g - \Delta V_c = (\beta - 3\alpha_a)V_c\Delta T = \left[(5.1\times10^{-4}/\text{C}°) - 3(23\times10^{-6}/\text{C}°)\right](100\,\text{cm}^3)(6.0\,°\text{C})$$

$$= 0.26\,\text{cm}^3.$$

16. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s \alpha_s \Delta T = (20.11\,\text{cm})(11\times10^{-6}/\text{C°})(270°\text{C} - 20°\text{C}) = 0.055\,\text{cm}.$$

Thus, the actual change in length for the rod is

$$\Delta L = (20.11\text{ cm} - 20.05\text{ cm}) + 0.055\text{ cm} = 0.115\text{ cm}.$$

The coefficient of thermal expansion for the material of which the rod is made is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115\text{ cm}}{270°\text{C} - 20°\text{C}} = 23\times10^{-6}/\text{C°}.$$

17. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0}\,\Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0}\,\Delta T$, where D_{s0} and D_{b0} are the original diameters, α_s and α_b are the coefficients of linear expansion, and ΔT is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means

$$D_{s0} + \alpha_s D_{s0}\,\Delta T = D_{b0} + \alpha_b D_{b0}\,\Delta T.$$

Therefore,

$$\Delta T = \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000\,\text{cm} - 2.992\,\text{cm}}{\left(19.00\times10^{-6}/\text{C}^\circ\right)\left(2.992\,\text{cm}\right) - \left(11.00\times10^{-6}/\text{C}^\circ\right)\left(3.000\,\text{cm}\right)}$$
$$= 335.0\,^\circ\text{C}.$$

The temperature is $T = (25.00°\text{C} + 335.0\ °\text{C}) = 360.0°\text{C}$.

18. (a) Since $A = \pi D^2/4$, we have the differential $dA = 2(\pi D/4)dD$. Dividing the latter relation by the former, we obtain $dA/A = 2\ dD/D$. In terms of Δ's, this reads

$$\frac{\Delta A}{A} = 2\frac{\Delta D}{D} \qquad \text{for} \quad \frac{\Delta D}{D} \ll 1.$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by 2(0.18%) = 0.36%.

(b) Assuming that all dimensions are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by 3(0.18%) = 0.54%.

(d) The mass does not change.

(e) The coefficient of linear expansion is

$$\alpha = \frac{\Delta D}{D\Delta T} = \frac{0.18\times10^{-2}}{100°\text{C}} = 1.8\times10^{-5}\,/\text{C}°.$$

19. The initial volume V_0 of the liquid is h_0A_0 where A_0 is the initial cross-section area and h_0 = 0.64 m. Its final volume is $V = hA$ where $h - h_0$ is what we wish to compute. Now, the area expands according to how the glass expands, which we analyze as follows: Using $A = \pi r^2$, we obtain

$$dA = 2\pi r\,dr = 2\pi r\left(r\alpha\,dT\right) = 2\alpha(\pi r^2)dT = 2\alpha A\,dT\ .$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0\left(1+\beta_{\text{liquid}}\Delta T\right)}{A_0\left(1+2\alpha_{\text{glass}}\Delta T\right)}.$$

Thus, with $V_0/A_0 = h_0$ we obtain

$$h - h_0 = h_0\left(\frac{1+\beta_{\text{liquid}}\Delta T}{1+2\alpha_{\text{glass}}\Delta T} - 1\right) = (0.64)\left(\frac{1+\left(4\times10^{-5}\right)\left(10°\right)}{1+2\left(1\times10^{-5}\right)\left(10°\right)}\right) = 1.3\times10^{-4}\ \text{m}.$$

20. We divide Eq. 18-9 by the time increment Δt and equate it to the (constant) speed $v = 100 \times 10^{-9}$ m/s.

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where $L_0 = 0.0200$ m and $\alpha = 23 \times 10^{-6}$/C°. Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217 \frac{\text{C}°}{\text{s}} = 0.217 \frac{\text{K}}{\text{s}}.$$

21. Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha\ell_0\Delta T$. The old position of the half-bar, its new position, and the distance x that one end is displaced form a right triangle, with a hypotenuse of length ℓ, one side of length ℓ_0, and the other side of length x. The Pythagorean theorem yields

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha\Delta T)^2 - \ell_0^2.$$

Since the change in length is small we may approximate $(1 + \alpha\,\Delta T\,)^2$ by $1 + 2\alpha\,\Delta T$, where the small term $(\alpha\,\Delta T\,)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\,\Delta T - \ell_0^2 = 2\ell_0^2\alpha\,\Delta T$$

and

$$x = \ell_0\sqrt{2\alpha\,\Delta T} = \frac{3.77\,\text{m}}{2}\sqrt{2\left(25\times10^{-6}/\text{C}^\circ\right)\left(32^\circ\text{C}\right)} = 7.5\times10^{-2}\ \text{m}.$$

22. (a) The specific heat is given by $c = Q/m(T_f - T_i)$, where Q is the heat added, m is the mass of the sample, T_i is the initial temperature, and T_f is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\,\text{J}}{\left(30.0\times10^{-3}\,\text{kg}\right)\left(45.0°\text{C} - 25.0°\text{C}\right)} = 523\,\text{J/kg}\cdot\text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N\left(T_f - T_i\right)} = \frac{314\,\text{J}}{\left(0.600\,\text{mol}\right)\left(45.0°\text{C} - 25.0°\text{C}\right)} = 26.2\,\text{J/mol}\cdot\text{K}.$$

(c) If N is the number of moles of the substance and M is the mass per mole, then $m = NM$, so

$$N = \frac{m}{M} = \frac{30.0\times10^{-3}\,\text{kg}}{50\times10^{-3}\,\text{kg/mol}} = 0.600\,\text{mol}.$$

23. We use $Q = cm\Delta T$. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass m of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500 \times 10^3 \text{ cal}}{(1 \text{ g/cal} \cdot \text{C}°)(37.0°\text{C} - 0.0°\text{C})} = 94.6 \times 10^4 \text{ g},$$

which is equivalent to 9.46×10^4 g/(1000 g/liter) = 94.6 liters of water. This is certainly too much to drink in a single day!

24. The amount of water m that is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2\,\text{kJ}}{333\,\text{kJ/kg}} = 0.151\,\text{kg} = 151\,\text{g}.$$

Therefore the amount of water which remains unfrozen is 260 g – 151 g = 109 g.

25. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236\,\mathrm{J/kg\cdot K})(0.130\,\mathrm{kg})(1235°\mathrm{C} - 288°\mathrm{C}) = 2.91\times10^4\ \mathrm{J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q = mL_F = \left(0.130\,\mathrm{kg}\right)\left(105\times10^3\ \mathrm{J/kg}\right) = 1.36\times10^4\ \mathrm{J}.$$

The total heat required is (2.91×10^4 J + 1.36×10^4 J) = 4.27×10^4 J.

26. (a) The water (of mass m) releases energy in two steps, first by lowering its temperature from 20°C to 0°C, and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m\Delta T + L_F m = (4190\,\text{J/kg}\cdot\text{K})(125\,\text{kg})(20°\text{C}) + (333\,\text{kJ/kg})(125\,\text{kg}) = 5.2\times10^{7}\,\text{J}.$$

(b) Before all the water freezes, the lowest temperature possible is 0°C, below which the water must have already turned into ice.

27. The mass m = 0.100 kg of water, with specific heat c = 4190 J/kg·K, is raised from an initial temperature T_i = 23°C to its boiling point T_f = 100°C. The heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater P multiplied by the time t; $Q = Pt$. Thus,

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(100^\circ\mathrm{C} - 23^\circ\mathrm{C})}{200\,\mathrm{J/s}} = 160\,\mathrm{s}.$$

28. The work the man has to do to climb to the top of Mt. Everest is given by

$$W = mgy = (73.0 \text{ kg})(9.80 \text{ m/s}^2)(8840 \text{ m}) = 6.32 \times 10^6 \text{ J}.$$

Thus, the amount of butter needed is

$$m = \frac{(6.32\times10^6 \text{ J}) \left(\frac{1.00\,\text{cal}}{4.186\,\text{J}}\right)}{6000\,\text{cal/g}} \approx 250\,\text{g}.$$

29. Let the mass of the steam be m_s and that of the ice be m_i. Then

$$L_F m_c + c_w m_c (T_f - 0.0°\text{C}) = m_s L_s + m_s c_w (100°\text{C} - T_f),$$

where T_f = 50°C is the final temperature. We solve for m_s:

$$m_s = \frac{L_F m_c + c_w m_c (T_f - 0.0°\text{C})}{L_s + c_w (100°\text{C} - T_f)} = \frac{(79.7\,\text{cal/g})(150\,\text{g}) + (1\,\text{cal/g}\cdot°\text{C})(150\text{g})(50°\text{C} - 0.0°\text{C})}{539\,\text{cal/g} + (1\,\text{cal/g}\cdot\text{C°})(100°\text{C} - 50°\text{C})}$$

$$= 33\,\text{g}.$$

30. (a) Using Eq. 18-17, the heat transferred to the water is

$$Q_w = c_w m_w \Delta T + L_V m_s = (1\,\text{cal/g}\cdot\text{C°})(220\,\text{g})(100°\text{C} - 20.0°\text{C}) + (539\,\text{cal/g})(5.00\,\text{g})$$
$$= 20.3\,\text{kcal}.$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923\,\text{cal/g}\cdot\text{C°})(150\,\text{g})(100°\text{C} - 20.0°\text{C}) = 1.11\,\text{kcal}.$$

(c) If the original temperature of the cylinder be T_i, then $Q_w + Q_b = c_c m_c (T_i - T_f)$, which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3\,\text{kcal} + 1.11\,\text{kcal}}{(0.0923\,\text{cal/g}\cdot\text{C°})(300\,\text{g})} + 100°\text{C} = 873°\text{C}.$$

31. We note from Eq. 18-12 that 1 Btu = 252 cal. The heat relates to the power, and to the temperature change, through $Q = Pt = cm\Delta T$. Therefore, the time t required is

$$t = \frac{cm\Delta T}{P} = \frac{(1000\,\text{cal/kg}\cdot\text{C}°)(40\,\text{gal})(1000\,\text{kg}/264\,\text{gal})(100°\text{F} - 70°\text{F})(5°\text{C}/9°\text{F})}{(2.0\times 10^5\,\text{Btu/h})(252.0\,\text{cal/Btu})(1\,\text{h}/60\,\text{min})}$$
$$= 3.0\,\text{min}.$$

The metric version proceeds similarly:

$$t = \frac{c\rho V\Delta T}{P} = \frac{(4190\text{ J/kg}\cdot\text{C}°)(1000\text{ kg/m}^3)(150\text{ L})(1\text{ m}^3/1000\text{ L})(38°\text{C} - 21°\text{C})}{(59000\text{ J/s})(60\text{ s}/1\text{ min})}$$
$$= 3.0\,\text{min}.$$

32. We note that the heat capacity of sample B is given by the reciprocal of the slope of the line in Figure 18-32(b) (compare with Eq. 18-14). Since the reciprocal of that slope is 16/4 = 4 kJ/kg·C°, then c_B = 4000 J/kg·C° = 4000 J/kg·K (since a change in Celsius is equivalent to a change in Kelvins). Now, following the same procedure as shown in Sample Problem 18-4, we find

$$c_A\, m_A\, (T_f - T_A) + c_B\, m_B\, (T_f - T_B) = 0$$

$$c_A\, (5.0 \text{ kg})(40°\text{C} - 100°\text{C}) + (4000 \text{ J/kg·C°})(1.5 \text{ kg})(40°\text{C} - 20°\text{C}) = 0$$

which leads to $c_A = 4.0\times10^2$ J/kg·K.

33. The power consumed by the system is

$$P=\left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t}=\left(\frac{1}{20\%}\right)\frac{(4.18\,\text{J/g}\cdot{}^\circ\text{C})(200\times10^3\,\text{cm}^3)(1\,\text{g/cm}^3)(40^\circ\text{C}-20^\circ\text{C})}{(1.0\,\text{h})(3600\,\text{s/h})}$$
$$=2.3\times10^4\ \text{W}.$$

The area needed is then $A=\dfrac{2.3\times10^4\ \text{W}}{700\,\text{W/m}^2}=33\,\text{m}^2$.

34. While the sample is in its liquid phase, its temperature change (in absolute values) is $|\Delta T| = 30\,°\text{C}$. Thus, with $m = 0.40$ kg, the absolute value of Eq. 18-14 leads to

$$|Q| = c\,m\,|\Delta T| = (3000\ \text{J/kg}\cdot°\text{C})(0.40\ \text{kg})(30\,°\text{C}) = 36000\ \text{J}\,.$$

The rate (which is constant) is

$$P = |Q|\,/\,t = (36000\ \text{J})/(40\ \text{min}) = 900\ \text{J/min},$$

which is equivalent to 15 Watts.

(a) During the next 30 minutes, a phase change occurs which is described by Eq. 18-16:

$$|Q| = P\,t = (900\ \text{J/min})(30\ \text{min}) = 27000\ \text{J} = L\,m\,.$$

Thus, with $m = 0.40$ kg, we find $L = 67500$ J/kg $\approx$ 68 kJ/kg.

(b) During the final 20 minutes, the sample is solid and undergoes a temperature change (in absolute values) of $|\Delta T| = 20\,\text{C}°$. Now, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\,|\Delta T|} = \frac{P\,t}{m\,|\Delta T|} = \frac{(900)(20)}{(0.40)(20)} = 2250\ \frac{\text{J}}{\text{kg}\cdot\text{C}°} \approx\ 2.3\ \frac{\text{kJ}}{\text{kg}\cdot\text{C}°}\,.$$

35. We denote the ice with subscript I and the coffee with c, respectively. Let the final temperature be T_f. The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w\,(T_f - 0°\text{C}),$$

and the heat given away by the coffee is $|Q_c| = m_w c_w\,(T_I - T_f)$. Setting $Q_I = |Q_c|$, we solve for T_f:

$$T_f = \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c)c_w} = \frac{(130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})\,(80.0°\text{C}) - (333\times10^3\ \text{J/g})(12.0\,\text{g})}{(12.0\,\text{g} + 130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})}$$
$$= 66.5°\text{C}.$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T| = 80.0°\text{C} - 66.5°\text{C} = 13.5\text{C°}$.

36. (a) Eq. 18-14 (in absolute value) gives

$$|Q| = (4190 \text{ J/ kg}\cdot°\text{C})(0.530 \text{ kg})(40 °\text{C}) = 88828 \text{ J}.$$

Since $\frac{dQ}{dt}$ is assumed constant (we will call it P) then we have

$$P = \frac{88828 \text{ J}}{40 \text{ min}} = \frac{88828 \text{ J}}{2400 \text{ s}} = 37 \text{ W} .$$

(b) During that same time (used in part (a)) the ice warms by 20 C°. Using Table 18-3 and Eq. 18-14 again we have

$$m_{\text{ice}} = \frac{Q}{c_{\text{ice}}\Delta T} = \frac{88828}{(2220)(20°)} = 2.0 \text{ kg} .$$

(c) To find the ice produced (by freezing the water that has already reached 0°C – so we concerned with the $40 \text{ min} < t < 60 \text{ min}$ time span), we use Table 18-4 and Eq. 18-16:

$$m_{\text{water becoming ice}} = \frac{Q_{20 \text{ min}}}{L_F} = \frac{44414}{333000} = 0.13 \text{ kg}.$$

37. To accomplish the phase change at 78°C,

$$Q = L_V m = (879 \text{ kJ/kg})\,(0.510 \text{ kg}) = 448.29 \text{ kJ}$$

must be removed. To cool the liquid to –114°C,

$$Q = cm|\Delta T| = (2.43 \text{ kJ/ kg}\cdot\text{K})\,(0.510 \text{ kg})\,(192 \text{ K}) = 237.95 \text{ kJ},$$

must be removed. Finally, to accomplish the phase change at –114°C,

$$Q = L_F m = (109 \text{ kJ/kg})\,(0.510 \text{ kg}) = 55.59 \text{ kJ}$$

must be removed. The grand total of heat removed is therefore (448.29 + 237.95 + 55.59) kJ = 742 kJ.

38. The heat needed is found by integrating the heat capacity:

$$Q = \int_{T_i}^{T_f} cm\ dT = m\int_{T_i}^{T_f} c dT = (2.09)\int_{5.0°C}^{15.0°C} (0.20 + 0.14T + 0.023T^2)\,dT$$

$$= (2.0)(0.20T + 0.070T^2 + 0.00767T^3)\Big|_{5.0}^{15.0}\ \text{(cal)}$$

$$= 82\,\text{cal.}$$

39. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is T_f then the energy absorbed as heat by the ice is

$$Q_I = L_F m_I + c_w m_I (T_f - 0°\text{C}),$$

while the energy transferred as heat from the water is $Q_w = c_w m_w (T_f - T_i)$. The system is insulated, so $Q_w + Q_I = 0$, and we solve for T_f:

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_C) c_w}.$$

(a) Now $T_i = 90°\text{C}$ so

$$T_f = \frac{(4190\,\text{J/kg}\cdot\text{C°})(0.500\,\text{kg})(90°\text{C}) - (333\times10^3\,\text{J/kg})(0.500\,\text{kg})}{(0.500\,\text{kg} + 0.500\,\text{kg})(4190\,\text{J/kg}\cdot\text{C°})} = 5.3°\text{C}.$$

(b) Since no ice has remained at $T_f = 5.3°C$, we have $m_f = 0$.

(c) If we were to use the formula above with $T_i = 70°\text{C}$, we would get $T_f < 0$, which is impossible. In fact, not all the ice has melted in this case and the equilibrium temperature is $T_f = 0°\text{C}$.

(d) The amount of ice that melts is given by

$$m_I' = \frac{c_w m_w (T_i - 0°\text{C})}{L_F} = \frac{(4190\,\text{J/kg}\cdot\text{C°})(0.500\,\text{kg})(70\,\text{C°})}{333\times10^3\,\text{J/kg}} = 0.440\,\text{kg}.$$

Therefore, the amount of (solid) ice remaining is $m_f = m_I - m'_I = 500\text{ g} - 440\text{ g} = 60.0\text{ g}$, and (as mentioned) we have $T_f = 0°\text{C}$ (because the system is an ice-water mixture in thermal equilibrium).

40. (a) Using Eq. 18-32, we find the rate of energy conducted upward to be

$$P_{\text{cond}} = \frac{Q}{t} = kA\frac{T_H - T_C}{L} = (0.400\ \text{W/m}\cdot{}^\circ\text{C})A\frac{5.0\ {}^\circ\text{C}}{0.12\ \text{m}} = (16.7A)\ \text{W}.$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

(b) The heat of fusion in this process is $Q = L_F m$, where $L_F = 3.33\times10^5$ J/kg . Differentiating the expression with respect to t and equating the result with P_{cond}, we have

$$P_{\text{cond}} = \frac{dQ}{dt} = L_F\frac{dm}{dt}.$$

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{\text{cond}}}{L_F} = \frac{16.7A\ \text{W}}{3.33\times10^5\ \text{J/kg}} = (5.02\times10^{-5}A)\ \text{kg/s}.$$

(c) Since $m = \rho V = \rho A h$, differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}(\rho A h) = \rho A\frac{dh}{dt}.$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A}\frac{dm}{dt} = \frac{5.02\times10^{-5}\,\text{kg/m}^2\cdot\text{s}}{1000\ \text{kg/m}^3} = 5.02\times10^{-8}\ \text{m/s}$$

41. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

• None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

• The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

• All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, suppose that no ice melts. The temperature of the water decreases from T_{Wi} = 25°C to some final temperature T_f and the temperature of the ice increases from T_{Ii} = –15°C to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W (T_{Wi} - T_f).$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})} \\ &= 16.6°\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at T_f = 0°C, with mass m (< m_I) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + mL_F,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F.$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25°\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15°\mathrm{C})}{333\times 10^3\,\mathrm{J/kg}} \\ &= 5.3\times 10^{-2}\,\mathrm{kg} = 53\,\mathrm{g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain T_f = 2.5°C.

42. If the ring diameter at 0.000°C is D_{r0} then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}\,(1 + \alpha_c T_f),$$

where T_f is the final temperature and α_c is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at T_i (= 100.0°C) is D_{s0} then its diameter at the final temperature is

$$D_s = D_{s0}\,[1 + \alpha_a\,(T_f - T_i)],$$

where α_a is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a (T_f - T_i)].$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})(23\times10^{-6}/\text{C°})(100.0°\text{C})}{(2.54508\,\text{cm})(23\times10^{-6}/\text{C°}) - (2.54000\,\text{cm})\,(17\times10^{-6}/\text{C°})} \\ &= 50.38°\text{C}. \end{aligned}$$

The expansion coefficients are from Table 18-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is $Q = c_c m_r T_f$, where c_c is the specific heat of copper and m_r is the mass of the ring. The heat rejected up by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where c_a is the specific heat of aluminum and m_s is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s\,(T_i - T_f),$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a (T_i - T_f)} = \frac{(386\,\text{J/kg}\cdot\text{K})(0.0200\,\text{kg})(50.38°\text{C})}{(900\,\text{J/kg}\cdot\text{K})(100°\text{C} - 50.38°\text{C})} = 8.71\times10^{-3}\,\text{kg}.$$

43. Over a cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$. Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write

$$p = \frac{10}{3}\ \text{Pa} + \left(\frac{20}{3}\ \text{Pa/m}^3\right)V,$$

where the coefficients were chosen so that $p = 10$ Pa when $V = 1.0\ \text{m}^3$ and $p = 30$ Pa when $V = 4.0\ \text{m}^3$. The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p\,dV = \int_1^4 \left(\frac{10}{3} + \frac{20}{3}V\right) dV = \left(\frac{10}{3}V + \frac{10}{3}V^2\right)\Bigg|_1^4 \\ &= \left(\frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3}\right)\text{J} = 60\ \text{J}. \end{aligned}$$

The BC portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p\Delta V = (30\ \text{Pa})(1.0\ \text{m}^3 - 4.0\ \text{m}^3) = -90\ \text{J}.$$

The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA} = 60\ \text{J} - 90\ \text{J} + 0 = -30\ \text{J}$$

and the total heat absorbed is $Q = W = -30$ J. This means the gas loses 30 J of energy in the form of heat.

44. (a) Since work is done *on* the system (perhaps to compress it) we write $W = -200$ J.

(b) Since heat leaves the system, we have $Q = -70.0$ cal $= -293$ J.

(c) The change in internal energy is $\Delta E_{\text{int}} = Q - W = -293 \text{ J} - (-200 \text{ J}) = -93$ J.

45. (a) One part of path A represents a constant pressure process. The volume changes from 1.0 m^3 to 4.0 m^3 while the pressure remains at 40 Pa. The work done is

$$W_A = p\Delta V = (40\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 1.2\times 10^2\ \text{J}.$$

(b) The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is 120 J. To find the work done over path B we need to know the pressure as a function of volume. Then, we can evaluate the integral $W = \int p\, dV$. According to the graph, the pressure is a linear function of the volume, so we may write $p = a + bV$, where a and b are constants. In order for the pressure to be 40 Pa when the volume is 1.0 m^3 and 10 Pa when the volume is 4.00 m^3 the values of the constants must be a = 50 Pa and b = –10 Pa/m^3. Thus,

$$p = 50\ \text{Pa} - (10\ \text{Pa/m}^3)V$$

and

$$W_B = \int_1^4 p\, dV = \int_1^4 (50 - 10V)\, dV = \left(50V - 5V^2\right)\Big|_1^4 = 200\,\text{J} - 50\,\text{J} - 80\,\text{J} + 5.0\,\text{J} = 75\,\text{J}.$$

(c) One part of path C represents a constant pressure process in which the volume changes from 1.0 m^3 to 4.0 m^3 while p remains at 10 Pa. The work done is

$$W_C = p\Delta V = (10\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 30\,\text{J}.$$

The other part of the process is at constant volume and no work is done. The total work is 30 J. We note that the work is different for different paths.

46. During process $A \to B$, the system is expanding, doing work on its environment, so $W > 0$, and since $\Delta E_{int} > 0$ is given then $Q = W + \Delta E_{int}$ must also be positive.

(a) $Q > 0$.

(b) $W > 0$.

During process $B \to C$, the system is neither expanding nor contracting. Thus,

(c) $W = 0$.

(d) The sign of ΔE_{int} must be the same (by the first law of thermodynamics) as that of Q which is given as positive. Thus, $\Delta E_{int} > 0$.

During process $C \to A$, the system is contracting. The environment is doing work on the system, which implies $W < 0$. Also, $\Delta E_{int} < 0$ because $\sum \Delta E_{int} = 0$ (for the whole cycle) and the other values of ΔE_{int} (for the other processes) were positive. Therefore, $Q = W + \Delta E_{int}$ must also be negative.

(e) $Q < 0$.

(f) $W < 0$.

(g) $\Delta E_{int} < 0$.

(h) The area of a triangle is $\frac{1}{2}$ (base)(height). Applying this to the figure, we find $|W_{net}| = \frac{1}{2}(2.0\,\text{m}^3)(20\,\text{Pa}) = 20\,\text{J}$. Since process $C \to A$ involves larger negative work (it occurs at higher average pressure) than the positive work done during process $A \to B$, then the net work done during the cycle must be negative. The answer is therefore $W_{net} = -20$ J.

47. We note that there is no work done in the process going from d to a, so $Q_{da} = \Delta E_{\text{int } da}$ = 80 J. Also, since the total change in internal energy around the cycle is zero, then

$$\Delta E_{\text{int } ac} + \Delta E_{\text{int } cd} + \Delta E_{\text{int } da} = 0$$

$$-200 \text{ J} + \Delta E_{\text{int } cd} + 80 \text{ J} = 0$$

which yields $\Delta E_{\text{int } cd} = 120$ J. Thus, applying the first law of thermodynamics to the c to d process gives the work done as

$$W_{cd} = Q_{cd} - \Delta E_{\text{int } cd} = 180 \text{ J} - 120 \text{ J} = 60 \text{ J}.$$

48. (a) We note that process a to b is an expansion, so $W > 0$ for it. Thus, $W_{ab} = +5.0$ J. We are told that the change in internal energy during that process is +3.0 J, so application of the first law of thermodynamics for that process immediately yields $Q_{ab} = +8.0$ J.

(b) The net work (+1.2 J) is the same as the net heat ($Q_{ab} + Q_{bc} + Q_{ca}$), and we are told that $Q_{ca} = +2.5$ J. Thus we readily find $Q_{bc} = (1.2 - 8.0 - 2.5)\text{ J} = -9.3$ J.

49. (a) The change in internal energy ΔE_{int} is the same for path *iaf* and path *ibf*. According to the first law of thermodynamics, $\Delta E_{int} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along *iaf*

$$\Delta E_{int} = Q - W = 50 \text{ cal} - 20 \text{ cal} = 30 \text{ cal}.$$

Along *ibf* ,

$$W = Q - \Delta E_{int} = 36 \text{ cal} - 30 \text{ cal} = 6.0 \text{ cal}.$$

(b) Since the curved path is traversed from f to i the change in internal energy is –30 cal and $Q = \Delta E_{int} + W = -30 \text{ cal} - 13 \text{ cal} = -43 \text{ cal}$.

(c) Let $\Delta E_{int} = E_{int,\,f} - E_{int,\,i}$. Then, $E_{int,\,f} = \Delta E_{int} + E_{int,\,i} = 30 \text{ cal} + 10 \text{ cal} = 40 \text{ cal}$.

(d) The work W_{bf} for the path *bf* is zero, so $Q_{bf} = E_{int,\,f} - E_{int,\,b} = 40 \text{ cal} - 22 \text{ cal} = 18 \text{ cal}$.

(e) For the path *ibf*, $Q = 36$ cal so $Q_{ib} = Q - Q_{bf} = 36 \text{ cal} - 18 \text{ cal} = 18 \text{ cal}$.

50. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and

$$Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0\ \text{J} - 20.0\ \text{J} - 0 = -5.0\ \text{J}.$$

This means 5.0 J of energy leaves the gas in the form of heat.

51. The rate of heat flow is given by

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L},$$

where k is the thermal conductivity of copper (401 W/m·K), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C. Thus,

$$P_{\text{cond}} = \frac{(401\,\text{W/m}\cdot\text{K})(90.0\times10^{-4}\,\text{m}^2)(125^\circ\text{C} - 10.0^\circ\text{C})}{0.250\,\text{m}} = 1.66\times10^3\ \text{J/s}.$$

The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

52. (a) We estimate the surface area of the average human body to be about 2 m^2 and the skin temperature to be about 300 K (somewhat less than the internal temperature of 310 K). Then from Eq. 18-37

$$P_r = \sigma\varepsilon AT^4 \approx \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(0.9)\left(2.0\,\text{m}^2\right)(300\,\text{K})^4 = 8\times10^2\ \text{W}.$$

(b) The energy lost is given by

$$\Delta E = P_r\Delta t = \left(8\times10^2\ \text{W}\right)(30\,\text{s}) = 2\times10^4\ \text{J}.$$

53. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(401\,\text{W/m}\cdot\text{K})(4.8\times10^{-4}\,\text{m}^2)(100\,°\text{C})}{1.2\,\text{m}} = 16\ \text{J/s}.$$

(b) Using Table 18-4, the rate at which ice melts is

$$\left|\frac{dm}{dt}\right| = \frac{P_{\text{cond}}}{L_F} = \frac{16\,\text{J/s}}{333\,\text{J/g}} = 0.048\,\text{g/s}.$$

54. We refer to the polyurethane foam with subscript p and silver with subscript s. We use Eq. 18–32 to find $L = kR$.

(a) From Table 18-6 we find $k_p = 0.024$ W/m·K so

$$\begin{aligned} L_p &= k_p R_p \\ &= (0.024\,\mathrm{W/m\cdot K})(30\,\mathrm{ft^2\cdot F^\circ\cdot h/Btu})(1\,\mathrm{m}/3.281\,\mathrm{ft})^2(5\mathrm{C}^\circ/9\mathrm{F}^\circ)(3600\,\mathrm{s/h})(1\,\mathrm{Btu}/1055\,\mathrm{J}) \\ &= 0.13\,\mathrm{m}. \end{aligned}$$

(b) For silver $k_s = 428$ W/m·K, so

$$L_s = k_s R_s = \left(\frac{k_s R_s}{k_p R_p}\right) L_p = \left[\frac{428(30)}{0.024(30)}\right](0.13\,\mathrm{m}) = 2.3\times10^3\ \mathrm{m}.$$

55. We use Eqs. 18-38 through 18-40. Note that the surface area of the sphere is given by $A = 4\pi r^2$, where $r = 0.500$ m is the radius.

(a) The temperature of the sphere is $T = (273.15 + 27.00)$ K = 300.15 K. Thus

$$P_r = \sigma\varepsilon AT^4 = \left(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\right)(0.850)(4\pi)(0.500\,\mathrm{m})^2(300.15\,\mathrm{K})^4$$
$$= 1.23\times10^3\ \mathrm{W}.$$

(b) Now, $T_{env} = 273.15 + 77.00 = 350.15$ K so

$$P_a = \sigma\varepsilon AT_{env}^4 = (5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4})(0.850)(4\pi)(0.500\,\mathrm{m})^2(350.15\,\mathrm{K})^4 = 2.28\times10^3\ \mathrm{W}.$$

(c) From Eq. 18-40, we have

$$P_n = P_a - P_r = 2.28\times10^3\ \mathrm{W} - 1.23\times10^3\ \mathrm{W} = 1.05\times10^3\ \mathrm{W}.$$

56. (a) The surface area of the cylinder is given by

$$A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5\times10^{-2}\,\text{m})^2 + 2\pi(2.5\times10^{-2}\,\text{m})(5.0\times10^{-2}\,\text{m}) = 1.18\times10^{-2}\,\text{m}^2,$$

its temperature is T_1 = 273 + 30 = 303 K, and the temperature of the environment is T_{env} = 273 + 50 = 323 K. From Eq. 18-39 we have

$$P_1 = \sigma\varepsilon A_1\left(T_{\text{env}}^4 - T^4\right) = (0.85)\left(1.18\times10^{-2}\,\text{m}^2\right)\left((323\,\text{K})^4 - (303\,\text{K})^4\right) = 1.4\,\text{W}.$$

(b) Let the new height of the cylinder be h_2. Since the volume V of the cylinder is fixed, we must have $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$. We solve for h_2:

$$h_2 = \left(\frac{r_1}{r_2}\right)^2 h_1 = \left(\frac{2.5\,\text{cm}}{0.50\,\text{cm}}\right)^2 (5.0\,\text{cm}) = 125\,\text{cm} = 1.25\,\text{m}.$$

The corresponding new surface area A_2 of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi(0.50\times10^{-2}\,\text{m})^2 + 2\pi(0.50\times10^{-2}\,\text{m})(1.25\,\text{m}) = 3.94\times10^{-2}\,\text{m}^2.$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94\times10^{-2}\,\text{m}^2}{1.18\times10^{-2}\,\text{m}^2} = 3.3.$$

57. We use $P_{\text{cond}} = kA\Delta T/L \propto A/L$. Comparing cases (*a*) and (*b*) in Figure 18–44, we have

$$P_{\text{cond }b} = \left(\frac{A_b L_a}{A_a L_b}\right) P_{\text{cond }a} = 4P_{\text{cond }a}.$$

Consequently, it would take 2.0 min/4 = 0.50 min for the same amount of heat to be conducted through the rods welded as shown in Fig. 18-44(*b*).

58. (a) As in Sample Problem 18-6, we take the rate of conductive heat transfer through each layer to be the same. Thus, the rate of heat transfer across the entire wall P_w is equal to the rate across layer 2 (P_2). Using Eq. 18-37 and canceling out the common factor of area A, we obtain

$$\frac{T_H - T_c}{(L_1/k_1 + L_2/k_2 + L_3/k_3)} = \frac{\Delta T_2}{(L_2/k_2)} \quad \Rightarrow \quad \frac{45\ \text{C}°}{(1 + 7/9 + 35/80)} = \frac{\Delta T_2}{(7/9)}$$

which leads to $\Delta T_2 = 15.8$ °C.

(b) We expect (and this is supported by the result in the next part) that greater conductivity should mean a larger rate of conductive heat transfer.

(c) Repeating the calculation above with the new value for k_2, we have

$$\frac{45\ \text{C}°}{(1 + 7/11 + 35/80)} = \frac{\Delta T_2}{(7/11)}$$

which leads to $\Delta T_2 = 13.8$ °C. This is less than our part (a) result which implies that the temperature gradients across layers 1 and 3 (the ones where the parameters did not change) are greater than in part (a); those larger temperature gradients lead to larger conductive heat currents (which is basically a statement of "Ohm's law as applied to heat conduction").

59. (a) We use

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as 1.0 W/m·K. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72°\text{F} - (-20°\text{F}) = 92\ °\text{F}$$

is equivalent to $\frac{5}{9}(92) = 51.1\text{C}°$. This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k\frac{T_H - T_C}{L} = (1.0\,\text{W/m}\cdot\text{K})\left(\frac{51.1°\text{C}}{3.0\times10^{-3}\text{m}}\right) = 1.7\times10^4\ \text{W/m}^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate P is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If L_g is the thickness of a glass layer, L_a is the thickness of the air layer, k_g is the thermal conductivity of glass, and k_a is the thermal conductivity of air, then the denominator is

$$\sum\frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\frac{P_{\text{cond}}}{A} = \frac{(T_H - T_C)k_a k_g}{2L_g k_a + L_a k_g} = \frac{(51.1°\text{C})(0.026\,\text{W/m}\cdot\text{K})(1.0\,\text{W/m}\cdot\text{K})}{2(3.0\times10^{-3}\text{m})(0.026\,\text{W/m}\cdot\text{K}) + (0.075\,\text{m})(1.0\,\text{W/m}\cdot\text{K})}$$

$$= 18\,\text{W/m}^2.$$

60. The surface area of the ball is $A = 4\pi R^2 = 4\pi(0.020 \text{ m})^2 = 5.03\times10^{-3} \text{ m}^2$. Using Eq. 18-37 with $T_i = 35+273 = 308 \text{ K}$ and $T_f = 47+273 = 320 \text{ K}$, the power required to maintain the temperature is

$$P_r = \sigma\varepsilon A(T_f^4 - T_i^4) \approx (5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(0.80)(5.03\times10^{-3} \text{ m}^2)\left[(320 \text{ K})^4 - (308 \text{ K})^4\right]$$
$$= 0.34 \text{ W}.$$

Thus, the heat each bee must produce during the 20-minutes interval is

$$\frac{Q}{N} = \frac{P_r t}{N} = \frac{(0.34 \text{ W})(20 \text{ min})(60 \text{ s/min})}{500} = 0.81 \text{ J}.$$

61. We divide both sides of Eq. 18-32 by area A, which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{\text{cond}}}{A} = k_1 \frac{T_H - T_1}{L_1} = k_4 \frac{T - T_C}{L_4}$$

where $T_H = 30°\text{C}$, $T_1 = 25°\text{C}$ and $T_C = -10°\text{C}$. We solve for the unknown T.

$$T = T_C + \frac{k_1 L_4}{k_4 L_1}\left(T_H - T_1\right) = -4.2°\text{C}.$$

62. (a) For each individual penguin, the surface area that radiates is the sum of the top surface area and the sides:

$$A_r = a + 2\pi rh = a + 2\pi\sqrt{\frac{a}{\pi}}h = a + 2h\sqrt{\pi a}\ ,$$

where we have used $r = \sqrt{a/\pi}$ (from $a = \pi r^2$) for the radius of the cylinder. For the huddled cylinder, the radius is $r' = \sqrt{Na/\pi}$ (since $Na = \pi r'^2$), and the total surface area is

$$A_h = Na + 2\pi r'h = Na + 2\pi\sqrt{\frac{Na}{\pi}}h = Na + 2h\sqrt{N\pi a}\ .$$

Since the power radiated is proportional to the surface area, we have

$$\frac{P_h}{NP_r} = \frac{A_h}{NA_r} = \frac{Na + 2h\sqrt{N\pi a}}{N(a + 2h\sqrt{\pi a})} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}}\ .$$

With $N = 1000$, $a = 0.34\ \text{m}^2$ and $h = 1.1\ \text{m},$ the ratio is

$$\frac{P_h}{NP_r} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}} = \frac{1 + 2(1.1\ \text{m})\sqrt{\pi/(1000 \cdot 0.34\ \text{m}^2)}}{1 + 2(1.1\ \text{m})\sqrt{\pi/(0.34\ \text{m}^2)}} = 0.16\ .$$

(b) The total radiation loss is reduced by $1.00 - 0.16 = 0.84$, or 84%.

63. We assume (although this should be viewed as a "controversial" assumption) that the top surface of the ice is at T_C = –5.0°C. Less controversial are the assumptions that the bottom of the body of water is at T_H = 4.0°C and the interface between the ice and the water is at T_X = 0.0°C. The primary mechanism for the heat transfer through the total distance L = 1.4 m is assumed to be conduction, and we use Eq. 18-34:

$$\frac{k_{\text{water}} A(T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}} A(T_X - T_C)}{L_{\text{ice}}} \quad \Rightarrow \quad \frac{(0.12)A\left(4.0^\circ - 0.0^\circ\right)}{1.4 - L_{\text{ice}}} = \frac{(0.40)A\left(0.0^\circ + 5.0^\circ\right)}{L_{\text{ice}}}.$$

We cancel the area A and solve for thickness of the ice layer: L_{ice} = 1.1 m.

64. (a) Using Eq. 18-32, the rate of energy flow through the surface is

$$P_{\text{cond}} = \frac{kA\left(T_s - T_w\right)}{L} = (0.026\ \text{W/m}\cdot\text{K})(4.00\times10^{-6}\ \text{m}^2)\frac{300°\text{C} - 100°\text{C}}{1.0\times10^{-4}\ \text{m}} = 0.208\text{W} \approx 0.21\ \text{W}.$$

(Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.)

(b) With $P_{\text{cond}}t = L_V m = L_V(\rho V) = L_V(\rho A h),$ the drop will last a duration of

$$t = \frac{L_V \rho A h}{P_{\text{cond}}} = \frac{(2.256\times10^6\ \text{J/kg})(1000\ \text{kg/m}^3)(4.00\times10^{-6}\ \text{m}^2)(1.50\times10^{-3}\ \text{m})}{0.208\text{W}} = 65\ \text{s}.$$

65. Let h be the thickness of the slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{h}$$

where k is the thermal conductivity of ice, T_H is the temperature of the water (0°C), and T_C is the temperature of the air above the ice (–10°C). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho A h$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt}.$$

We equate the two expressions for P_{cond} and solve for dh/dt:

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h}.$$

Since 1 cal = 4.186 J and 1 cm = 1×10^{-2} m, the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s·cm·K})\ (4.186 \text{ J/cal})/(1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m·K}.$$

The density of ice is $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$. Thus,

$$\frac{dh}{dt} = \frac{(1.674\ \text{W/m}\cdot\text{K})(0°\text{C} + 10°\text{C})}{(333\times10^3\ \text{J/kg})(0.92\times10^3\ \text{kg/m}^3)(0.050\ \text{m})} = 1.1\times10^{-6}\ \text{m/s} = 0.40\ \text{cm/h}.$$

66. The condition that the energy lost by the beverage can due to evaporation equals the energy gained via radiation exchange implies

$$L_V \frac{dm}{dt} = P_{\text{rad}} = \sigma\varepsilon A(T_{\text{env}}^4 - T^4).$$

The total area of the top and side surfaces of the can is

$$A = \pi r^2 + 2\pi rh = \pi(0.022 \text{ m})^2 + 2\pi(0.022 \text{ m})(0.10 \text{ m}) = 1.53\times10^{-2} \text{ m}^2.$$

With $T_{\text{env}} = 32°\text{C} = 305 \text{ K}$, $T = 15°\text{C} = 288 \text{ K}$ and $\varepsilon = 1$, the rate of water mass loss is

$$\frac{dm}{dt} = \frac{\sigma\varepsilon A}{L_V}(T_{\text{env}}^4 - T^4) = \frac{(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1.0)(1.53\times10^{-2} \text{ m}^2)}{2.256\times10^6 \text{ J/kg}}\left[(305 \text{ K})^4 - (288 \text{ K})^4\right]$$

$$= 6.82\times10^{-7} \text{ kg/s} \approx 0.68 \text{ mg/s}.$$

67. We denote the total mass M and the melted mass m. The problem tells us that Work/M = p/ρ, and that all the work is assumed to contribute to the phase change Q = Lm where $L = 150 \times 10^3$ J/kg. Thus,

$$\frac{p}{\rho}M = Lm \quad \Rightarrow \quad m = \frac{5.5\times10^6}{1200}\frac{M}{150\times10^3}$$

which yields $m = 0.0306M$. Dividing this by 0.30 M (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage 0.0306/0.30 = 10%.

68. As is shown in the textbook for Sample Problem 18-4, we can express the final temperature in the following way:

$$T_f = \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} = \frac{c_A T_A + c_B T_B}{c_A + c_B}$$

where the last equality is made possible by the fact that $m_A = m_B$. Thus, in a graph of T_f versus T_A, the "slope" must be $c_A/(c_A + c_B)$, and the "y intercept" is $c_B/(c_A + c_B)T_B$. From the observation that the "slope" is equal to 2/5 we can determine, then, not only the ratio of the heat capacities but also the coefficient of T_B in the "y intercept"; that is,

$$c_B/(c_A + c_B)T_B = (1 - \text{“slope”})T_B .$$

(a) We observe that the "y intercept" is 150 K, so

$$T_B = 150/(1 - \text{“slope”}) = 150/(3/5)$$

which yields $T_B = 2.5\times10^2$ K.

(b) As noted already, $c_A/(c_A + c_B) = \frac{2}{5}$, so $5\,c_A = 2c_A + 2c_B$, which leads to $c_B/c_A = \frac{3}{2} = 1.5$.

69. The graph shows that the absolute value of the temperature change is $|\Delta T| = 25\ °C$. Since a Watt is a Joule per second, we reason that the energy removed is

$$|Q| = (2.81\ \text{J/s})(20\ \text{min})(60\ \text{s/min}) = 3372\ \text{J}\ .$$

Thus, with $m = 0.30$ kg, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\ |\Delta T|} = 4.5\times10^2\ \text{J/kg}\cdot\text{K}\ .$$

70. Let m_w = 14 kg, m_c = 3.6 kg, m_m = 1.8 kg, T_{i1} = 180°C, T_{i2} = 16.0°C, and T_f = 18.0°C. The specific heat c_m of the metal then satisfies

$$\left(m_w c_w + m_c c_m\right)\left(T_f - T_{i2}\right) + m_m c_m \left(T_f - T_{i1}\right) = 0$$

which we solve for c_m:

$$c_m = \frac{m_w c_w \left(T_{i2} - T_f\right)}{m_c \left(T_f - T_{i2}\right) + m_m \left(T_f - T_{i1}\right)} = \frac{(14\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(16.0°\text{C} - 18.0°\text{C})}{(3.6\,\text{kg})(18.0°\text{C} - 16.0°\text{C}) + (1.8\,\text{kg})(18.0°\text{C} - 180°\text{C})}$$
$$= 0.41\,\text{kJ/kg}\cdot\text{C°} = 0.41\,\text{kJ/kg}\cdot\text{K}.$$

71. Its initial volume is $5^3 = 125\ \text{cm}^3$, and using Table 18-2, Eq. 18-10 and Eq. 18-11, we find

$$\Delta V = (125\,\text{m}^3)\,(3\times 23\times 10^{-6}\,/\,\text{C°})\,(50.0\,\text{C°}) = 0.432\,\text{cm}^3.$$

72. (a) We denote T_H = 100°C, T_C = 0°C, the temperature of the copper-aluminum junction by T_1. and that of the aluminum-brass junction by T_2. Then,

$$P_{\text{cond}} = \frac{k_c A}{L}(T_H - T_1) = \frac{k_a A}{L}(T_1 - T_2) = \frac{k_b A}{L}(T_2 - T_c).$$

We solve for T_1 and T_2 to obtain

$$T_1 = T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_a k_b} = 100°\text{C} + \frac{0.00°\text{C} - 100°\text{C}}{1 + 401(235 + 109)/[(235)(109)]} = 84.3°\text{C}$$

(b) and

$$T_2 = T_c + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} = 0.00°\text{C} + \frac{100°\text{C} - 0.00°\text{C}}{1 + 109(235 + 401)/[(235)(401)]}$$
$$= 57.6°\text{C}.$$

73. The work (the "area under the curve") for process 1 is $4p_iV_i$, so that

$$U_b - U_a = Q_1 - W_1 = 6p_iV_i$$

by the First Law of Thermodynamics.

(a) Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$). With $W_2 = 4p_iV_i + p_iV_i = 5p_iV_i$, we obtain

$$Q_2 = W_2 + U_b - U_a = 5p_iV_i + 6p_iV_i = 11p_iV_i.$$

(b) Path 3 starts at a and ends at b so that $\Delta U = U_b - U_a = 6p_iV_i$.

74. We use $P_{\text{cond}} = kA(T_H - T_C)/L$. The temperature T_H at a depth of 35.0 km is

$$T_H = \frac{P_{\text{cond}} L}{kA} + T_C = \frac{\left(54.0\times10^{-3}\ \text{W/m}^2\right)\left(35.0\times10^{3}\ \text{m}\right)}{2.50\ \text{W/m}\cdot\text{K}} + 10.0°\text{C} = 766°\text{C}.$$

75. The volume of the disk (thought of as a short cylinder) is $\pi r^2 L$ where L = 0.50 cm is its thickness and r = 8.0 cm is its radius. Eq. 18-10, Eq. 18-11 and Table 18-2 (which gives $\alpha = 3.2 \times 10^{-6}$/C°) lead to

$$\Delta V = (\pi r^2 L)(3\alpha)(60°\text{C} - 10°\text{C}) = 4.83 \times 10^{-2} \text{ cm}^3 .$$

76. We use $Q = cm\Delta T$ and $m = \rho V$. The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C \Delta T} = \frac{\left(1.00\times10^{6}\ \text{kcal/day}\right)\left(5\,\text{days}\right)}{\left(1.00\times10^{3}\ \text{kg/m}^{3}\right)\left(1.00\,\text{kcal/kg}\right)\left(50.0^{\circ}\text{C}-22.0^{\circ}\text{C}\right)} = 35.7\,\text{m}^{3}.$$

77. We have $W = \int p\, dV$ (Eq. 18-24). Therefore,

$$W = a\int V^2 dV = \frac{a}{3}\left(V_f^3 - V_i^3\right) = 23\text{ J}.$$

78. (a) The rate of heat flow is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040\,\text{W/m}\cdot\text{K})(1.8\,\text{m}^2)(33°\text{C} - 1.0°\text{C})}{1.0\times10^{-2}\,\text{m}} = 2.3\times10^2\ \text{J/s}.$$

(b) The new rate of heat flow is

$$P'_{\text{cond}} = \frac{k' P_{\text{cond}}}{k} = \frac{(0.60\,\text{W/m}\cdot\text{K})(230\,\text{J/s})}{0.040\,\text{W/m}\cdot\text{K}} = 3.5\times10^3\ \text{J/s},$$

which is about 15 times as fast as the original heat flow.

79. We note that there is no work done in process $c \to b$, since there is no change of volume. We also note that the *magnitude* of work done in process $b \to c$ is given, but not its sign (which we identify as negative as a result of the discussion in §18-8). The total (or *net*) heat transfer is Q_{net} = [(–40) + (–130) + (+400)] J = 230 J. By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have

$$\begin{aligned} Q_{\text{net}} &= W_{\text{net}} \\ 230\,\text{J} &= W_{a\to c} + W_{c\to b} + W_{b\to a} \\ &= W_{a\to c} + 0 + (-80\,\text{J}) \end{aligned}$$

Therefore, $W_{a\to c} = 3.1\times10^2$ J.

80. If the window is L_1 high and L_2 wide at the lower temperature and $L_1 + \Delta L_1$ high and $L_2 + \Delta L_2$ wide at the higher temperature then its area changes from $A_1 = L_1 L_2$ to

$$A_2 = (L_1 + \Delta L_1)(L_2 + \Delta L_2) \approx L_1 L_2 + L_1 \ \Delta L_2 + L_2 \ \Delta L_1$$

where the term $\Delta L_1 \ \Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1 \ \Delta L_2 + L_2 \ \Delta L_1.$$

If ΔT is the change in temperature then $\Delta L_1 = \alpha L_1 \ \Delta T$ and $\Delta L_2 = \alpha L_2 \ \Delta T$, where α is the coefficient of linear expansion. Thus

$$\begin{aligned}\Delta A &= \alpha(L_1 L_2 + L_1 L_2)\ \Delta T = 2\alpha L_1 L_2 \Delta T \\ &= 2\left(9 \times 10^{-6} / \mathrm{C}°\right)(30\,\mathrm{cm})\,(20\,\mathrm{cm})\,(30°\mathrm{C}) \\ &= 0.32\,\mathrm{cm}^2.\end{aligned}$$

81. Following the method of Sample Problem 18-4 (particularly its third Key Idea), we have

$$\left(900\,\frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)(2.50\text{ kg})(T_f - 92.0°\text{C}) + \left(4190\,\frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)(8.00\text{ kg})(T_f - 5.0°\text{C}) = 0$$

where Table 18-3 has been used. Thus we find $T_f = 10.5°\text{C}$.

82. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$. In this case $\Delta E_{\text{int}} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\,\text{kJ}.$$

83. This is similar to Sample Problem 18-3. An important difference with part (b) of that sample problem is that, in this case, the final state of the H_2O is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process:

$$Q = m\,[c_{\text{ice}}(0\ \text{C}° - (-150\ \text{C}°)) + L_F + c_{\text{liquid}}(\,T_f - 0\ \text{C}°)]$$

Thus,

$$T_f = \frac{Q/m - (c_{\text{ice}}(150°) + L_F)}{c_{\text{liquid}}} = 79.5°\text{C}\ .$$

84. We take absolute values of Eq. 18-9 and Eq. 12-25:

$$|\Delta L| = L\alpha\,|\Delta T| \quad \text{and} \quad \left|\frac{F}{A}\right| = E\left|\frac{\Delta L}{L}\right|.$$

The ultimate strength for steel is $(F/A)_{\text{rupture}} = S_u = 400 \times 10^6$ N/m^2 from Table 12-1. Combining the above equations (eliminating the ratio $\Delta L/L$), we find the rod will rupture if the temperature change exceeds

$$|\Delta T| = \frac{S_u}{E\alpha} = \frac{400\times10^6\ \text{N/m}^2}{\left(200\times10^9\ \text{N/m}^2\right)\left(11\times10^{-6}\,/\,\text{C}°\right)} = 182°\text{C}.$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is $T = 25.0°\text{C} - 182°\text{C} = -157°\text{C}$.

85. The problem asks for 0.5% of E, where $E = Pt$ with $t = 120$ s and P given by Eq. 18-38. Therefore, with $A = 4\pi r^2 = 5.0 \times 10^{-3}$ m^2, we obtain

$$(0.005)Pt = (0.005)\sigma\varepsilon A T^4 t = 8.6 \text{ J}.$$

86. From the law of cosines, with $\phi = 59.95^\circ$, we have

$$L_{\text{Invar}}^2 = L_{\text{alum}}^2 + L_{\text{steel}}^2 - 2L_{\text{alum}}L_{\text{steel}} \cos\phi$$

Plugging in $L = L_0\,(1 + \alpha\Delta T)$, dividing by L_0 (which is the same for all sides) and ignoring terms of order $(\Delta T)^2$ or higher, we obtain

$$1 + 2\alpha_{\text{Invar}}\Delta T = 2 + 2\,(\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T - 2\,(1 + (\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T)\cos\phi\ .$$

This is rearranged to yield

$$\Delta T = \frac{\cos\phi - \tfrac{1}{2}}{(\alpha_{\text{alum}} + \alpha_{\text{steel}})\,(1 - \cos\phi) - \alpha_{\text{Invar}}} = \approx 46\,^\circ\text{C}\,,$$

so that the final temperature is $T = 20.0^\circ + \Delta T = 66^\circ$ C. Essentially the same argument, but arguably more elegant, can be made in terms of the differential of the above cosine law expression.

87. We assume the ice is at 0°C to being with, so that the only heat needed for melting is that described by Eq. 18-16 (which requires information from Table 18-4). Thus,

$$Q = Lm = (333 \text{ J/g})(1.00 \text{ g}) = 333 \text{ J} .$$

88. Let the initial water temperature be T_{wi} and the initial thermometer temperature be T_{ti}. Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_t m_t \left(T_f - T_{ti}\right) = c_w m_w \left(T_{wi} - T_f\right).$$

We solve for the initial temperature of the water:

$$T_{wi} = \frac{c_t m_t \left(T_f - T_{ti}\right)}{c_w m_w} + T_f = \frac{(0.0550\,\text{kg})(0.837\,\text{kJ/kg}\cdot\text{K})(44.4 - 15.0)\,\text{K}}{(4.18\,\text{kJ / kg}\cdot\text{C°})(0.300\,\text{kg})} + 44.4\text{°C}$$
$$= 45.5\text{°C}.$$

89. For a cylinder of height h, the surface area is $A_c = 2\pi rh$, and the area of a sphere is $A_o = 4\pi R^2$. The net radiative heat transfer is given by Eq. 18-40.

(a) We estimate the surface area A of the body as that of a cylinder of height 1.8 m and radius $r = 0.15$ m plus that of a sphere of radius $R = 0.10$ m. Thus, we have $A \approx A_c + A_o = 1.8\ \text{m}^2$. The emissivity $\varepsilon = 0.80$ is given in the problem, and the Stefan-Boltzmann constant is found in §18-11: $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. The "environment" temperature is $T_{\text{env}} = 303$ K, and the skin temperature is $T = \frac{5}{9}(102 - 32) + 273 = 312$ K. Therefore,

$$P_{\text{net}} = \sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right) = -86\ \text{W}.$$

The corresponding sign convention is discussed in the textbook immediately after Eq. 18-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

(b) Half the body surface area is roughly $A = 1.8/2 = 0.9\ \text{m}^2$. Now, with $T_{\text{env}} = 248$ K, we find

$$|P_{\text{net}}| = |\sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right)| \approx 2.3\times10^2\ \text{W}.$$

(c) Finally, with $T_{\text{env}} = 193$ K (and still with $A = 0.9\ \text{m}^2$) we obtain $|P_{\text{net}}| = 3.3\times10^2$ W.

90. One method is to simply compute the change in length in each edge (x_0 = 0.200 m and y_0 = 0.300 m) from Eq. 18-9 ($\Delta x = 3.6 \times 10^{-5}$ m and $\Delta y = 5.4 \times 10^{-5}$ m) and then compute the area change:

$$A - A_0 = (x_0 + \Delta x)(y_0 + \Delta y) - x_0 y_0 = 2.16 \times 10^{-5} \text{ m}^2.$$

Another (though related) method uses $\Delta A = 2\alpha A_0 \Delta T$ (valid for $\Delta A/A \ll 1$) which can be derived by taking the differential of $A = xy$ and replacing d's with Δ's.

91. (a) Let the number of weight lift repetitions be N. Then $Nmgh = Q$, or (using Eq. 18-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500\,\text{Cal})(4186\,\text{J/Cal})}{(80.0\,\text{kg})\,(9.80\,\text{m/s}^2)\,(1.00\,\text{m})} \approx 1.87\times10^4.$$

(b) The time required is

$$t = (18700)(2.00\,\text{s})\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right) = 10.4\,\text{h}.$$

92. The heat needed is

$$Q = (10\%)mL_F = \left(\frac{1}{10}\right)(200{,}000\,\text{metric tons})\ (1000\,\text{kg}/\text{metric ton})\ (333\,\text{kJ/kg})$$
$$= 6.7\times10^{12}\ \text{J}.$$

93. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate ± sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in pV diagrams:

$$\text{for straight line Work} = \frac{p_i + p_f}{2} \Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the "triangle" BC consists of three processes:

• "tilted" straight line from (1.0 m^3, 40 Pa) to (4.0 m^3, 10 Pa), with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\,\text{J}$$

• horizontal line from (4.0 m^3, 10 Pa) to (1.0 m^3, 10 Pa), with

$$\text{Work} = \left(10\,\text{Pa}\right)\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\,\text{J}$$

• vertical line from (1.0 m^3, 10 Pa) to (1.0 m^3, 40 Pa), with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

(a) and (b) Thus, the total work during the BC cycle is (75 – 30) J = 45 J. During the BA cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work = (40 Pa)(–3.0 m^3) = –120 J. Therefore, the total work during the BA cycle is (75 – 120) J = – 45 J.

94. For isotropic materials, the coefficient of linear expansion α is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$ (Eq. 18-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\Delta R_E = R_E \alpha \Delta T = \left(6.4\times10^3\ \text{km}\right)\left(\frac{1}{3}\right)\left(3.0\times10^{-5}\ /\ \text{K}\right)\ (3000\,\text{K} - 300\,\text{K}) = 1.7\times10^2\ \text{km}.$$

95. (a) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step "path": $a \to b$ followed by $b \to c$. During the latter part of this "path" there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since ΔU for process $c \to a$ is –160 J, then $U_c - U_a = 160$ J. Therefore, using the First Law of Thermodynamics, we have

$$\begin{aligned} 160 &= U_c - U_b + U_b - U_a \\ &= Q_{b\to c} - W_{b\to c} + Q_{a\to b} - W_{a\to b} \\ &= 40 - 0 + 200 - W_{a\to b} \end{aligned}$$

Therefore, $W_{a\to b\to c} = W_{a\to b} = 80$ J.

(b) $W_{a\to b} = 80$ J.

96. Since the combination "p_1V_1" appears frequently in this derivation we denote it as "x. Thus for process 1, the heat transferred is $Q_1 = 5x = \Delta E_{\text{int }1} + W_1$, and for path 2 (which consists of two steps, one at constant volume followed by an expansion accompanied by a linear pressure decrease) it is $Q_2 = 5.5x = \Delta E_{\text{int }2} + W_2$. If we subtract these two expressions and make use of the fact that internal energy is state function (and thus has the same value for path 1 as for path 2) then we have

$$5.5x - 5x = W_2 - W_1 = \text{"area" inside the triangle} = \frac{1}{2}(2\,V_1)(p_2 - p_1)\,.$$

Thus, dividing both sides by x ($= p_1V_1$), we find

$$0.5 = \frac{p_2}{p_1} - 1$$

which leads immediately to the result: $p_2/p_1 = 1.5$.

97. The cube has six faces, each of which has an area of $(6.0 \times 10^{-6}\text{ m})^2$. Using Kelvin temperatures and Eq. 18-40, we obtain

$$\begin{aligned} P_{\text{net}} &= \sigma \varepsilon A\,(T_{\text{env}}^4 - T^4) \\ &= \left(5.67\times10^{-8}\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.75)\left(2.16\times10^{-10}\text{ m}^2\right)\left((123.15\text{ K})^4 \;-\; (173.15\text{ K})^4\right) \\ &= -6.1\times10^{-9}\text{ W}. \end{aligned}$$

98. We denote the density of the liquid as ρ, the rate of liquid flowing in the calorimeter as μ, the specific heat of the liquid as c, the rate of heat flow as P, and the temperature change as ΔT. Consider a time duration dt, during this time interval, the amount of liquid being heated is $dm = \mu\rho dt$. The energy required for the heating is

$$dQ = Pdt = c(dm)\,\Delta T = c\mu\Delta T dt.$$

Thus,

$$c = \frac{P}{\rho\mu\Delta T} = \frac{250\text{ W}}{\left(8.0\times10^{-6}\text{ m}^3/\text{s}\right)\left(0.85\times10^{3}\text{ kg/m}^3\right)\left(15°\text{C}\right)}$$
$$= 2.5\times10^{3}\text{ J/kg}\cdot\text{C}° = 2.5\times10^{3}\text{ J/kg}\cdot\text{K}.$$

99. Consider the object of mass m_1 falling through a distance h. The loss of its mechanical energy is $\Delta E = m_1 gh$. This amount of energy is then used to heat up the temperature of water of mass m_2: $\Delta E = m_1 gh = Q = m_2 c\Delta T$. Thus, the maximum possible rise in water temperature is

$$\Delta T = \frac{m_1 gh}{m_2 c} = \frac{(6.00\,\text{kg})(9.8\,\text{m/s}^2)(50.0\,\text{m})}{(0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{C}^\circ)} = 1.17\,^\circ\text{C}.$$

Chapter 19

1. (a) Eq. 19-3 yields $n = M_{\text{sam}}/M = 2.5/197 = 0.0127$ mol.

(b) The number of atoms is found from Eq. 19-2:

$$N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}.$$

2. Each atom has a mass of $m = M/N_A$, where M is the molar mass and N_A is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or 74.9×10^{-3} kg/mol. Therefore, 7.50×10^{24} arsenic atoms have a total mass of

$$(7.50 \times 10^{24})\ (74.9 \times 10^{-3}\ \text{kg/mol})/(6.02 \times 10^{23}\ \text{mol}^{-1}) = 0.933\ \text{kg}.$$

3. With $V = 1.0 \times 10^{-6}$ m^3, $p = 1.01 \times 10^{-13}$ Pa, and $T = 293$ K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01\times10^{-13}\ \text{Pa}\right)\left(1.0\times10^{-6}\ \text{m}^3\right)}{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(293\ \text{K}\right)} = 4.1\times10^{-23}\ \text{mole.}$$

Consequently, Eq. 19-2 yields $N = nN_A = 25$ molecules. We can express this as a ratio (with V now written as 1 cm^3) $N/V = 25$ molecules/cm^3.

4. (a) We solve the ideal gas law $pV = nRT$ for n:

$$n = \frac{pV}{RT} = \frac{(100\,\text{Pa})\left(1.0\times10^{-6}\,\text{m}^3\right)}{(8.31\,\text{J/mol}\cdot\text{K})(220\,\text{K})} = 5.47\times10^{-8}\,\text{mol}.$$

(b) Using Eq. 19-2, the number of molecules N is

$$N = nN_\text{A} = \left(5.47\times10^{-6}\,\text{mol}\right)\left(6.02\times10^{23}\,\text{mol}^{-1}\right) = 3.29\times10^{16}\ \text{molecules}.$$

5. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is p_i = 266 kPa. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out — see Sample Problem 19-1), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

which yields

$$p_f = p_i\left(\frac{V_i}{V_f}\right)\left(\frac{T_f}{T_i}\right) = (266\,\text{kPa})\left(\frac{1.64\times10^{-2}\ \text{m}^3}{1.67\times10^{-2}\ \text{m}^3}\right)\left(\frac{300\,\text{K}}{273\,\text{K}}\right) = 287\ \text{kPa}\,.$$

Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

6. (a) With $T = 283$ K, we obtain

$$n = \frac{pV}{RT} = \frac{\left(100 \times 10^3 \text{ Pa}\right)\left(2.50\,\text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(283\,\text{K}\right)} = 106\,\text{mol.}$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as in Sample Problem 19-1 (where $n_i = n_f$ and thus cancels out):

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

which yields a final volume of

$$V_f = V_i\left(\frac{p_i}{p_f}\right)\left(\frac{T_f}{T_i}\right) = \left(2.50\,\text{m}^3\right)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right) = 0.892\ \text{m}^3.$$

7. (a) In solving $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T = (40.0 + 273.15)\,\text{K} = 313.15\ \text{K}$. And we convert the volume to SI units: 1000 cm^3 = $1000 \times 10^{-6}\ \text{m}^3$. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{\left(1.01\times10^5\ \text{Pa}\right)\left(1000\times10^{-6}\ \text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(313.15\,\text{K}\right)} = 3.88\times10^{-2}\ \text{mol}.$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{\left(1.06\times10^5\ \text{Pa}\right)\left(1500\times10^{-6}\ \text{m}^3\right)}{\left(3.88\times10^{-2}\ \text{mol}\right)\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 493\,\text{K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

8. The pressure p_1 due to the first gas is $p_1 = n_1RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1RT}{V} + \frac{n_2RT}{V} = \left(n_1 + n_2\right)\frac{RT}{V}.$$

The fraction of P due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2RT/V}{\left(n_1+n_2\right)\left(RT/V\right)} = \frac{n_2}{n_1 + n_2} = \frac{0.5}{2 + 0.5} = 0.2.$$

9. (a) Eq. 19-45 (which gives 0) implies $Q = W$. Then Eq. 19-14, with T = (273 + 30.0)K leads to gives $Q = -3.14 \times 10^3$ J, or $|Q| = 3.14 \times 10^3$ J.

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

10. The initial and final temperatures are $T_i = 5.00°\text{C} = 278\text{ K}$ and $T_f = 75.0°\text{C} = 348\text{ K}$, respectively. Using ideal-gas law with $V_i = V_f$, we find the final pressure to be

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad \Rightarrow \quad p_f = \frac{T_f}{T_i} p_i = \left(\frac{348\,\text{K}}{278\,\text{K}}\right)(1.00\text{ atm}) = 1.25\text{ atm}.$$

11. Using Eq. 19-14, we note that since it is an isothermal process (involving an ideal gas) then $Q = W = nRT \ln(V_f/V_i)$ applies at any point on the graph. An easy one to read is $Q = 1000$ J and $V_f = 0.30$ m^3, and we can also infer from the graph that $V_i = 0.20$ m^3. We are told that $n = 0.825$ mol, so the above relation immediately yields $T = 360$ K.

12. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where T_1=315 K is the initial temperature, A =24.9 J/K and B=0.00662 J/K^2. The final volume is $V_2 = (AT_2 - BT_2^2)/p$, where T_2=315 K. Thus

$$\begin{aligned} W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\ &= (24.9\ \text{J/K})(325\ \text{K} - 315\ \text{K}) - (0.00662\ \text{J/K}^2)[(325\ \text{K})^2 - (315\ \text{K})^2] = 207\ \text{J}. \end{aligned}$$

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_i V_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is 1.03×10^5 Pa,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = \left(2.04 \times 10^5 \text{ Pa}\right)\left(0.14\,\text{m}^3\right) \ln\left(\frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}}\right) = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$W = p_f\left(V_i - \frac{p_iV_i}{p_f}\right) = \left(p_f - p_i\right)V_i = \left(1.013\times10^5 \text{ Pa} - 2.04\times10^5 \text{ Pa}\right)\left(0.14\,\text{m}^3\right)$$
$$= -1.44\times10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = 2.00 \times 10^4 \text{ J} - 1.44 \times 10^4 \text{ J} = 5.60 \times 10^3 \text{ J}.$$

14. (a) At the surface, the air volume is

$$V_1 = Ah = \pi(1.00\text{ m})^2(4.00\text{ m}) = 12.57\text{ m}^3 \approx 12.6\text{ m}^3.$$

(b) The temperature and pressure of the air inside the submarine at the surface are $T_1 = 20°\text{C} = 293\text{ K}$ and $p_1 = p_0 = 1.00\text{ atm}$. On the other hand, at depth $h = 80$ m, we have $T_2 = -30°\text{C} = 243\text{ K}$ and

$$p_2 = p_0 + \rho gh = 1.00\text{ atm} + (1024\text{ kg/m}^3)(9.80\text{ m/s}^2)(80.0\text{ m})\frac{1.00\text{ atm}}{1.01\times10^5\text{ Pa}}$$
$$= 1.00\text{ atm} + 7.95\text{ atm} = 8.95\text{ atm}.$$

Therefore, using ideal-gas law, $pV = NkT$, the air volume at this depth would be

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2} \Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)V_1 = \left(\frac{1.00\text{ atm}}{8.95\text{ atm}}\right)\left(\frac{243\text{K}}{293\text{K}}\right)\left(12.57\text{ m}^3\right) = 1.16\text{ m}^3.$$

(c) The decrease in volume is $\Delta V = V_1 - V_2 = 11.44\text{ m}^3$. Using Eq. 19-5, the amount of air this volume corresponds to is

$$n = \frac{p\Delta V}{RT} = \frac{(8.95\text{ atm})\left(1.01\times10^5\text{ Pa/atm}\right)\left(11.44\text{m}^3\right)}{\left(8.31\text{ J/mol}\cdot\text{K}\right)\left(243\text{K}\right)} = 5.10\times10^3\text{ mol}.$$

Thus, in order for the submarine to maintain the original air volume in the chamber, $5.10\times10^3\text{ mol}$ of air must be released.

15. (a) At point a, we know enough information to compute n:

$$n = \frac{pV}{RT} = \frac{(2500\,\text{Pa})(1.0\,\text{m}^3)}{(8.31\ \text{J/mol}\cdot\text{K})\,(200\,\text{K})} = 1.5\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law as in Sample Problem 19-1 in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a} \Rightarrow T_b = (200\,\text{K})\left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at b of $T_b = 1.8\times10^3$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form (see Sample Problem 19-1):

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a} \Rightarrow T_c = (200\,\text{K})\left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at c of $T_c = 6.0\times10^2$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the pV diagram shown in Fig. 19-21). This, in turn, is related to ± "area" inside that triangle (with area $= \frac{1}{2}$(base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2}\left(2.0\,\text{m}^3\right)\left(5.0\times10^3\ \text{Pa}\right) = 5.0\times10^3\ \text{J}.$$

16. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1V_1 = nRT_1$, the number of moles of gas in the bubble is

$$n = p_1V_1/RT_1 = (p_0 + \rho g d)V_1/RT_1,$$

where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for n to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1}\frac{p_0 + \rho g d}{p_0}V_1 \\ &= \left(\frac{293\,\text{K}}{277\,\text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + \left(0.998\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(40\,\text{m}\right)}{1.013\times10^5\ \text{Pa}}\right)\left(20\,\text{cm}^3\right) \\ &= 1.0\times10^2\ \text{cm}^3. \end{aligned}$$

17. When the valve is closed the number of moles of the gas in container A is $n_A = p_A V_A / R T_A$ and that in container B is $n_B = 4 p_B V_A / R T_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4 p_B}{T_B}\right) = \text{const.}$$

After the valve is opened the pressure in container A is $p'_A = R n'_A T_A / V_A$ and that in container B is $p'_B = R n'_B T_B / 4 V_A$. Equating p'_A and p'_B, we obtain $R n'_A T_A / V_A = R n'_B T_B / 4 V_A$, or $n'_B = (4 T_A / T_B) n'_A$. Thus,

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4 p_B}{T_B}\right).$$

We solve the above equation for n'_A:

$$n'_A = \frac{V}{R}\frac{\left(p_A/T_A + 4 p_B/T_B\right)}{\left(1 + 4T_A/T_B\right)}.$$

Substituting this expression for n'_A into $p' V_A = n'_A R T_A$, we obtain the final pressure:

$$p' = \frac{n'_A R T_A}{V_A} = \frac{p_A + 4 p_B T_A / T_B}{1 + 4 T_A / T_B} = 2.0 \times 10^5 \text{ Pa}.$$

18. Appendix F gives $M = 4.00 \times 10^{-3}$ kg/mol (Table 19-1 gives this to fewer significant figures). Using Eq. 19-22, we obtain

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\,(8.31\ \text{J/mol}\cdot\text{K})\,(1000\,\text{K})}{4.00\times10^{-3}\ \text{kg/mol}}} = 2.50\times10^{3}\ \text{m/s}.$$

19. According to kinetic theory, the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where T is the temperature and M is the molar mass. See Eq. 19-34. According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\,\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02 \times 10^{-3}\,\text{kg/mol}}} = 1.8{\times}10^{2}\ \text{m/s}.$$

20. The molar mass of argon is 39.95 g/mol. Eq. 19–22 gives

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(313\,\text{K})}{39.95\times 10^{-3}\ \text{kg/mol}}} = 442\,\text{m/s}.$$

21. Table 19-1 gives M = 28.0 g/mol for nitrogen. This value can be used in Eq. 19-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With T_2 = (20.0 + 273.15) K ≈ 293 K, we obtain

$$v_2 = (517\,\text{m/s})\sqrt{\frac{293\,\text{K}}{300\,\text{K}}} = 511\,\text{m/s}.$$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\,\text{K})\left(\frac{1}{2}\right)^2 = 73.0\,\text{K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\text{K})(4) = 1.17\times10^3\,\text{K}$$

which is equivalent to 899°C.

22. First we rewrite Eq. 19-22 using Eq. 19-4 and Eq. 19-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(kN_{\text{A}})T}{(mN_{\text{A}})}} = \sqrt{\frac{3kT}{M}}.$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\text{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is Δt, then the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the N molecules on the wall, and the pressure is the average force per unit area:

$$
\begin{aligned}
p &= \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta \\
&= \left(\frac{2}{2.0\times 10^{-4}\,\mathrm{m}^2}\right)\left(1.0\times 10^{23}\,\mathrm{s}^{-1}\right)\left(3.3\times 10^{-27}\,\mathrm{kg}\right)\left(1.0\times 10^{3}\,\mathrm{m/s}\right)\cos 55^\circ \\
&= 1.9\times 10^{3}\,\mathrm{Pa}.
\end{aligned}
$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to m^2.

24. We can express the ideal gas law in terms of density using $n = M_{\text{sam}}/M$:

$$pV = \frac{M_{\text{sam}}RT}{M} \Rightarrow \rho = \frac{pM}{RT} .$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}} .$$

(a) We convert to SI units: $\rho = 1.24 \times 10^{-2}$ kg/m^3 and $p = 1.01 \times 10^3$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

(b) We find M from $\rho = pM/RT$ with $T = 273$ K.

$$M = \frac{\rho RT}{p} = \frac{(0.0124\,\text{kg/m}^3)\left(8.31\,\text{J/mol}\cdot\text{K}\right)(273\,\text{K})}{1.01 \times 10^3\ \text{Pa}} = 0.0279\ \text{kg/mol} = 27.9\ \text{g/mol}.$$

(c) From Table 19.1, we identify the gas to be N_2.

25. (a) Eq. 19-24 gives $K_{\text{avg}} = \frac{3}{2}\left(1.38\times10^{-23}\text{ J/K}\right)(273\,\text{K}) = 5.65\times10^{-21}\text{ J}$.

(b) For $T = 373$ K, the average translational kinetic energy is $K_{\text{avg}} = 7.72\times10^{-21}$ J.

(c) The unit mole may be thought of as a (large) collection: 6.02×10^{23} molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\text{mole}} = N_A K_{\text{avg}} = (6.02\times10^{23})(5.65\times10^{-21}\text{ J}) = 3.40\times10^{3}\text{ J}.$$

(d) Similarly, the result from part (b) leads to

$$K_{\text{mole}} = (6.02\times10^{23})(7.72\times10^{-21}\text{ J}) = 4.65\times10^{3}\text{ J}.$$

26. The average translational kinetic energy is given by $K_{\text{avg}} = \frac{3}{2}kT$, where k is the Boltzmann constant (1.38×10^{-23} J/K) and T is the temperature on the Kelvin scale. Thus

$$K_{\text{avg}} = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(1600 \text{ K}) = 3.31 \times 10^{-20} \text{ J}.$$

27. (a) We use $\varepsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol so the molar mass of H_2O is (1.0 + 1.0 + 16) = 18 g/mol. There are $N_A = 6.02 \times 10^{23}$ molecules in a mole so the number of molecules in a gram of water is $(6.02 \times 10^{23}\ \text{mol}^{-1})/(18\ \text{g/mol}) = 3.34 \times 10^{22}$ molecules/g. Thus

$$\varepsilon = (539\ \text{cal/g})/(3.34 \times 10^{22}/\text{g}) = 1.61 \times 10^{-20}\ \text{cal} = 6.76 \times 10^{-20}\ \text{J}.$$

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})[(32.0 + 273.15)\ \text{K}] = 6.32 \times 10^{-21}\ \text{J}.$$

The ratio $\varepsilon/K_{\text{avg}}$ is $(6.76 \times 10^{-20}\ \text{J})/(6.32 \times 10^{-21}\ \text{J}) = 10.7$.

28. We solve Eq. 19-25 for *d*:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80\times 10^{5}\ \text{cm})\ \pi\sqrt{2}\,(2.7\times 10^{19}/\text{cm}^{3})}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

29. (a) According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of a molecule and N is the number of molecules in volume V. Substitute $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m^3 to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\text{ m})^2 (1 \times 10^6\text{ m}^{-3})} = 6 \times 10^{12}\text{ m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

30. Using $v = f\lambda$ with $v = 331$ m/s (see Table 17-1) with Eq. 19-2 and Eq. 19-25 leads to

$$f = \frac{v}{\left(\dfrac{1}{\sqrt{2}\pi d^2 (N/V)}\right)} = (331\,\text{m/s})\pi\sqrt{2}\ (3.0\times10^{-10}\ \text{m})^2 \left(\frac{nN_A}{V}\right)$$
$$= \left(8.0\times10^7\ \frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right) = \left(8.0\times10^7\ \frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01\times10^5\ \text{Pa}}{(8.31\ \text{J/mol}\cdot\text{K})(273.15\,\text{K})}\right)$$
$$= 3.5\times10^9\ \text{Hz}.$$

where we have used the ideal gas law and substituted $n/V = p/RT$. If we instead use v = 343 m/s (the "default value" for speed of sound in air, used repeatedly in Ch. 17), then the answer is 3.7×10^9 Hz.

31. (a) We use the ideal gas law $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333 \text{ Pa}) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.333\times10^{-4} \text{ Pa}}{(1.38\times10^{-23} \text{ J/K})(295 \text{ K})} = 3.27\times10^{16} \text{ molecules/m}^3 = 3.27\times10^{10} \text{ molecules/cm}^3 .$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 19-25, the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi(2.00\times10^{-10} \text{ m})^2 (3.27\times10^{16} \text{ m}^{-3})} = 172 \text{ m}.$$

32. (a) We set up a ratio using Eq. 19-25:

$$\frac{\lambda_{Ar}}{\lambda_{N_2}} = \frac{1/\left(\pi\sqrt{2}d_{Ar}^2\,(N/V)\right)}{1/\left(\pi\sqrt{2}d_{N_2}^2\,(N/V)\right)} = \left(\frac{d_{N_2}}{d_{Ar}}\right)^2 .$$

Therefore, we obtain

$$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}} = \sqrt{\frac{27.5\times10^{-6}\ \text{cm}}{9.9\times10^{-6}\ \text{cm}}} = 1.7.$$

(b) Using Eq. 19-2 and the ideal gas law, we substitute $N/V = N_A n/V = N_A p/RT$ into Eq. 19–25 and find

$$\lambda = \frac{RT}{\pi\sqrt{2}d^2 p N_A}.$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right).$$

With $\lambda_1 = 9.9 \times 10^{-6}$ cm, $T_1 = 293$ K (the same as T_2 in this part), $p_1 = 750$ torr and $p_2 = 150$ torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2 = 233$ K and $p_2 = 750$ torr.

33. (a) The average speed is

$$v_{avg} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s}.$$

(b) The rms speed is

$$v_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s}$$

(c) Yes, $v_{rms} > v_{avg}$.

34. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{[2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)]\ \text{cm/s}}{2+4+6+8+2} = 3.2\,\text{cm/s}.$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i v_i^2 / \sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2+4(2.0)^2+6(3.0)^2+8(4.0)^2+2(5.0)^2}{2+4+6+8+2}}\ \text{cm/s} = 3.4\,\text{cm/s}.$$

(c) There are eight particles at v = 4.0 cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

35. (a) The average speed is $\bar{v} = \dfrac{\sum v}{N}$, where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\text{km/s}}{10} = 6.5\,\text{km/s}.$$

(b) The rms speed is given by $v_{\text{rms}} = \sqrt{\dfrac{\sum v^2}{N}}$. Now

$$\begin{aligned}\sum v^2 &= [(2.0)^2+(3.0)^2+(4.0)^2+(5.0)^2+(6.0)^2 \\ &\quad +(7.0)^2+(8.0)^2+(9.0)^2+(10.0)^2+(11.0)^2]\ \text{km}^2/\text{s}^2 = 505\ \text{km}^2/\text{s}^2\end{aligned}$$

so

$$v_{\text{rms}} = \sqrt{\frac{505\ \text{km}^2/\text{s}^2}{10}} = 7.1\ \text{km/s}.$$

36. (a) From the graph we see that $v_p = 400$ m/s. Using the fact that $M = 28$ g/mol = 0.028 kg/mol for nitrogen (N_2) gas, Eq. 19-35 can then be used to determine the absolute temperature. We obtain $T = \frac{1}{2} M v_p^2 / R = 2.7 \times 10^2$ K.

(b) Comparing with Eq. 19-34, we conclude $v_{rms} = \sqrt{3/2}\, v_p = 4.9 \times 10^2$ m/s.

37. The rms speed of molecules in a gas is given by $v_{rms} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the gas. See Eq. 19-34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at Earth's surface and r_e (= 6.37×10^6 m) is the radius of Earth. To derive this expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass m at Earth's surface is

$$U = -GMm/r_e^2 = -mgr_e ,$$

where $g = GM/r_e^2$ was used. If v is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means E = 0 and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_eM/3R$.

(a) The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2\left(9.8\,\text{m/s}^2\right)\left(6.37\times10^6\,\text{m}\right)\left(2.02\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 1.0\times10^4\,\text{K}.$$

(b) The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2\left(9.8\,\text{m/s}^2\right)\left(6.37\times10^6\,\text{m}\right)\left(32.0\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 1.6\times10^5\,\text{K}.$$

(c) Now, $T = 2g_mr_mM / 3R$, where $r_m = 1.74 \times 10^6$ m is the radius of the Moon and $g_m = 0.16g$ is the acceleration due to gravity at the Moon's surface. For hydrogen, the temperature is

$$T = \frac{2(0.16)\left(9.8\,\text{m/s}^2\right)\left(1.74\times10^6\,\text{m}\right)\left(2.02\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 4.4\times10^2\,\text{K}.$$

(d) For oxygen, the temperature is

$$T = \frac{2(0.16)\left(9.8\,\text{m/s}^2\right)\left(1.74\times10^6\,\text{m}\right)\left(32.0\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 7.0\times10^3\,\text{K}.$$

(e) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen.

(f) On the other hand, very few oxygen atoms escape. So there should be much oxygen high in Earth’s upper atmosphere.

38. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which, for $v_{\text{avg2}} = 2v_{\text{rms1}}$, leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} = 4.7\,.$$

39. (a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. See Eq. 19-34. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(4000\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 7.0\times10^{3}\,\text{m/s}.$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii:

$$d = r_1 + r_2 = 0.5 \times 10^{-8}\,\text{cm} + 1.5 \times 10^{-8}\,\text{cm} = 2.0 \times 10^{-8}\,\text{cm}.$$

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt, where v is its speed. That is, the number of collisions is $\pi d^2 vtN/V$, where, N/V is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 vN}{V} = \pi\left(2.0\times10^{-10}\,\text{m}\right)^2\left(7.0\times10^{3}\,\text{m/s}\right)\left(4.0\times10^{25}\,\text{m}^{-3}\right) = 3.5\times10^{10}\,\text{collisions/s}.$$

40. We divide Eq. 19-35 by Eq. 19-22:

$$\frac{v_P}{v_{\text{rms}}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which, for $v_P = v_{\text{rms}}$, leads to

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{v_P}{v_{\text{rms}}}\right)^2 = \frac{3}{2} .$$

41. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\, dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 19-24. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0 ,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) The average speed is given by $v_{\text{avg}} = \int vP(v)\,dv$. For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2 dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \;\Rightarrow\; \frac{v_{\text{avg}}}{v_0} = 1.22 .$$

(c) The mean-square speed is given by $v_{\text{rms}}^2 = \int v^2 P(v)\,dv$. The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3 dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2 dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \;\Rightarrow\; \frac{v_{\text{rms}}}{v_0} = 1.31 .$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N\,av_0 = N/3,$$

where $2/3v_0$ was substituted for a. In other words, the fraction of particles in this range is 1/3 or 0.33.

42. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K}) = 3.4\times10^{3}\ \text{J}.$$

43. (a) The work is zero in this process since volume is kept fixed.

(b) Since $C_V = \frac{3}{2}R$ for an ideal monatomic gas, then Eq. 19-39 gives $Q = +374$ J.

(c) $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) Two moles are equivalent to $N = 12 \times 10^{23}$ particles. Dividing the result of part (c) by N gives the average translational kinetic energy change per atom: 3.11×10^{-22} J.

44. (a) Since the process is a constant-pressure expansion,

$$W = p\Delta V = nR\Delta T = (2.02\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(15\,\text{K}) = 249\,\text{J}.$$

(b) Now, $C_p = \frac{5}{2}R$ in this case, so $Q = nC_p\Delta T = +623$ J by Eq. 19-46.

(c) The change in the internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is

$$\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}\text{ J}.$$

45. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1 C_1\ \Delta T$, the internal energy of the second gas changes by $n_2 C_2\ \Delta T$, and the internal energy of the third gas changes by $n_3 C_3\ \Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1\ C_1 + n_2\ C_2 + n_3\ C_3)\ \Delta T.$$

This must be $(n_1 + n_2 + n_3)\ C_V\ \Delta T$, where C_V is the molar specific heat of the mixture. Thus,

$$C_V = \frac{n_1 C_1 + n_2 C_2 + n_3 C_3}{n_1 + n_2 + n_3}.$$

With n_1=2.40 mol, C_{V1}=12.0 J/mol·K for gas 1, n_2=1.50 mol, C_{V2}=12.8 J/mol·K for gas 2, and n_3=3.20 mol, C_{V3}=20.0 J/mol·K for gas 3, we obtain C_V=15.8 J/mol·K for the mixture.

46. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the pV diagram — the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N / m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int }c} - E_{\text{int }a} &= \frac{5}{2}\left(p_c V_c - p_a V_a\right) = \frac{5}{2}\left(\left(2.0\times10^3\ \text{Pa}\right)\left(4.0\,\text{m}^3\right) - \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)\right) \\ &= -5.0\times10^3\ \text{J}. \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)\left(V_c - V_a\right) = \left(3.5\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7.0\times10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0\times10^3 + 7.0\times10^3)\ \text{J} = 2.0\times10^3\ \text{J}.$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{\text{int}} = E_{\text{int }c} - E_{\text{int }a} = -5.0\times10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right) + 0 = 1.0\times10^4\ \text{J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0\times10^3 + 1.0\times10^4)\ \text{J} = 5.0\times10^3\ \text{J}.$$

47. Argon is a monatomic gas, so $f = 3$ in Eq. 19-51, which provides

$$C_V = \frac{3}{2}R = \frac{3}{2}(8.31\ \text{J/mol}\cdot\text{K})\left(\frac{1\ \text{cal}}{4.186\ \text{J}}\right) = 2.98\frac{\text{cal}}{\text{mol}\cdot\text{C}^\circ}$$

where we have converted Joules to calories, and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) M is effectively a conversion factor between grams and moles, we see that c_V (see units specified in the problem statement) is related to C_V by $C_V = c_V\ M$ where $M = mN_\text{A}$, and m is the mass of a single atom (see Eq. 19-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_\text{A}} = \frac{C_V / c_V}{N_\text{A}} = \frac{2.98/0.075}{6.02\times10^{23}} = 6.6\times10^{-23}\ \text{g}.$$

(b) The molar mass is found to be $M = C_V/c_V = 2.98/0.075 = 39.7$ g/mol which should be rounded to 40 g/mol since the given value of c_V is specified to only two significant figures.

48. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$\Delta E_{\text{int}} = Q - p\Delta V = 20.9\ \text{J} - \left(1.01\times10^{5}\ \text{Pa}\right)\left(100\ \text{cm}^{3} - 50\ \text{cm}^{3}\right)\left(\frac{1\times10^{-6}\ \text{m}^{3}}{1\ \text{cm}^{3}}\right) = 15.9\ \text{J}.$$

(b) The molar specific heat at constant pressure is

$$C_p = \frac{Q}{n\Delta T} = \frac{Q}{n\left(p\Delta V/nR\right)} = \frac{R}{p}\frac{Q}{\Delta V} = \frac{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(20.9\ \text{J}\right)}{\left(1.01\times10^{5}\ \text{Pa}\right)\left(50\times10^{-6}\ \text{m}^{3}\right)} = 34.4\ \text{J/mol}\cdot\text{K}.$$

(c) Using Eq. 19-49, $C_V = C_p - R = 26.1$ J/mol·K.

49. (a) From Table 19-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 19-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3.49\times10^3 \text{ J}.$$

(b) Eq. 19-45 leads to

$$\Delta E_{\text{int}} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2.49\times10^3 \text{ J}.$$

(c) From either $W = Q - \Delta E_{\text{int}}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Eq. 19-24 is written in more convenient form (for this problem) in Eq. 19-38. Thus, the increase in kinetic energy is

$$\Delta K_{\text{trans}} = \Delta\left(NK_{\text{avg}}\right) = n\left(\frac{3}{2}R\right)\Delta T \approx 1.49\times10^3 \text{ J}.$$

Since $\Delta E_{\text{int}} = \Delta K_{\text{trans}} + \Delta K_{\text{rot}}$, the increase in rotational kinetic energy is

$$\Delta K_{\text{rot}} = \Delta E_{\text{int}} - \Delta K_{\text{trans}} = 2.49\times10^3 \text{ J} - 1.49\times10^3 \text{ J} = 1.00\times10^3 \text{ J}.$$

Note that had there been no rotation, all the energy would have gone into the translational kinetic energy.

50. Referring to Table 19-3, Eq. 19-45 and Eq. 19-46, we have

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{5}{2} nR\Delta T$$

$$Q = nC_p \Delta T = \frac{7}{2} nR\Delta T.$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7}.$$

Thus, the given value Q = 70 J leads to $\Delta E_{\text{int}} = 50$ J.

51. The fact that they rotate but do not oscillate means that the value of f given in Table 19-3 is relevant. Thus, Eq. 19-46 leads to

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\left(T_f - T_i\right) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where T_i = 273 K and n = 1.0 mol. The ratio of absolute temperatures is found from the gas law in ratio form (see Sample Problem 19-1). With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2.$$

Therefore, the energy added as heat is

$$Q = (1.0\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(273\,\text{K})\left(\frac{7}{2}\right)(2-1) \approx 8.0\times10^3\text{ J}.$$

52. (a) Using $M = 32.0$ g/mol from Table 19-1 and Eq. 19-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0\text{ g}}{32.0\text{ g/mol}} = 0.375\text{ mol}.$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 19-46 and Table 19-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T = (0.375\text{ mol})\left(\frac{7}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(100\,\text{K}) = 1.09\times10^3\text{ J}.$$

(c) We could compute a value of ΔE_{int} from Eq. 19-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\,\Delta T}{n\left(\frac{7}{2}R\right)\,\Delta T} = \frac{5}{7} \approx 0.714.$$

53. (a) Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus,

$$Q = \frac{7}{2}nR\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times10^3\ \text{J}.$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\,\text{J/mol.K})(60.0\,\text{K}) = 4.99\times10^3\ \text{J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98\times10^3\ \text{J} - 4.99\times10^3\ \text{J} = 1.99\times10^3\ \text{J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times10^3\ \text{J}.$$

54. (a) We use Eq. 19-54 with $V_f / V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = (2.00)^{1.3}$$

which yields $p_f = (2.46)(1.0 \text{ atm}) = 2.46$ atm.

(b) Similarly, Eq. 19-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (273\,\text{K})(1.23) = 336\,\text{K}.$$

(c) We use the gas law in ratio form (see Sample Problem 19-1) and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273\,\text{K}}{336\,\text{K}} = 0.813.$$

The volume V_1 is half the original volume of one liter, so

$$V_2 = 0.813(0.500\,\text{L}) = 0.406\,\text{L}.$$

55. (a) Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f, V_f, and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_i V_i^{\gamma} = p_f V_f^{\gamma}$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^{\gamma} p_i = \left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4} (1.2\,\text{atm}) = 13.6\,\text{atm} \approx 14\text{ atm.}$$

We note that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_iV_i/p_fV_f = T_i/T_f$ and

$$T_f = \frac{p_f V_f}{p_i V_i} T_i = \left[\frac{(13.6\,\text{atm})(0.76\,\text{L})}{(1.2\,\text{atm})(4.3\,\text{L})}\right](310\,\text{K}) = 6.2\times 10^2\text{ K.}$$

56. The fact that they rotate but do not oscillate means that the value of f given in Table 19-3 is relevant. In §19-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(300\,\text{K})}{1.01\times10^5\text{ N/m}^2} = 0.049\text{ m}^3 .$$

Consequently,

$$pV^{\gamma} = \left(1.01\times10^5\text{ N/m}^2\right)\left(0.049\,\text{m}^3\right)^{1.4} = 1.5\times10^3\text{ N}\cdot\text{m}^{2.2}.$$

57. Since ΔE_{int} does not depend on the type of process,

$$\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = \left(\Delta E_{\text{int}}\right)_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$\left(\Delta E_{\text{int}}\right)_{\text{path 1}} = \sum \left(\Delta E_{\text{int}}\right)_{\text{adiabat}}.$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$\begin{aligned}\left(\Delta E_{\text{int}}\right)_{\text{adiabatic expansion}} &= -W = -40 \text{ J} \\ \left(\Delta E_{\text{int}}\right)_{\text{adiabatic compression}} &= -W = -(-25)\text{ J} = 25 \text{ J}.\end{aligned}$$

Therefore, $\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = -40\text{ J} + 25\text{ J} = -15\text{ J}$.

58. Let p_1, V_1 and T_1 represent the pressure, volume, and temperature of the air at $y_1 = 4267$ m. Similarly, let p, V and T be the pressure, volume, and temperature of the air at $y = 1567$ m. Since the process is adiabatic $p_1 V_1^\gamma = pV^\gamma$. Combining with ideal-gas law, $pV = NkT$, we obtain

$$pV^\gamma = p(T/p)^\gamma = p^{1-\gamma}T^\gamma = \text{constant} \quad \Rightarrow \quad p^{1-\gamma}T^\gamma = p_1^{1-\gamma}T_1^\gamma$$

With $p = p_0 e^{-ay}$ and $\gamma = 4/3$ (which gives $(1-\gamma)/\gamma = -1/4$), the temperature at the end of the decent is

$$T = \left(\frac{p_1}{p}\right)^{\frac{1-\gamma}{\gamma}} T_1 = \left(\frac{p_0 e^{-ay_1}}{p_0 e^{-ay}}\right)^{\frac{1-\gamma}{\gamma}} T_1 = e^{-a(y-y_1)/4} T_1 = e^{-(1.16\times 10^{-4}/\text{m})(1567\text{ m}-4267\text{ m})/4}(268\text{ K})$$
$$= (1.08)(268\text{ K}) = 290\text{ K} = 17°\text{C}$$

59. The aim of this problem is to emphasize what it means for the internal energy to be a state function. Since path 1 and path 2 start and stop at the same places, then the internal energy change along path 1 is equal to that along path 2. Now, during isothermal processes (involving an ideal gas) the internal energy change is zero, so the only step in path 1 that we need to examine is step 2. Eq. 19-28 then immediately yields –20 J as the answer for the internal energy change.

60. Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas, and let p_f, V_f, and T_f be the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_iV_i^\gamma = p_fV_f^\gamma$. Combining with ideal-gas law, $pV = NkT$, we obtain

$$p_iV_i^\gamma = p_i(T_i/p_i)^\gamma = p_i^{1-\gamma}T_i^\gamma = \text{constant} \quad \Rightarrow \quad p_i^{1-\gamma}T_i^\gamma = p_f^{1-\gamma}T_f^\gamma$$

With $\gamma = 4/3$ which gives $(1-\gamma)/\gamma = -1/4$, the temperature at the end of the adiabatic expansion is

$$T_f = \left(\frac{p_i}{p_f}\right)^{\frac{1-\gamma}{\gamma}} T_i = \left(\frac{5.00\text{ atm}}{1.00\text{ atm}}\right)^{-1/4} (278\text{ K}) = 186\text{ K} = -87°\text{C}.$$

61. (a) Eq. 19-54, $p_i V_i^\gamma = p_f V_f^{\ \gamma}$, leads to

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma \Rightarrow \quad 4.00\text{ atm} = (1.00\text{ atm})\left(\frac{200\text{ L}}{74.3\text{ L}}\right)^\gamma$$

which can be solved to yield

$$\gamma = \frac{\ln(p_f/p_i)}{\ln(V_i/V_f)} = \frac{\ln(4.00\text{ atm}/1.00\text{ atm})}{\ln(200\text{ L}/74.3\text{ L})} = 1.4 = \frac{7}{5}.$$

This implies that the gas is diatomic (see Table 19-3).

(b) One can now use either Eq. 19-56 (as illustrated in part (a) of Sample Problem 19-9) or use the ideal gas law itself. Here we illustrate the latter approach:

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i} \quad \Rightarrow \quad T_f = 446\text{ K}.$$

(c) Again using the ideal gas law: $n = P_i V_i / RT_i = 8.10$ moles. The same result would, of course, follow from $n = P_f V_f / RT_f$.

62. Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} .$$

Using Eq. 19-54 we can write this as

$$W = p_i V_i \frac{1-(p_f / p_i)^{1-1/\gamma}}{1-\gamma}$$

In this problem, $\gamma = 7/5$ (see Table 19-3) and $P_f/P_i = 2$. Converting the initial pressure to Pascals we find $P_i V_i = 24240$ J. Plugging in, then, we obtain $W = -1.33 \times 10^4$ J.

63. In the following $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

The process $1 \to 2$ takes place at constant volume.

(a) The heat added is

$$Q = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74\times10^3\ \text{J}.$$

(b) Since the process takes place at constant volume the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74\times10^3\ \text{J}.$$

(c) The work W done by the gas is zero.

The process $2 \to 3$ is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K} - 600\,\text{K}) = -1.81\times10^3\ \text{J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81\times10^3\,\text{J}.$$

The process $3 \to 1$ takes place at constant pressure.

(g) The heat added is

$$Q = nC_p\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}(1.00\ \text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -3.22\times10^3\ \text{J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -1.93\times10^3\ \text{J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22\times10^3 \text{ J } + 1.93\times10^3 \text{ J } = -1.29\times10^3 \text{ J}.$$

(j) For the entire process the heat added is

$$Q = 3.74\times10^3 \text{ J } + 0 - 3.22\times10^3 \text{ J } = 520 \text{ J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74\times10^3 \text{ J} - 1.81\times10^3 \text{ J} - 1.93\times10^3 \text{ J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81\times10^3 \text{ J} - 1.29\times10^3 \text{ J } = 520 \text{ J}.$$

(m) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{(1.013\times10^5 \text{ Pa})} = 2.46\times10^{-2} \text{ m}^3.$$

(n) Since $1 \to 2$ is a constant volume process $V_2 = V_1 = 2.46 \times 10^{-2}$ m^3. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(600 \text{ K})}{2.46\times10^{-2} \text{ m}^3} = 2.02\times10^5 \text{ Pa}.$$

This is approximately equal to 2.00 atm.

(o) $3 \to 1$ is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(455 \text{ K})}{1.013\times10^5 \text{ Pa}} = 3.73\times10^{-2} \text{ m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5$ Pa (1.00 atm)

64. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50.0)}{1.00\times10^{-8}} = 4.16\times10^{10}\ \text{m}^3.$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = \frac{(1)\left(6.02\times10^{23}\right)}{4.16\times10^{10}} = 1.45\times10^{13}\ \frac{\text{molecules}}{\text{m}^3}.$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 19-25 yields

$$\lambda = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)} = 38.8\ \text{m}.$$

65. We note that $\Delta K = n\left(\frac{3}{2}R\right)\Delta T$ according to the discussion in §19-5 and §19-9. Also, $\Delta E_{\text{int}} = nC_V\Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 19-49 leads to $C_p = C_V + R \approx 8.0$ cal/mol·K after we convert Joules to calories in the ideal gas constant value (Eq. 19-6): $R \approx 2.0$ cal/mol·K. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

• Constant volume process with $\Delta T = 50$ K and $n = 3.0$ mol.

(a) Since the change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal, and the work done by the gas is $W = 0$ for constant volume processes, the first law gives $Q = 900 + 0 = 900$ cal.

(b) As shown in part (a), $W = 0$.

(c) The change in the internal energy is, from part (a), $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(d) The change in the total translational kinetic energy is

$$\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$$

• Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.

(e) $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law)

$$W = nR\Delta T = (3.0)(2.0)(50) = 300 \text{ cal}.$$

The first law gives $Q = (900 + 300)$ cal $= 1200$ cal.

(f) From (e), we have W=300 cal.

(g) The change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(h) The change in the translational kinetic energy is $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$

• Adiabiatic process with $\Delta T = 50$ K and $n = 3.0$ mol.

(i) $Q = 0$ by definition of "adiabatic."

(j) The first law leads to $W = Q - E_{\text{int}} = 0 - 900 \text{ cal} = -900$ cal.

(k) The change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(l) As in part (d) and (h), $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$

66. The ratio is

$$\frac{mgh}{mv_{\rm rms}^2/2}=\frac{2gh}{v_{\rm rms}^2}=\frac{2Mgh}{3RT}$$

where we have used Eq. 19-22 in that last step. With $T = 273$ K, $h = 0.10$ m and $M = 32$ g/mol = 0.032 kg/mol, we find the ratio equals 9.2×10^{-6}.

67. In this solution we will use non-standard notation: writing ρ for *weight*-density (instead of mass-density), where ρ_c refers to the cool air and ρ_h refers to the hot air. Then the condition required by the problem is

$$F_{\text{net}} = F_{\text{buoyant}} - \text{hot-air-weight} - \text{balloon-weight}$$

$$2.67 \times 10^3 \text{ N} = \rho_c V - \rho_h V - 2.45 \times 10^3 \text{ N}$$

where $V = 2.18 \times 10^3$ m^3 and $\rho_c = 11.9$ N/m^3. This condition leads to $\rho_h = 9.55$ N/m^3. Using the ideal gas law to write ρ_h as PMg/RT where $P = 101000$ Pascals and $M = 0.028$ kg/m^3 (as suggested in the problem), we conclude that the temperature of the enclosed air should be 349 K.

68. (a) In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3.00 V_0} = \frac{1}{3.00} p_0 \Rightarrow \frac{p_1}{p_0} = \frac{1}{3.00} = 0.333.$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$, i.e.,

$$\frac{1}{3.00} p_0 (3.00 V_0)^{\gamma} = (3.00)^{\frac{1}{3}} p_0 V_0^{\gamma}$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^{1/3} = 1.44.$$

69. (a) By Eq. 19-28, $W = -374$ J (since the process is an adiabatic compression).

(b) $Q = 0$ since the process is adiabatic.

(c) By first law of thermodynamics, the change in internal energy is $\Delta E_{int} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is

$$\Delta K_{avg} = \Delta E_{int}/N = +3.11 \times 10^{-22} \text{ J}.$$

70. (a) With work being given by

$$W = p\Delta V = (250)(-0.60)\text{ J} = -150\text{ J},$$

and the heat transfer given as –210 J, then the change in internal energy is found from the first law of thermodynamics to be [–210 – (–150)] J = –60 J.

(b) Since the pressures (and also the number of moles) don't change in this process, then the volume is simply proportional to the (absolute) temperature. Thus, the final temperature is ¼ of the initial temperature. The answer is 90 K.

71. This is very similar to Sample Problem 19-4 (and we use similar notation here) except for the use of Eq. 19-31 for v_{avg} (whereas in that Sample Problem, its value was just assumed). Thus,

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{avg}}{\lambda} = \frac{p\,d^2}{k}\sqrt{\frac{16\pi R}{MT}} \ .$$

Therefore, with $p = 2.02 \times 10^3$ Pa, $d = 290 \times 10^{-12}$ m and $M = 0.032$ kg/mol (see Table 19-1), we obtain $f = 7.03 \times 10^9\ \text{s}^{-1}$.

72. Eq. 19-25 gives the mean free path:

$$\lambda = \frac{1}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, (N/V)} = \frac{n\, R\, T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\ P\, N}$$

where we have used the ideal gas law in that last step. Thus, the change in the mean free path is

$$\Delta\lambda = \frac{n\, R\, \Delta T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\ P\, N} = \frac{R\, Q}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\ P\, N\, C_p}$$

where we have used Eq. 19-46. The constant pressure molar heat capacity is (7/2)R in this situation, so (with $N = 9 \times 10^{23}$ and $d = 250 \times 10^{-12}$ m) we find

$$\Delta\lambda = 1.52 \times 10^{-9}\,\text{m} = 1.52\ \text{nm}\ .$$

73. (a) The volume has increased by a factor of 3, so the pressure must decrease accordingly (since the temperature does not change in this process). Thus, the final pressure is one-third of the original 6.00 atm. The answer is 2.00 atm.

(b) We note that Eq. 19-14 can be written as $P_iV_i \ln(V_f/V_i)$. Converting "atm" to "Pa" (a Pascal is equivalent to a N/m^2) we obtain $W = 333$ J.

(c) The gas is monatomic so $\gamma = 5/3$. Eq. 19-54 then yields $P_f = 0.961$ atm.

(d) Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_iV_i^{\gamma}\int_{V_i}^{V_f} V^{-\gamma}dV = p_iV_i^{\gamma}\,\frac{V_f^{1-\gamma}-V_i^{1-\gamma}}{1-\gamma} = \frac{p_fV_f - p_iV_i}{1-\gamma}$$

where in the last step Eq. 19-54 has been used. Converting "atm" to "Pa", we obtain $W = 236$ J.

74. (a) With $P_1 = (20.0)(1.01 \times 10^5 \text{ Pa})$ and $V_1 = 0.0015 \text{ m}^3$, the ideal gas law gives

$$P_1V_1 = nRT_1 \quad \Rightarrow \quad T_1 = 121.54 \text{ K} \approx 122 \text{ K}.$$

(b) From the information in the problem, we deduce that $T_2 = 3T_1 = 365$ K.

(c) We also deduce that $T_3 = T_1$ which means $\Delta T = 0$ for this process. Since this involves an ideal gas, this implies the change in internal energy is zero here.

75. (a) We use $p_i V_i^{\gamma} = p_f V_f^{\gamma}$ to compute γ:

$$\gamma = \frac{\ln\left(p_i/p_f\right)}{\ln\left(V_f/V_i\right)} = \frac{\ln\left(1.0\,\text{atm}/1.0\times10^5\ \text{atm}\right)}{\ln\left(1.0\times10^3\ \text{L}/1.0\times10^6\ \text{L}\right)} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form (see Sample Problem 19-1), the final temperature is

$$T_f = T_i \frac{p_f V_f}{p_i V_i} = (273\,\text{K}) \frac{\left(1.0\times10^5\ \text{atm}\right)\left(1.0\times10^3\ \text{L}\right)}{\left(1.0\,\text{atm}\right)\left(1.0\times10^6\,\text{L}\right)} = 2.7\times10^4\ \text{K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{\left(1.01\times10^5\ \text{Pa}\right)\left(1.0\times10^3\ \text{cm}^3\right)}{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right)} = 4.5\times10^4\ \text{mol}.$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2} RT_i = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right) = 3.4\times10^3\ \text{J}.$$

(e) After the compression,

$$K_f = \frac{3}{2} RT_f = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(2.7\times10^4\ \text{K}\right) = 3.4\times10^5\ \text{J}.$$

(f) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\,\text{K}}{2.7\times10^4\ \text{K}} = 0.010.$$

76. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with V_2 = 4 m^3; then continues on to state 3 isothermally, with V_3 = 10 m^3; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \to 2$ $p_1V_1^\gamma = p_2V_2^\gamma$, for the isothermal process 2 → 3 $p_2V_2 = p_3V_3$, and finally for the adiabatic process $3 \to 4$ $p_3V_3^\gamma = p_4V_4^\gamma$. These equations yield

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^\gamma = p_2\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1\left(\frac{V_1}{V_2}\right)^\gamma\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma.$$

We substitute this expression for p_4 into the equation $p_1V_1 = p_4V_4$ (since $T_1 = T_4$) to obtain $V_1V_3 = V_2V_4$. Solving for V_4 we obtain

$$V_4 = \frac{V_1V_3}{V_2} = \frac{(2.0\,\mathrm{m}^3)(10\,\mathrm{m}^3)}{4.0\,\mathrm{m}^3} = 5.0\,\mathrm{m}^3.$$

77. (a) The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm},$$

(b) For the isothermal process the final temperature of the gas is $T_f = T_i = 300$ K.

(c) The work done is

$$W = nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = (32\,\text{atm})\left(1.01\times10^5\ \text{Pa/atm}\right)\left(1.0\times10^{-3}\ \text{m}^3\right)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right)$$
$$= 4.4\times10^3\ \text{J}.$$

For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

(d) The final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K}\ .$$

(f) The work done is

$$W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}\left(p_f V_f - p_i V_i\right)$$
$$= -\frac{3}{2}\left[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right)$$
$$= 2.9\times10^3\ \text{J}\ .$$

(g) If the gas is diatomic, then γ= 1.4, and the final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\,\text{atm}\,.$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\,\text{K}\,.$$

(i) The work done is

$$
\begin{aligned}
W &= Q - \Delta E_{\text{int}} = -\frac{5}{2} nR\Delta T = -\frac{5}{2}\left(p_f V_f - p_i V_i\right) \\
&= -\frac{5}{2}\left[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^{5}\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) \\
&= 3.4\times10^{3}\ \text{J}.
\end{aligned}
$$

78. We write $T = 273$ K and use Eq. 19-14:

$$W = (1.00\,\text{mol})\,(8.31\ \text{J/mol}\cdot\text{K})\,(273\,\text{K})\ \ln\left(\frac{16.8}{22.4}\right)$$

which yields $W = -653$ J. Recalling the sign conventions for work stated in Chapter 18, this means an external agent does 653 J of work *on* the ideal gas during this process.

79. (a) We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\,\text{g}}{17\ \text{g/mol}}\right)(8.31\ \text{J/mol}\cdot\text{K})(350\,\text{K})}{1.35\times10^{6}\ \text{Pa}} = 3.8\times10^{-2}\ \text{m}^3 = 38\,\text{L}.$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7\times10^{5}\ \text{Pa})(3.8\times10^{-2}\ \text{m}^3)}{(8.31\ \text{J/mol}\cdot\text{K})(293\,\text{K})} = 13.5\,\text{mol}.$$

The mass of the gas that leaked out is then Δm = 300 g – (13.5 mol)(17 g/mol) = 71 g.

80. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\,\text{K})}{M_{\text{hydrogen}}}}$$

for T. With the molar masses found in Table 19-1, we obtain

$$T = (293\,\text{K})\left(\frac{4.0}{2.02}\right) = 580\,\text{K}$$

which is equivalent to 307°C.

81. It is recommended to look over §19-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_o} P(v)\,dv = 1 \Rightarrow C = \frac{3}{v_o^3}.$$

(b) The average speed is

$$\int_0^{v_o} vP(v)\,dv = \int_0^{v_o} v\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{4}v_o\,.$$

(c) The rms speed is the square root of

$$\int_0^{v_o} v^2\ P(v)\,dv = \int_0^{v_o} v^2\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{5}v_o^2\,.$$

Therefore, $v_{\text{rms}} = \sqrt{3/5}v_{\circ} \approx 0.775v_{\circ}$.

82. To model the "uniform rates" described in the problem statement, we have expressed the volume and the temperature functions as follows:

$$V = V_i + \left(\frac{V_f - V_i}{\tau_f}\right) t \qquad \text{and} \qquad T = T_i + \left(\frac{T_f - T_i}{\tau_f}\right) t$$

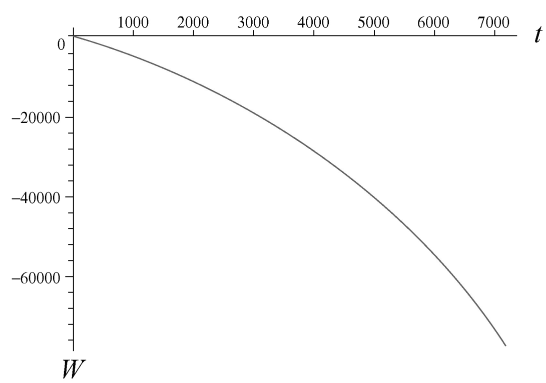
where $V_i = 0.616\ \text{m}^3$, $V_f = 0.308\ \text{m}^3$, $\tau_f = 7200$ s, $T_i = 300$ K and $T_f = 723$ K.

(a) We can take the derivative of V with respect to t and use that to evaluate the cumulative work done (from $t = 0$ until $t = \tau$):

$$W = \int p\, dV = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) dt = 12.2\,\tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $W = -77169$ J ≈ -77.2 kJ, or $|W| \approx$ 77.2 kJ.

The graph of cumulative work is shown below. The graph for work done is purely negative because the gas is being compressed (work is being done *on* the gas).

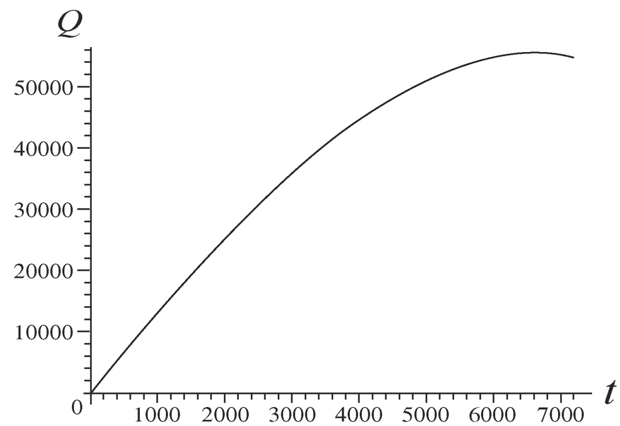


(b) With $C_V = \frac{3}{2}R$ (since it's a monatomic ideal gas) then the (infinitesimal) change in internal energy is $nC_V\, dT = \frac{3}{2}nR\left(\frac{dT}{dt}\right) dt$ which involves taking the derivative of the temperature expression listed above. Integrating this and adding this to the work done gives the cumulative heat absorbed (from $t = 0$ until $t = \tau$):

$$Q = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) + \frac{3}{2}nR\left(\frac{dT}{dt}\right) dt = 30.5\,\tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $Q_{\text{total}} = 54649$ J $\approx 5.46\times10^4$ J.

The graph cumulative heat is shown below. We see that $Q > 0$ since the gas is absorbing heat.



(c) Defining $C = \dfrac{Q_{\text{total}}}{n(T_f - T_i)}$ we obtain $C = 5.17$ J/mol·K. We note that this is considerably smaller than the constant-volume molar heat C_V.

We are now asked to consider this to be a two-step process (time dependence is no longer an issue) where the first step is isothermal and the second step occurs at constant volume (the ending values of pressure, volume and temperature being the same as before).

(d) Eq. 19-14 readily yields $W = -43222$ J $\approx -4.32 \times 10^4$ J (or $|W| \approx 4.32 \times 10^4$ J), where it is important to keep in mind that no work is done in a process where the volume is held constant.

(e) In step 1 the heat is equal to the work (since the internal energy does not change during an isothermal ideal gas process), and step 2 the heat is given by Eq. 19-39. The total heat is therefore $88595 \approx 8.86 \times 10^4$ J.

(f) Defining a molar heat capacity in the same manner as we did in part (c), we now arrive at $C = 8.38$ J/ mol·K.

83. (a) The temperature is 10.0°C → $T = 283$ K. Then, with $n = 3.50$ mol and $V_f/V_0 = 3/4$, we use Eq. 19-14:

$$W = nRT \ln\left(\frac{V_f}{V_0}\right) = -2.37\,\text{kJ}.$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.37$ kJ. The negative value implies that the heat transfer is from the sample to its environment.

84. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = N_A\left(\frac{p}{RT}\right)(6.02\times10^{23})\frac{1.01\times10^5\text{ Pa}}{\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\text{ K})} = 2.5\times10^{25}\,\frac{\text{molecules}}{\text{m}^3}.$$

(b) Three-fourths of the 2.5×10^{25} value found in part (a) are nitrogen molecules with M = 28.0 g/mol (using Table 19-1), and one-fourth of that value are oxygen molecules with M = 32.0 g/mol. Consequently, we generalize the $M_{sam} = NM/N_A$ expression for these two species of molecules and write

$$\frac{3}{4}(2.5\times10^{25})\frac{28.0}{6.02\times10^{23}} + \frac{1}{4}(2.5\times10^{25})\frac{32.0}{6.02\times10^{23}} = 1.2\times10^3\text{ g}.$$

85. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that $\sum_{\text{cycle}} E = 0$, which gives

$$\Delta E_{A\to B} + \Delta E_{B\to C} + \Delta E_{C\to D} + \Delta E_{D\to E} + \Delta E_{E\to A} = 0.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0.$$

Now, with $\Delta E_{E\to A}$ = 8.0 J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} + 8.0\text{ J} = 0.$$

In an adiabatic process, $\Delta E = -W$, which leads to $-5.0\text{ J} + \Delta E_{C\to D} + 8.0\text{ J} = 0$, and we obtain $\Delta E_{C\to D} = -3.0$ J.

86. (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\text{N/m}^2\right)(1.8\,\text{m}^3 - 3.0\,\text{m}^3) = -30\,\text{J}.$$

The sign conventions discussed in the textbook for Q indicate that we should write –75 J for the energy which leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

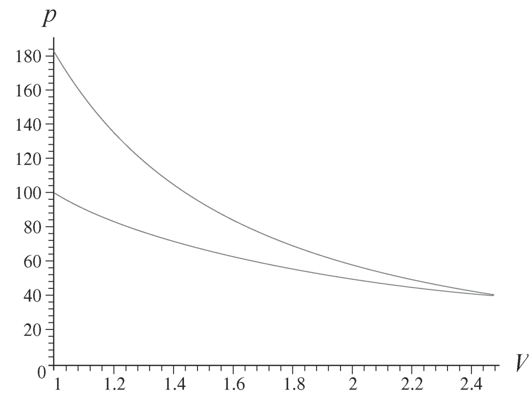
$$\Delta E_{\text{int}} = Q - W = (-75\text{ J}) - (-30\text{ J}) = -45\text{ J}.$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form (see Sample Problem 19-1) leads to

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (300\,\text{K})\left(\frac{1.8\,\text{m}^3}{3.0\,\text{m}^3}\right) = 1.8\times10^2\text{ K}.$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 19-45, one arrives at this same value for the final temperature).

87. (a) The *p*-*V* diagram is shown below. Note that o obtain the above graph, we have chosen $n = 0.37$ moles for concreteness, in which case the horizontal axis (which we note starts not at zero but at 1) is to be interpreted in units of cubic centimeters, and the vertical axis (the absolute pressure) is in kilopascals. However, the constant volume temp-increase process described in the third step (see problem statement) is difficult to see in this graph since it coincides with the pressure axis.



(b) We note that the change in internal energy is zero for an ideal gas isothermal process, so (since the net change in the internal energy must be zero for the entire cycle) the increase in internal energy in step 3 must equal (in magnitude) its decease in step 1. By Eq. 19-28, we see this number must be 125 J.

(c) As implied by Eq. 19-29, this is equivalent to heat being added *to the gas*.

88. (a) The ideal gas law leads to

$$V = \frac{nRT}{p} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})\,(273\,\text{K})}{1.01\times10^5\ \text{Pa}}$$

which yields $V = 0.0225\ \text{m}^3 = 22.5$ L. If we use the standard pressure value given in Appendix D, 1 atm = 1.013×10^5 Pa, then our answer rounds more properly to 22.4 L.

(b) From Eq. 19-2, we have $N = 6.02 \times 10^{23}$ molecules in the volume found in part (a) (which may be expressed as $V = 2.24 \times 10^4\ \text{cm}^3$), so that

$$\frac{N}{V} = \frac{6.02\times10^{23}}{2.24\times10^4\ \text{cm}^3} = 2.69\times10^{19}\ \text{molecules/cm}^3\,.$$

Chapter 20

1. An isothermal process is one in which $T_i = T_f$ which implies $\ln(T_f/T_i) = 0$. Therefore, with $V_f/V_i = 2.00$, Eq. 20-4 leads to

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = (2.50 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})\ln(2.00) = 14.4 \text{ J/K}.$$

2. From Eq. 20-2, we obtain

$$Q = T\Delta S = (405\ \text{K})(46.0\ \text{J/K}) = 1.86\times10^{4}\ \text{J}.$$

3. We use the following relation derived in Sample Problem 20-2:

$$\Delta S = mc \ln\left(\frac{T_f}{T_i}\right).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386 \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(2.00 \text{ kg})(75 \text{ K}) = 5.79\times10^4 \text{ J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With $T_f = 373.15$ K and $T_i = 298.15$ K, we obtain

$$\Delta S = (2.00 \text{ kg})\left(386 \frac{\text{J}}{\text{kg}\cdot\text{K}}\right) \ln\left(\frac{373.15}{298.15}\right) = 173 \text{ J/K}.$$

4. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333$ J/g from Table 19-4. Consequently,

$$\Delta S = \frac{(333\ \text{J/g})(12.0\ \text{g})}{273\ \text{K}} = 14.6\ \text{J/K}.$$

(b) The situation is similar to that described in part (a), except with $L = 2256$ J/g, $m = 5.00$ g, and $T = 373$ K. We therefore find $\Delta S = 30.2$ J/K.

5. (a) Since the gas is ideal, its pressure p is given in terms of the number of moles n, the volume V, and the temperature T by $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1}.$$

We substitute $V_2 = 2.00V_1$ to obtain

$$W = nRT \ln 2.00 = (4.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(400 \text{ K}) \ln 2.00 = 9.22\times10^3 \text{ J}.$$

(b) Since the expansion is isothermal, the change in entropy is given by

$$\Delta S = \int (1/T)\, dQ = Q/T,$$

where Q is the heat absorbed. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J/K}.$$

(c) $\Delta S = 0$ for all reversible adiabatic processes.

6. An isothermal process is one in which $T_i = T_f$ which implies ln $(T_f/T_i) = 0$. Therefore, Eq. 20-4 leads to

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \Rightarrow n = \frac{22.0}{(8.31)\ln(3.4/1.3)} = 2.75 \text{ mol.}$$

7. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a(T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w(T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a \left(T_{ai} - T_f\right) = m_w c_w \left(T_f - T_{wi}\right) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})(100°\text{C}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})(20°\text{C})}{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})} \\ &= 57.0°\text{C} = 330\text{ K}. \end{aligned}$$

(b) Now temperatures must be given in Kelvins: T_{ai} = 393 K, T_{wi} = 293 K, and T_f = 330 K. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} = (0.200\text{ kg})(900\text{ J/kg}\cdot\text{K}) \ln\left(\frac{330\text{ K}}{373\text{ K}}\right) \\ &= -22.1\text{ J/K}. \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} = (0.0500\text{ kg})(4190\text{ J/kg.K}) \ln\left(\frac{330\,\text{K}}{293\,\text{K}}\right) \\ &= +24.9\,\text{J/K}. \end{aligned}$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1\text{ J/K} + 24.9\text{ J/K} = +2.8\text{ J/K}.$$

8. We follow the method shown in Sample Problem 20-2. Since

$$\Delta S = mc\int_{T_i}^{T_f}\frac{dT}{T} = mc\,\ln(T_f/T_i)\ ,$$

then with ΔS = 50 J/K, T_f = 380 K, T_i = 280 K and m = 0.364 kg, we obtain $c = 4.5\times10^2$ J/kg·K.

9. This problem is similar to Sample Problem 20-2. The only difference is that we need to find the mass m of each of the blocks. Since the two blocks are identical the final temperature T_f is the average of the initial temperatures:

$$T_f = \frac{1}{2}\left(T_i + T_f\right) = \frac{1}{2}(305.5\ \text{K} + 294.5\ \text{K}) = 300.0\ \text{K}.$$

Thus from $Q = mc\Delta T$ we find the mass m:

$$m = \frac{Q}{c\Delta T} = \frac{215\ \text{J}}{(386\ \text{J}/\text{kg}\cdot\text{K})(300.0\ \text{K} - 294.5\ \text{K})} = 0.101\ \text{kg}.$$

(a) The change in entropy for block L is

$$\Delta S_L = mc\ \ln\left(\frac{T_f}{T_{iL}}\right) = (0.101\ \text{kg})(386\ \text{J/kg}\cdot\text{K})\ln\left(\frac{300.0\ \text{K}}{305.5\ \text{K}}\right) = -0.710\ \text{J/K}.$$

(b) Since the temperature of the reservoir is virtually the same as that of the block, which gives up the same amount of heat as the reservoir absorbs, the change in entropy $\Delta S'_L$ of the reservoir connected to the left block is the opposite of that of the left block: $\Delta S'_L$ = $-\Delta S_L = +0.710$ J/K.

(c) The entropy change for block R is

$$\Delta S_R = mc\ln\left(\frac{T_f}{T_{iR}}\right) = (0.101\ \text{kg})(386\ \text{J/kg}\cdot\text{K})\ln\left(\frac{300.0\ \text{K}}{294.5\ \text{K}}\right) = +0.723\ \text{J/K}.$$

(d) Similar to the case in part (b) above, the change in entropy $\Delta S'_R$ of the reservoir connected to the right block is given by $\Delta S'_R = -\Delta S_R = -0.723$ J/K.

(e) The change in entropy for the two-block system is

$$\Delta S_L + \Delta S_R = -0.710\ \text{J/K} + 0.723\ \text{J/K} = +0.013\ \text{J/K}.$$

(f) The entropy change for the entire system is given by

$$\Delta S = \Delta S_L + \Delta S'_L + \Delta S_R + \Delta S'_R = \Delta S_L - \Delta S_L + \Delta S_R - \Delta S_R = 0,$$

which is expected of a reversible process.

10. We concentrate on the first term of Eq. 20-4 (the second term is zero because the final and initial temperatures are the same, and because ln(1) = 0). Thus, the entropy change is

$$\Delta S = nR \ln(V_f / V_i) \ .$$

Noting that $\Delta S = 0$ at $V_f = 0.40 \text{ m}^3$, we are able to deduce that $V_i = 0.40 \text{ m}^3$. We now examine the point in the graph where $\Delta S = 32$ J/K and $V_f = 1.2 \text{ m}^3$; the above expression can now be used to solve for the number of moles. We obtain $n = 3.5$ mol.

11. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature T_f satisfies

$$m_1c_1(T_f - T_{i,1}) + m_2c_2(T_f - T_{i2}) = 0,$$

which we solve for T_f:

$$T_f = \frac{m_1c_1T_{i,1} + m_2c_2T_{i,2}}{m_1c_1 + m_2c_2} = \frac{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K})(400\text{ K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})(200\text{ K})}{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})}$$
$$= 320\text{ K}.$$

(b) Since the two-block system in thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1c_1\ln\left(\frac{T_f}{T_{i,1}}\right) + m_2c_2\ln\left(\frac{T_f}{T_{i,2}}\right)$$
$$= (50.0\text{ g})(386\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{400\text{ K}}\right) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{200\text{ K}}\right)$$
$$= +1.72\text{ J/K}.$$

12. We use Eq. 20-1:

$$\Delta S = \int \frac{nC_V \, dT}{T} = nA \int_{5.00}^{10.0} T^2 dT = \frac{nA}{3}\left[(10.0)^3 - (5.00)^3 \right] = 0.0368 \text{ J/K}.$$

13. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting $f = 5$ in Eq. 19-51. Thus, $C_V = 5R/2$, $C_p = 7R/2$, and $\gamma = 7/5$. In addition to various equations from Chapter 19, we also make use of Eq. 20-4 of this chapter. We note that we are asked to use the ideal gas constant as R and not plug in its numerical value. We also recall that isothermal means constant-temperature, so $T_2 = T_1$ for the $1 \to 2$ process. The statement (at the end of the problem) regarding "per mole" may be taken to mean that n may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form (see Sample Problem 19-1) is used to obtain

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) = \frac{p_1}{3} \quad \Rightarrow \quad \frac{p_2}{p_1} = \frac{1}{3} = 0.333\,.$$

(b) The adiabatic relations Eq. 19-54 and Eq. 19-56 lead to

$$p_3 = p_1\left(\frac{V_1}{V_3}\right)^{\gamma} = \frac{p_1}{3^{1.4}} \quad \Rightarrow \quad \frac{p_3}{p_1} = \frac{1}{3^{1.4}} = 0.215\,.$$

(c) Similarly, we find

$$T_3 = T_1\left(\frac{V_1}{V_3}\right)^{\gamma-1} = \frac{T_1}{3^{0.4}} \quad \Rightarrow \quad \frac{T_3}{T_1} = \frac{1}{3^{0.4}} = 0.644.$$

• process $1 \to 2$

(d) The work is given by Eq. 19-14:

$$W = nRT_1 \ln (V_2/V_1) = RT_1 \ln 3 = 1.10RT_1.$$

Thus, $W/\,nRT_1 = \ln 3 = 1.10$.

(e) The internal energy change is $\Delta E_{\text{int}} = 0$ since this is an ideal gas process without a temperature change (see Eq. 19-45). Thus, the energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$, or $Q/\,nRT_1 = \ln 3 = 1.10$.

(f) $\Delta E_{\text{int}} = 0$ or $\Delta E_{\text{int}}\,/\,nRT_1 = 0$

(g) The entropy change is $\Delta S = Q/T_1 = 1.10R$, or $\Delta S/R = 1.10$.

• process $2 \to 3$

(h) The work is zero since there is no volume change. Therefore, $W/nRT_1 = 0$

(i) The internal energy change is

$$\Delta E_{\text{int}} = nC_V\left(T_3 - T_2\right) = (1)\left(\frac{5}{2}R\right)\left(\frac{T_1}{3^{0.4}} - T_1\right) \approx -0.889\,RT_1 \;\;\Rightarrow\;\; \frac{\Delta E_{\text{int}}}{nRT_1} \approx -0.889.$$

This ratio (–0.889) is also the value for Q/nRT_1 (by either the first law of thermodynamics or by the definition of C_V).

(j) $\Delta E_{\text{int}}/nRT_1 = -0.889$.

(k) For the entropy change, we obtain

$$\frac{\Delta S}{R} = n\ln\left(\frac{V_3}{V_1}\right) + n\frac{C_V}{R}\ln\left(\frac{T_3}{T_1}\right) = (1)\;\ln(1) + (1)\left(\frac{5}{2}\right)\ln\left(\frac{T_1/3^{0.4}}{T_1}\right) = 0 + \frac{5}{2}\ln(3^{-0.4}) \approx -1.10\;.$$

• process 3 → 1

(l) By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process 1 → 2), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then,

$$W = Q - \Delta E_{\text{int}} = -0.889RT_1,$$

or $W/nRT_1 = -0.889$.

(m) $Q = 0$ in an adiabatic process.

(n) $\Delta E_{\text{int}}/nRT_1 = +0.889$.

(o) $\Delta S/nR = 0$.

14. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or "area under the curve") of a curve in a *TS* diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a "straight-line" path in the *TS* diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2}\right) \Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process which forms a straight line in a graph that plots T versus S). This leads to

$$Q = (300 \text{ K})\ (15 \text{ J/K}) = 4.5\times10^3 \text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\Delta T = (2.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(200 \text{ K} - 400 \text{ K}) = -5.0\times10^3 \text{ J}.$$

(c) By the first law of thermodynamics,

$$W = Q - \Delta E_{\text{int}} = 4.5 \text{ kJ} - (-5.0 \text{ kJ}) = 9.5 \text{ kJ}.$$

15. The ice warms to 0°C, then melts, and the resulting water warms to the temperature of the lake water, which is 15°C. As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i (= 263 K) is the initial temperature and T_f (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln \frac{T_f}{T_i} = (0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{273 \text{ K}}{263 \text{ K}}\right) = 0.828 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F, where L_F is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = 12.20 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln \frac{T_f}{T_i},$$

where c_w is the specific heat of water (4190 J/kg · K). Thus,

$$\Delta S = (0.010 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{288 \text{ K}}{273 \text{ K}}\right) = 2.24 \text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828 \text{ J/K} + 12.20 \text{ J/K} + 2.24 \text{ J/K} = 15.27 \text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0°C,

$$Q = -mc_I (T_f - T_i) = -(0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{ K})(10 \text{ K}) = -222 \text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010 \text{ kg})(333 \times 10^3 \text{ J / kg}) = -3.33 \times 10^3 \text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w (T_f - T_i) = -(0.010 \text{ kg})(4190 \text{ J / kg} \cdot \text{ K})(15 \text{ K}) = -629 \text{ J}.$$

The total energy leaving the lake water is

$$Q = -222\text{ J} - 3.33 \times 10^3\text{ J} - 6.29 \times 10^2\text{ J} = -4.18 \times 10^3\text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18 \times 10^3\text{ J}}{288\text{ K}} = -14.51\text{ J/K}.$$

The change in the entropy of the ice-lake system is $\Delta S = (15.27 - 14.51)\text{ J/K} = 0.76\text{ J/K}$.

16. (a) Work is done only for the *ab* portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W = \int_{V_0}^{4V_0} p_0\,dV = p_0(4.00V_0 - 1.00V_0) = 3.00p_0V_0 \;\Rightarrow\; \frac{W}{p_0V} = 3.00$$

(b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy Q absorbed by the gas as heat is $Q = nC_V\,\Delta T$, where C_V is the molar specific heat at constant volume and ΔT is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = 3R/2$. Use the ideal gas law to find that the initial temperature is

$$T_b = \frac{p_bV_b}{nR} = \frac{4p_0V_0}{nR}$$

and that the final temperature is

$$T_c = \frac{p_cV_c}{nR} = \frac{(2p_0)(4V_0)}{nR} = \frac{8p_0V_0}{nR}.$$

Thus,

$$Q = \frac{3}{2}nR\left(\frac{8p_0V_0}{nR} - \frac{4p_0V_0}{nR}\right) = 6.00p_0V_0.$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0V_0$ or $\Delta E_{\text{int}}/p_0V_0 = 6.00$. Since $n = 1$ mol, this can also be written $Q = 6.00RT_0$.

(c) For a complete cycle, $\Delta E_{\text{int}} = 0$

(d) Since the process is at constant volume, use $dQ = nC_V\,dT$ to obtain

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln\frac{T_c}{T_b}.$$

Substituting $C_V = \frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b} = \frac{p_cV_c}{p_bV_b} = \frac{(2p_0)(4V_0)}{p_0(4V_0)} = 2.$$

Thus, $\Delta S = \frac{3}{2}nR\ln 2$. Since $n = 1$, this is $\Delta S = \frac{3}{2}R\ln 2 = 8.64$ J/K..

(e) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

17. (a) The final mass of ice is (1773 g + 227 g)/2 = 1000 g. This means 773 g of water froze. Energy in the form of heat left the system in the amount mL_F, where m is the mass of the water that froze and L_F is the heat of fusion of water. The process is isothermal, so the change in entropy is

$$\Delta S = Q/T = -mL_F/T = -(0.773\text{ kg})(333 \times 10^3\text{ J/kg})/(273\text{ K}) = -943\text{ J/K}.$$

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943\text{ J/K}.$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water-ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir-water-ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner-water-ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

18. In coming to equilibrium, the heat lost by the 100 cm^3 of liquid water (of mass m_w = 100 g and specific heat capacity c_w = 4190 J/kg·K) is absorbed by the ice (of mass m_i which melts and reaches $T_f > 0$ °C). We begin by finding the equilibrium temperature:

$$\sum Q = 0$$

$$Q_{\text{warm water cools}} + Q_{\text{ice warms to }0^\circ} + Q_{\text{ice melts}} + Q_{\text{melted ice warms}} = 0$$

$$c_w m_w \left(T_f - 20^\circ\right) + c_i m_i \left(0^\circ - \left(-10^\circ\right)\right) + L_F m_i + c_w m_i \left(T_f - 0^\circ\right) = 0$$

which yields, after using L_F = 333000 J/kg and values cited in the problem, T_f = 12.24 ° which is equivalent to T_f = 285.39 K. Sample Problem 19-2 shows that

$$\Delta S_{\text{temp change}} = mc \ln\left(\frac{T_2}{T_1}\right)$$

for processes where $\Delta T = T_2 - T_1$, and Eq. 20-2 gives

$$\Delta S_{\text{melt}} = \frac{L_F\, m}{T_{\text{o}}}$$

for the phase change experienced by the ice (with T_{o} = 273.15 K). The total entropy change is (with T in Kelvins)

$$\begin{aligned}\Delta S_{\text{system}} &= m_w c_w \ln\left(\frac{285.39}{293.15}\right) + m_i c_i \ln\left(\frac{273.15}{263.15}\right) + m_i c_w \ln\left(\frac{285.39}{273.15}\right) + \frac{L_F\, m_i}{273.15} \\ &= (-11.24 + 0.66 + 1.47 + 9.75)\text{J/K} = 0.64 \text{ J/K.}\end{aligned}$$

19. We consider a three-step reversible process as follows: the supercooled water drop (of mass m) starts at state 1 (T_1 = 268 K), moves on to state 2 (still in liquid form but at T_2 = 273 K), freezes to state 3 ($T_3 = T_2$), and then cools down to state 4 (in solid form, with $T_4 = T_1$). The change in entropy for each of the stages is given as follows:

$$\Delta S_{12} = mc_w \ln (T_2/T_1),$$

$$\Delta S_{23} = -mL_F/T_2,$$

$$\Delta S_{34} = mc_I \ln (T_4/T_3) = mc_I \ln (T_1/T_2) = -mc_I \ln (T_2/T_1).$$

Thus the net entropy change for the water drop is

$$\begin{aligned}\Delta S &= \Delta S_{12} + \Delta S_{23} + \Delta S_{34} = m(c_w - c_I)\ln\left(\frac{T_2}{T_1}\right) - \frac{mL_F}{T_2} \\ &= (1.00\text{ g})(4.19\text{ J/g}\cdot\text{K} - 2.22\text{ J/g}\cdot\text{K})\ln\left(\frac{273\text{ K}}{268\text{ K}}\right) - \frac{(1.00\text{ g})(333\text{ J/g})}{273\text{ K}} \\ &= -1.18\text{ J/K}.\end{aligned}$$

20. (a) We denote the mass of the ice (which turns to water and warms to T_f) as m and the mass of original-water (which cools from 80º down to T_f) as m'. From $\Sigma Q = 0$ we have

$$L_F\, m + cm\,(T_f - 0^\circ) + cm'\,(T_f - 80^\circ) = 0\ .$$

Since $L_F = 333 \times 10^3$ J/kg, $c = 4190$ J/(kg·Cº), $m' = 0.13$ kg and $m = 0.012$ kg, we find T_f = 66.5ºC, which is equivalent to 339.67 K.

(b) Using Eq. 20-2, the process of ice at 0º C turning to water at 0º C involves an entropy change of

$$\frac{Q}{T} = \frac{L_F\, m}{273.15\ \text{K}} = 14.6\ \text{J/K}\ .$$

(c) Using Eq. 20-1, the process of $m = 0.012$ kg of water warming from 0º C to 66.5º C involves an entropy change of

$$\int_{273.15}^{339.67} \frac{cm\,dT}{T} = cm \ln\left(\frac{339.67}{273.15}\right) = 11.0\ \text{J/K}\ .$$

(d) Similarly, the cooling of the original-water involves an entropy change of

$$\int_{353.15}^{339.67} \frac{cm'\,dT}{T} = cm' \ln\left(\frac{339.67}{353.15}\right) = -21.2\ \text{J/K}$$

(e) The net entropy change in this calorimetry experiment is found by summing the previous results; we find (by using more precise values than those shown above) ΔS_{net} = 4.39 J/K.

21. We note that the connection between molar heat capacity and the degrees of freedom of a monatomic gas is given by setting $f = 3$ in Eq. 19-51. Thus, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$.

(a) Since this is an ideal gas, Eq. 19-45 holds, which implies $\Delta E_{\text{int}} = 0$ for this process. Eq. 19-14 also applies, so that by the first law of thermodynamics,

$$Q = 0 + W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2 \rightarrow \quad Q/p_1V_1 = \ln 2 = 0.693.$$

(b) The gas law in ratio form (see Sample Problem 19-1) implies that the pressure decreased by a factor of 2 during the isothermal expansion process to $V_2=2.00V_1$, so that it needs to increase by a factor of 4 in this step in order to reach a final pressure of $p_2=2.00p_1$. That same ratio form now applied to this constant-volume process, yielding $4.00 = T_2T_1$ which is used in the following:

$$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_2 - T_1) = \frac{3}{2}nRT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{3}{2}p_1V_1(4-1) = \frac{9}{2}p_1V_1$$

or $Q/p_1V_1 = 9/2 = 4.50$.

(c) The work done during the isothermal expansion process may be obtained by using Eq. 19-14:

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2.00 \rightarrow \quad W/p_1V_1 = \ln 2 = 0.693.$$

(d) In step 2 where the volume is kept constant, $W = 0$.

(e) The change in internal energy can be calculated by combining the above results and applying the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(p_1V_1 \ln 2 + \frac{9}{2}p_1V_1\right) - \left(p_1V_1 \ln 2 + 0\right) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$.

(f) The change in entropy may be computed by using Eq. 20-4:

$$\begin{aligned}\Delta S &= R\ln\left(\frac{2.00V_1}{V_1}\right) + C_V \ln\left(\frac{4.00T_1}{T_1}\right) = R\ln 2.00 + \left(\frac{3}{2}R\right)\ln(2.00)^2 \\ &= R\ln 2.00 + 3R\ln 2.00 = 4R\ln 2.00 = 23.0 \text{ J/K}.\end{aligned}$$

The second approach consists of an isothermal (constant T) process in which the volume halves, followed by an isobaric (constant p) process.

(g) Here the gas law applied to the first (isothermal) step leads to a volume half as big as the original. Since $\ln(1/2.00) = -\ln 2.00$, the reasoning used above leads to

$$Q = -p_1V_1 \ln 2.00 \Rightarrow \; Q/p_1V_1 = -\ln 2.00 = -0.693.$$

(h) To obtain a final volume twice as big as the original, in this step we need to increase the volume by a factor of 4.00. Now, the gas law applied to this isobaric portion leads to a temperature ratio $T_2/T_1 = 4.00$. Thus,

$$Q = C_p\Delta T = \frac{5}{2}R(T_2 - T_1) = \frac{5}{2}RT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{5}{2}p_1V_1(4-1) = \frac{15}{2}p_1V_1$$

or $Q/p_1V_1 = 15/2 = 7.50$.

(i) During the isothermal compression process, Eq. 19-14 gives

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln(-1/2.00) = -p_1V_1 \ln 2.00 \;\Rightarrow\; W/p_1V_1 = -\ln 2 = -0.693.$$

(j) The initial value of the volume, for this part of the process, is $V_i = V_1/2$, and the final volume is $V_f = 2V_1$. The pressure maintained during this process is $p' = 2.00p_1$. The work is given by Eq. 19-16:

$$W = p'\Delta V = p'(V_f - V_i) = (2.00p_1)\left(2.00V_1 - \frac{1}{2}V_1\right) = 3.00p_1V_1 \;\Rightarrow\; W/p_1V_1 = 3.00.$$

(k) Using the first law of thermodynamics, the change in internal energy is

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(\frac{15}{2}p_1V_1 - p_1V_1\ln 2.00\right) - (3p_1V_1 - p_1V_1\ln 2.00) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$. The result is the same as that obtained in part (e).

(l) Similarly, $\Delta S = 4R\ln 2.00 = 23.0$ J/K. the same as that obtained in part (f).

22. (a) The final pressure is

$$p_f = (5.00\text{ kPa})e^{(V_i - V_f)/a} = (5.00\text{ kPa})e^{(1.00\text{ m}^3 - 2.00\text{ m}^3)/1.00\text{ m}^3} = 1.84\text{ kPa}.$$

(b) We use the ratio form of the gas law (see Sample Problem 19-1) to find the final temperature of the gas:

$$T_f = T_i\left(\frac{p_f V_f}{p_i V_i}\right) = (600\text{ K})\frac{(1.84\text{ kPa})(2.00\text{ m}^3)}{(5.00\text{ kPa})(1.00\text{ m}^3)} = 441\text{ K}.$$

For later purposes, we note that this result can be written "exactly" as $T_f = T_i\,(2e^{-1})$. In our solution, we are avoiding using the "one mole" datum since it is not clear how precise it is.

(c) The work done by the gas is

$$\begin{aligned} W &= \int_i^f p\,dV = \int_{V_i}^{V_f} (5.00\text{ kPa})e^{(V_i - V)/a}\,dV = (5.00\text{ kPa})e^{V_i/a}\cdot\left[-ae^{-V/a}\right]_{V_i}^{V_f} \\ &= (5.00\text{ kPa})e^{1.00}\left(1.00\text{ m}^3\right)\left(e^{-1.00} - e^{-2.00}\right) \\ &= 3.16\text{ kJ}. \end{aligned}$$

(d) Consideration of a two-stage process, as suggested in the hint, brings us simply to Eq. 20-4. Consequently, with $C_V = \frac{3}{2}R$ (see Eq. 19-43), we find

$$\begin{aligned} \Delta S &= nR\ln\left(\frac{V_f}{V_i}\right) + n\left(\frac{3}{2}R\right)\ln\left(\frac{T_f}{T_i}\right) = nR\left(\ln 2 + \frac{3}{2}\ln\left(2e^{-1}\right)\right) = \frac{p_i V_i}{T_i}\left(\ln 2 + \frac{3}{2}\ln 2 + \frac{3}{2}\ln e^{-1}\right) \\ &= \frac{(5000\text{ Pa})(1.00\text{ m}^3)}{600\text{ K}}\left(\frac{5}{2}\ln 2 - \frac{3}{2}\right) \\ &= 1.94\text{ J/K}. \end{aligned}$$

23. We solve (b) first.

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20-13. Therefore,

$$T_{\mathrm{H}} = \frac{T_{\mathrm{H}} - T_{\mathrm{L}}}{\varepsilon} = \frac{75\ \mathrm{K}}{0.22} = 341\ \mathrm{K}$$

which is equivalent to 68°C.

(a) The temperature of the cold reservoir is $T_{\mathrm{L}} = T_{\mathrm{H}} - 75 = 341\ \mathrm{K} - 75\ \mathrm{K} = 266\ \mathrm{K}$.

24. Eq. 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{373\text{ K}}{7\times10^8\text{ K}} = 0.9999995$$

quoting more figures than are significant. As a percentage, this is $\varepsilon = 99.99995\%$.

25. (a) The efficiency is

$$\varepsilon = \frac{T_{\mathrm{H}} - T_{\mathrm{L}}}{T_{\mathrm{H}}} = \frac{(235-115)\,\mathrm{K}}{(235+273)\,\mathrm{K}} = 0.236 = 23.6\% \ .$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_{\mathrm{H}}|$, the work done is given by

$$|W| = \varepsilon |Q_{\mathrm{H}}| = 0.236\,(6.30\times10^4\ \mathrm{J}) = 1.49\times10^4\ \mathrm{J}\ .$$

26. The answers to this exercise do not depend on the engine being of the Carnot design. Any heat engine that intakes energy as heat (from, say, consuming fuel) equal to $|Q_H|$ = 52 kJ and exhausts (or discards) energy as heat equal to $|Q_L|$ = 36 kJ will have these values of efficiency ε and net work W.

(a) Eq. 20-12 gives

$$\varepsilon = 1 - \left|\frac{Q_L}{Q_H}\right| = 0.31 = 31\% \ .$$

(b) Eq. 20-8 gives

$$W = |Q_H| - |Q_L| = 16 \text{ kJ} \ .$$

27. With T_L = 290 k, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow T_H = \frac{T_L}{1-\varepsilon} = \frac{290\ \text{K}}{1-0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: T_H = 483 K. If we replace ε = 0.40 in the above calculation with ε = 0.50, we obtain a (final) high temperature equal to $T'_H = 580\ \text{K}$. The difference is

$$T'_H - T_H = 580\ \text{K} - 483\ \text{K} = 97\ \text{K}.$$

28. (a) Eq. 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{333\ \text{K}}{373\ \text{K}} = 0.107.$$

We recall that a Watt is Joule-per-second. Thus, the (net) work done by the cycle per unit time is the given value 500 J/s. Therefore, by Eq. 20-11, we obtain the heat input per unit time:

$$\varepsilon = \frac{W}{|Q_H|} \quad \Rightarrow \quad \frac{0.500\ \text{kJ/s}}{0.107} = 4.67\ \text{kJ/s}\ .$$

(b) Considering Eq. 20-8 on a per unit time basis, we find (4.67 – 0.500) kJ/s = 4.17 kJ/s for the rate of heat exhaust.

29. (a) Energy is added as heat during the portion of the process from a to b. This portion occurs at constant volume (V_b), so $Q_{\text{in}} = nC_V\,\Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a)\, V_b.$$

Thus, $Q_{\text{in}} = \frac{3}{2}\left(p_b - p_a\right)V_b$. V_b and p_b are given. We need to find p_a. Now p_a is the same as p_c and points c and b are connected by an adiabatic process. Thus, $p_c V_c^\gamma = p_b V_b^\gamma$ and

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3}\left(1.013\times10^6 \text{ Pa}\right) = 3.167\times10^4 \text{ Pa}.$$

The energy added as heat is

$$Q_{\text{in}} = \frac{3}{2}\left(1.013\times10^6 \text{ Pa} - 3.167\times10^4 \text{ Pa}\right)\left(1.00\times10^{-3} \text{ m}^3\right) = 1.47\times10^3 \text{ J}.$$

(b) Energy leaves the gas as heat during the portion of the process from c to a. This is a constant pressure process, so

$$\begin{aligned} Q_{\text{out}} &= nC_p\Delta T = \frac{5}{2}\left(p_a V_a - p_c V_c\right) = \frac{5}{2}p_a\left(V_a - V_c\right) \\ &= \frac{5}{2}\left(3.167\times10^4 \text{ Pa}\right)\left(-7.00\right)\left(1.00\times10^{-3} \text{ m}^3\right) = -5.54\times10^2 \text{ J}, \end{aligned}$$

or $|Q_{\text{out}}| = 5.54\times10^2$ J. The substitutions $V_a - V_c = V_a - 8.00\,V_a = -\,7.00\,V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_{\text{in}} = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

30. From Fig. 20-28, we see $Q_H = 4000$ J at $T_H = 325$ K. Combining Eq. 20-11 with Eq. 20-13, we have

$$\frac{W}{Q_H} = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad W = 923 \text{ J} .$$

Now, for $T'_H = 550$ K, we have

$$\frac{W}{Q'_H} = 1 - \frac{T_C}{T'_H} \qquad \Rightarrow \quad Q'_H = 1692 \text{ J} \approx 1.7 \text{ kJ}$$

31. (a) The net work done is the rectangular "area" enclosed in the pV diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0 p_0.$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

(b) We compute the energy added as heat during the "heat-intake" portions of the cycle using Eq. 19-39, Eq. 19-43, and Eq. 19-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) = n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) = p_0V_0\left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \\ &= \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used (see Sample Problem 19-1). Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

(c) The efficiency is given by Eq. 20-11:

$$\varepsilon = \frac{W}{|Q_H|} = \frac{2}{13} = 0.154 = 15.4\%.$$

(d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used.

(e) This is greater than our result in part (c), as expected from the second law of thermodynamics.

32. (a) Using Eq. 19-54 for process $D \to A$ gives

$$p_D V_D^{\gamma} = p_A V_A^{\gamma} \quad \Rightarrow \quad \frac{p_0}{32}(8V_0)^{\gamma} = p_0 V_0^{\gamma}$$

which leads to $8^{\gamma} = 32 \Rightarrow \gamma = 5/3$. The result (see §19-9 and §19-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \to B$:

$$Q_{\mathrm{H}} = nC_p \Delta T = n\left(\frac{5}{2}R\right)T_A\left(\frac{T_B}{T_A} - 1\right) = nRT_A\left(\frac{5}{2}\right)(2-1) = p_0 V_0\left(\frac{5}{2}\right)$$

and the exhaust heat is that liberated during process $C \to D$:

$$Q_{\mathrm{L}} = nC_p \Delta T = n\left(\frac{5}{2}R\right)T_D\left(1 - \frac{T_{\mathrm{L}}}{T_D}\right) = nRT_D\left(\frac{5}{2}\right)(1-2) = -\frac{1}{4}p_0 V_0\left(\frac{5}{2}\right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4}T_A$ (from the gas law in ratio form — see Sample Problem 19-1). Therefore, Eq. 20-12 leads to

$$\varepsilon = 1 - \left|\frac{Q_{\mathrm{L}}}{Q_{\mathrm{H}}}\right| = 1 - \frac{1}{4} = 0.75 = 75\%.$$

33. (a) We use $\varepsilon = |W/Q_H|$. The heat absorbed is $|Q_{\rm H}| = \dfrac{|W|}{\varepsilon} = \dfrac{8.2\,\text{kJ}}{0.25} = 33\,\text{kJ}.$

(b) The heat exhausted is then $|Q_{\rm L}| = |Q_{\rm H}| - |W| = 33\,\text{kJ} - 8.2\,\text{kJ} = 25\,\text{kJ}.$

(c) Now we have $|Q_{\rm H}| = \dfrac{|W|}{\varepsilon} = \dfrac{8.2\,\text{kJ}}{0.31} = 26\,\text{kJ}.$

(d) Similarly, $|Q_{\rm C}| = |Q_{\rm H}| - |W| = 26\,\text{kJ} - 8.2\,\text{kJ} = 18\,\text{kJ}$.

34. All terms are assumed to be positive. The total work done by the two-stage system is $W_1 + W_2$. The heat-intake (from, say, consuming fuel) of the system is Q_1 so we have (by Eq. 20-11 and Eq. 20-8)

$$\varepsilon = \frac{W_1 + W_2}{Q_1} = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = 1 - \frac{Q_3}{Q_1}.$$

Now, Eq. 20-10 leads to

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

where we assume Q_2 is absorbed by the second stage at temperature T_2. This implies the efficiency can be written

$$\varepsilon = 1 - \frac{T_3}{T_1} = \frac{T_1 - T_3}{T_1}.$$

35. (a) The pressure at 2 is $p_2 = 3.00p_1$, as given in the problem statement. The volume is $V_2 = V_1 = nRT_1/p_1$. The temperature is

$$T_2 = \frac{p_2 V_2}{nR} = \frac{3.00\, p_1 V_1}{nR} = 3.00 T_1 \;\Rightarrow\; \frac{T_2}{T_1} = 3.00.$$

(b) The process 2 → 3 is adiabatic, so $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$. Using the result from part (a), $V_3 = 4.00V_1$, $V_2 = V_1$ and γ=1.30, we obtain

$$\frac{T_3}{T_1} = \frac{T_3}{T_2/3.00} = 3.00\left(\frac{V_2}{V_3}\right)^{\gamma-1} = 3.00\left(\frac{1}{4.00}\right)^{0.30} = 1.98\,.$$

(c) The process 4 → 1 is adiabatic, so $T_4 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$. Since $V_4 = 4.00V_1$, we have

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{1}{4.00}\right)^{0.30} = 0.660.$$

(d) The process 2 → 3 is adiabatic, so $p_2 V_2^{\gamma} = p_3 V_3^{\gamma}$ or $p_3 = \left(V_2/V_3\right)^{\gamma} p_2$. Substituting $V_3 = 4.00V_1$, $V_2 = V_1$, $p_2 = 3.00p_1$ and γ=1.30, we obtain

$$\frac{p_3}{p_1} = \frac{3.00}{(4.00)^{1.30}} = 0.495.$$

(e) The process 4 → 1 is adiabatic, so $p_4 V_4^{\gamma} = p_1 V_1^{\gamma}$ and

$$\frac{p_4}{p_1} = \left(\frac{V_1}{V_4}\right)^{\gamma} = \frac{1}{(4.00)^{1.30}} = 0.165,$$

where we have used $V_4 = 4.00V_1$.

(f) The efficiency of the cycle is $\varepsilon = W/Q_{12}$, where W is the total work done by the gas during the cycle and Q_{12} is the energy added as heat during the 1 → 2 portion of the cycle, the only portion in which energy is added as heat. The work done during the portion of the cycle from 2 to 3 is $W_{23} = \int p\, dV$. Substitute $p = p_2 V_2^{\gamma}/V^{\gamma}$ to obtain

$$W_{23} = p_2 V_2^{\gamma} \int_{V_2}^{V_3} V^{-\gamma} dV = \left(\frac{p_2 V_2^{\gamma}}{\gamma - 1}\right)\left(V_2^{1-\gamma} - V_3^{1-\gamma}\right).$$

Substitute $V_2 = V_1$, $V_3 = 4.00V_1$, and $p_3 = 3.00p_1$ to obtain

$$W_{23} = \left(\frac{3p_1V_1}{1-\gamma}\right)\left(1-\frac{1}{4^{\gamma-1}}\right) = \left(\frac{3nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

Similarly, the work done during the portion of the cycle from 4 to 1 is

$$W_{41} = \left(\frac{p_1V_1^{\gamma}}{\gamma-1}\right)\left(V_4^{1-\gamma} - V_1^{1-\gamma}\right) = -\left(\frac{p_1V_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right) = -\left(\frac{nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

No work is done during the $1 \to 2$ and $3 \to 4$ portions, so the total work done by the gas during the cycle is

$$W = W_{23} + W_{41} = \left(\frac{2nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

The energy added as heat is

$$Q_{12} = nC_V(T_2 - T_1) = nC_V(3T_1 - T_1) = 2nC_VT_1,$$

where C_V is the molar specific heat at constant volume. Now

$$\gamma = C_p/C_V = (C_V + R)/C_V = 1 + (R/C_V),$$

so $C_V = R/(\gamma - 1)$. Here C_p is the molar specific heat at constant pressure, which for an ideal gas is $C_p = C_V + R$. Thus, $Q_{12} = 2nRT_1/(\gamma - 1)$. The efficiency is

$$\varepsilon = \frac{2nRT_1}{\gamma-1}\left(1-\frac{1}{4^{\gamma-1}}\right)\frac{\gamma-1}{2nRT_1} = 1-\frac{1}{4^{\gamma-1}}.$$

With $\gamma = 1.30$, the efficiency is $\varepsilon = 0.340$ or 34.0%.

36. Eq. 20-10 still holds (particularly due to its use of absolute values), and energy conservation implies $|W| + Q_L = Q_H$. Therefore, with $T_L = 268.15$ K and $T_H = 290.15$ K, we find

$$|Q_H| = |Q_L|\left(\frac{T_H}{T_L}\right) = (|Q_H| - |W|)\left(\frac{290.15}{268.15}\right)$$

which (with $|W| = 1.0$ J) leads to $|Q_H| = |W|\left(\frac{1}{1 - 268.15/290.15}\right) = 13\,\text{J}$.

37. A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by

$$K = \frac{T_L}{T_H - T_L}.$$

For the refrigerator of this problem, $T_H = 96° \text{ F} = 309 \text{ K}$ and $T_L = 70° \text{ F} = 294 \text{ K}$, so

$$K = (294 \text{ K})/(309 \text{ K} - 294 \text{ K}) = 19.6.$$

The coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$. Thus,

$$|Q_L| = K|W| = (19.6)(1.0 \text{ J}) = 20 \text{ J}.$$

38. (a) Eq. 20-15 provides

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} \Rightarrow |Q_H| = |Q_L| \left(\frac{1 + K_C}{K_C} \right)$$

which yields $|Q_H| = 49$ kJ when $K_C = 5.7$ and $|Q_L| = 42$ kJ.

(b) From §20-5 we obtain

$$|W| = |Q_H| - |Q_L| = 49.4 \text{ kJ} - 42.0 \text{ kJ} = 7.4 \text{ kJ}$$

if we take the initial 42 kJ datum to be accurate to three figures. The given temperatures are not used in the calculation; in fact, it is possible that the given room temperature value is not meant to be the high temperature for the (reversed) Carnot cycle — since it does not lead to the given K_C using Eq. 20-16.

39. The coefficient of performance for a refrigerator is given by $K = |Q_L|/|W|$, where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$. In one hour,

$$|W| = \frac{7.54\,\text{MJ}}{3.8 + 1} = 1.57\,\text{MJ}.$$

The rate at which work is done is $(1.57 \times 10^6\ \text{J})/(3600\ \text{s}) = 440\ \text{W}$.

40. (a) Using Eq. 20-14 and Eq. 20-16, we obtain

$$|W| = \frac{|Q_L|}{K_C} = (1.0\ \text{J})\left(\frac{300\,\text{K} - 280\,\text{K}}{280\,\text{K}}\right) = 0.071\,\text{J}.$$

(b) A similar calculation (being sure to use absolute temperature) leads to 0.50 J in this case.

(c) With T_L = 100 K, we obtain $|W|$ = 2.0 J.

(d) Finally, with the low temperature reservoir at 50 K, an amount of work equal to $|W|$ = 5.0 J is required.

41. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be

$$\varepsilon = \frac{T_1 - T_2}{T_1} \quad \Rightarrow \quad \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be

$$K = \frac{T_4}{T_3 - T_4} \quad \Rightarrow \quad \frac{Q_4}{W} = \frac{T_4}{T_3 - T_4}.$$

Now $Q_4 = Q_3 - W$, so

$$(Q_3 - W)/W = T_4/(T_3 - T_4).$$

The work done by the engine is used to drive the refrigerator, so W is the same for the two. Solve the engine equation for W and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1\left(T_1 - T_2\right)} - 1.$$

Solving for Q_3/Q_1, we obtain

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1\right)\left(\frac{T_1 - T_2}{T_1}\right) = \left(\frac{T_3}{T_3 - T_4}\right)\left(\frac{T_1 - T_2}{T_1}\right) = \frac{1 - \left(T_2/T_1\right)}{1 - \left(T_4/T_3\right)}.$$

With $T_1 = 400$ K, $T_2 = 150$ K, $T_3 = 325$ K, and $T_4 = 225$ K, the ratio becomes $Q_3/Q_1 = 2.03$.

42. (a) Eq. 20-13 gives the Carnot efficiency as $1 - T_L/T_H$. This gives 0.222 in this case. Using this value with Eq. 20-11 leads to

$$W = (0.222)(750 \text{ J}) = 167 \text{ J}.$$

(b) Now, Eq. 20-16 gives $K_C = 3.5$. Then, Eq. 20-14 yields $|W| = 1200/3.5 = 343$ J.

43. We are told $K = 0.27K_C$ where

$$K_C = \frac{T_L}{T_H - T_L} = \frac{294\text{ K}}{307\text{ K} - 294\text{ K}} = 23$$

where the Fahrenheit temperatures have been converted to Kelvins. Expressed on a per unit time basis, Eq. 20-14 leads to

$$\frac{|W|}{t} = \frac{|Q_L|/t}{K} = \frac{4000\text{ Btu/h}}{(0.27)(23)} = 643\text{ Btu/h}.$$

Appendix D indicates 1 But/h = 0.0003929 hp, so our result may be expressed as $|W|/t$ = 0.25 hp.

44. The work done by the motor in t = 10.0 min is $|W| = Pt$ = (200 W)(10.0 min)(60 s/min) = 1.20×10^5 J. The heat extracted is then

$$|Q_L| = K|W| = \frac{T_L|W|}{T_H - T_L} = \frac{(270\,\text{K})(1.20\times10^5\,\text{J})}{300\,\text{K} - 270\,\text{K}} = 1.08\times10^6\,\text{J}.$$

45. We need nine labels:

Label	Number of molecules on side 1	Number of molecules on side 2
I	8	0
II	7	1
III	6	2
IV	5	3
V	4	4
VI	3	5
VII	2	6
VIII	1	7
IX	0	8

The multiplicity W is computing using Eq. 20-20. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 20-21)

$$S = k \ln W = \left(1.38\times10^{-23}\ \text{J/K}\right) \ln(56) = 5.6\times10^{-23}\ \text{J/K}.$$

In this way, we generate the following table:

Label	W	S
I	1	0
II	8	2.9×10^{-23} J/K
III	28	4.6×10^{-23} J/K
IV	56	5.6×10^{-23} J/K
V	70	5.9×10^{-23} J/K
VI	56	5.6×10^{-23} J/K
VII	28	4.6×10^{-23} J/K
VIII	8	2.9×10^{-23} J/K
IX	1	0

46. (a) We denote the configuration with n heads out of N trials as (n; N). We use Eq. 20-20:

$$W(25;50) = \frac{50!}{(25!)(50-25)!} = 1.26\times10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of N independent molecules, the total number of available states of the N-particle system is

$$N_{\text{total}} = 2\times2\times2\times\cdots\times2 = 2^N.$$

With $N = 50$, we obtain $N_{\text{total}} = 2^{50} = 1.13\times10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25;50) = \frac{W(25;50)}{2^{50}} = \frac{1.26\times10^{14}}{1.13\times10^{15}} = 11.1\%.$$

With $N = 100$, we obtain

(d) $W(N/2, N) = N!/[(N/2)!]^2 = 1.01\times10^{29}$,

(e) $N_{\text{total}} = 2^N = 1.27\times10^{30}$,

(f) and $p(N/2;N) = W(N/2, N)/ N_{\text{total}} = 8.0\%$.

Similarly, for $N = 200$, we obtain

(g) $W(N/2, N) = 9.25\times10^{58}$,

(h) $N_{\text{total}} = 1.61\times10^{60}$,

(i) and $p(N/2; N) = 5.7\%$.

(j) As N increases the number of available microscopic states increase as 2^N, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent in there decreases with an increase in N.

47. (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the right third, $n_C!$ are rearrangements of the n_C molecules in the center third, and $n_R!$ are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L!\,n_C!\,n_R!}.$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!\,(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!\,(N/3)!\,(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!\,(N/2)!}{(N/3)!\,(N/3)!\,(N/3)!}.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.2\times10^{16}.$$

48. Using Hooke's law, we find the spring constant to be

$$k = \frac{F_s}{x_s} = \frac{1.50 \text{ N}}{0.0350 \text{ m}} = 42.86 \text{ N/m} .$$

To find the rate of change of entropy with a small additional stretch, we use Eq. 20-7 (see also Sample Problem 20-3) and obtain

$$\left|\frac{dS}{dx}\right| = \frac{k\,|\,x\,|}{T} = \frac{(42.86 \text{ N/m})(0.0170 \text{ m})}{275 \text{ K}} = 2.65\times10^{-3} \text{ J/K}\cdot\text{m} .$$

49. Using Eq. 19-34 and Eq. 19-35, we arrive at

$$\Delta v = (\sqrt{3} - \sqrt{2})\sqrt{RT/M}$$

(a) We find, with M = 28 g/mol = 0.028 kg/mol (see Table 19-1), Δv_i=87 m/s at 250 K,

(b) and Δv_f=122 ≈1.2×10^2 m/s at 500 K.

(c) The expression above for Δv implies

$$T = \frac{M}{R(\sqrt{3} - \sqrt{2})^2}(\Delta v)^2$$

which we can plug into Eq. 20-4 to yield

$$\Delta S = nR\ln(V_f/V_i) + nC_V\ln(T_f/T_i) = 0 + nC_V\ln[(\Delta v_f)^2/(\Delta v_i)^2] = 2nC_V\ln(\Delta v_f/\Delta v_i).$$

Using Table 19-3 to get $C_V = 5R/2$ (see also Table 19-2) we then find, for n = 1.5 mol, ΔS = 22 J/K.

50. The net work is figured from the (positive) isothermal expansion (Eq. 19-14) and the (negative) constant-pressure compression (Eq. 19-48). Thus,

$$W_{\text{net}} = nRT_H \ln(V_{\max}/V_{\min}) + nR(T_L - T_H)$$

where $n = 3.4$, $T_H = 500$ K, $T_L = 200$ K and $V_{\max}/V_{\min} = 5/2$ (same as the ratio T_H/T_L). Therefore, $W_{\text{net}} = 4468$ J. Now, we identify the "input heat" as that transferred in steps 1 and 2:

$$Q_{\text{in}} = Q_1 + Q_2 = nC_V(T_H - T_L) + nRT_H \ln(V_{\max}/V_{\min})$$

where $C_V = 5R/2$ (see Table 19-3). Consequently, $Q_{\text{in}} = 34135$ J. Dividing these results gives the efficiency: $W_{\text{net}}/Q_{\text{in}} = 0.131$ (or about 13.1%).

51. (a) If T_H is the temperature of the high-temperature reservoir and T_L is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(800+40)\ \text{K}}{(800+273)\ \text{K}} = 0.78 \ \text{ or } \ 78\%.$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_H|$, where W is the work done by the engine and Q_H is the heat input. W is positive. Over a complete cycle, $Q_H = W + |Q_L|$, where Q_L is the heat output, so $\varepsilon = W/(W + |Q_L|)$ and $|Q_L| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_H - T_L)/T_H$, where T_H is the temperature of the high-temperature heat reservoir and T_L is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_L}{T_H - T_L} \ \text{ and } \ |Q_L| = \frac{WT_L}{T_H - T_L}.$$

The heat output is used to melt ice at temperature $T_i = -40°C$. The ice must be brought to 0°C, then melted, so

$$|Q_L| = mc(T_f - T_i) + mL_F,$$

where m is the mass of ice melted, T_f is the melting temperature (0°C), c is the specific heat of ice, and L_F is the heat of fusion of ice. Thus,

$$WT_L/(T_H - T_L) = mc(T_f - T_i) + mL_F.$$

We differentiate with respect to time and replace dW/dt with P, the power output of the engine, and obtain

$$PT_L/(T_H - T_L) = (dm/dt)[c(T_f - T_i) + L_F].$$

Therefore,

$$\frac{dm}{dt} = \left(\frac{PT_L}{T_H - T_L}\right)\left(\frac{1}{c(T_f - T_i) + L_F}\right).$$

Now, $P = 100 \times 10^6$ W, $T_L = 0 + 273 = 273$ K, $T_H = 800 + 273 = 1073$ K, $T_i = -40 + 273 = 233$ K, $T_f = 0 + 273 = 273$ K, $c = 2220$ J/kg·K, and $L_F = 333 \times 10^3$ J/kg, so

$$\frac{dm}{dt} = \left[\frac{(100\times10^6\ \text{J/s})(273\ \text{K})}{1073\ \text{K} - 273\ \text{K}}\right]\left[\frac{1}{(2220\ \text{J/kg}\cdot\text{K})(273\ \text{K} - 233\ \text{K}) + 333\times10^3\ \text{J/kg}}\right]$$
$$= 82\,\text{kg/s}.$$

We note that the engine is now operated between 0°C and 800°C.

52. (a) Combining Eq. 20-11 with Eq. 20-13, we obtain

$$|W| = |Q_{\mathrm{H}}| \left(1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}\right) = (500\,\mathrm{J}) \left(1 - \frac{260\,\mathrm{K}}{320\,\mathrm{K}}\right) = 93.8\,\mathrm{J}.$$

(b) Combining Eq. 20-14 with Eq. 20-16, we find

$$|W| = \frac{|Q_{\mathrm{L}}|}{\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{H}} - T_{\mathrm{L}}}\right)} = \frac{1000\,\mathrm{J}}{\left(\frac{260\,\mathrm{K}}{320\,\mathrm{K} - 260\,\mathrm{K}}\right)} = 231\ \mathrm{J}.$$

53. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = 40.9°\text{C},$$

which is equivalent to 314 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{copper}} = \int_{353}^{314} \frac{cm\,dT}{T} = (386)(0.600)\ln\left(\frac{314}{353}\right) = -27.1 \text{ J/K}.$$

(c) For water, the change in entropy is

$$\Delta S_{\text{water}} = \int_{283}^{314} \frac{cm\,dT}{T} = (4190)(0.0700)\ln\left(\frac{314}{283}\right) = 30.5 \text{ J/K}.$$

(d) The net result for the system is (30.5 – 27.1) J/K = 3.4 J/K. (Note: these calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

54. For an isothermal ideal gas process, we have $Q = W = nRT \ln(V_f/V_i)$. Thus,

$$\Delta S = Q/T = W/T = nR \ln(V_f/V_i)$$

(a) $V_f/V_i = (0.800)/(0.200) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34$ J/K.

(b) $V_f/V_i = (0.800)/(0.200) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34$ J/K.

(c) $V_f/V_i = (1.20)/(0.300) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34$ J/K.

(d) $V_f/V_i = (1.20)/(0.300) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34$ J/K.

55. Except for the phase change (which just uses Eq. 20-2), this has some similarities with Sample Problem 20-2. Using constants available in the Chapter 19 tables, we compute

$$\Delta S = m[c_{\text{ice}} \ln(273/253) + \frac{L_f}{273} + c_{\text{water}} \ln(313/273)] = 1.18 \times 10^3 \text{ J/K}.$$

56. Eq. 20-4 yields

$$\Delta S = nR\ln(V_f/V_i) + nC_V\ln(T_f/T_i) = 0 + nC_V\ln(425/380)$$

where $n = 3.20$ and $C_V = \frac{3}{2}R$ (Eq. 19-43). This gives 4.46 J/K.

57. (a) It is a reversible set of processes returning the system to its initial state; clearly, $\Delta S_{\text{net}} = 0$.

(b) Process 1 is adiabatic and reversible (as opposed to, say, a free expansion) so that Eq. 20-1 applies with $dQ = 0$ and yields $\Delta S_1 = 0$.

(c) Since the working substance is an ideal gas, then an isothermal process implies $Q = W$, which further implies (regarding Eq. 20-1) $dQ = p\,dV$. Therefore,

$$\int \frac{dQ}{T} = \int \frac{p\,dV}{\left(\frac{pV}{nR}\right)} = nR \int \frac{dV}{V}$$

which leads to $\Delta S_3 = nR\ln(1/2) = -23.0\ \text{J/K}$.

(d) By part (a), $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$. Then, part (b) implies $\Delta S_2 = -\Delta S_3$. Therefore, $\Delta S_2 =$ 23.0 J/K.

58. (a) The most obvious input-heat step is the constant-volume process. Since the gas is monatomic, we know from Chapter 19 that $C_V = \frac{3}{2}R$. Therefore,

$$Q_V = nC_V\Delta T = (1.0\text{ mol})\left(\frac{3}{2}\right)\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600\text{ K} - 300\text{ K}) = 3740\text{ J}.$$

Since the heat transfer during the isothermal step is positive, we may consider it also to be an input-heat step. The isothermal Q is equal to the isothermal work (calculated in the next part) because $\Delta E_{\text{int}} = 0$ for an ideal gas isothermal process (see Eq. 19-45). Borrowing from the part (b) computation, we have

$$Q_{\text{isotherm}} = nRT_{\text{H}}\ln 2 = (1\text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600\text{ K})\ln 2 = 3456\text{ J}.$$

Therefore, $Q_{\text{H}} = Q_V + Q_{\text{isotherm}} = 7.2 \times 10^3$ J.

(b) We consider the sum of works done during the processes (noting that no work is done during the constant-volume step). Using Eq. 19-14 and Eq. 19-16, we have

$$W = nRT_{\text{H}}\ln\left(\frac{V_{\max}}{V_{\min}}\right) + p_{\min}(V_{\min} - V_{\max})$$

where (by the gas law in ratio form, as illustrated in Sample Problem 19-1) the volume ratio is

$$\frac{V_{\max}}{V_{\min}} = \frac{T_{\text{H}}}{T_{\text{L}}} = \frac{600\text{ K}}{300\text{ K}} = 2.$$

Thus, the net work is

$$\begin{aligned} W &= nRT_{\text{H}}\ln 2 + p_{\min}V_{\min}\left(1 - \frac{V_{\max}}{V_{\min}}\right) = nRT_{\text{H}}\ln 2 + nRT_{\text{L}}(1-2) = nR(T_{\text{H}}\ln 2 - T_{\text{L}}) \\ &= (1\text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left((600\text{ K})\ln 2 - (300\text{ K})\right) \\ &= 9.6\times 10^2\text{ J}. \end{aligned}$$

(c) Eq. 20-11 gives

$$\varepsilon = \frac{W}{Q_{\text{H}}} = 0.134 \approx 13\%.$$

59. (a) Processes 1 and 2 both require the input of heat, which is denoted Q_H. Noting that rotational degrees of freedom are not involved, then, from the discussion in Chapter 19, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$. We further note that since the working substance is an ideal gas, process 2 (being isothermal) implies $Q_2 = W_2$. Finally, we note that the volume ratio in process 2 is simply 8/3. Therefore,

$$Q_H = Q_1 + Q_2 = nC_V\left(T' - T\right) + nRT'\ln\frac{8}{3}$$

which yields (for $T = 300$ K and $T' = 800$ K) the result $Q_H = 25.5 \times 10^3$ J.

(b) The net work is the net heat ($Q_1 + Q_2 + Q_3$). We find Q_3 from $nC_p\,(T - T') = -20.8 \times 10^3$ J. Thus, $W = 4.73 \times 10^3$ J.

(c) Using Eq. 20-11, we find that the efficiency is

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{4.73\times10^3}{25.5\times10^3} = 0.185 \text{ or } 18.5\%.$$

60. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = -44.2°\text{C},$$

which is equivalent to 229 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{tungsten}} = \int_{303}^{229} \frac{cm\,dT}{T} = (134)(0.045)\ln\left(\frac{229}{303}\right) = -1.69 \text{ J/K}.$$

(c) Also,

$$\Delta S_{\text{silver}} = \int_{153}^{229} \frac{cm\,dT}{T} = (236)(0.0250)\ln\left(\frac{229}{153}\right) = 2.38 \text{ J/K}.$$

(d) The net result for the system is (2.38 – 1.69) J/K = 0.69 J/K. (Note: these calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

61. From the formula for heat conduction, Eq. 19-32, using Table 19-6, we have

$$H = kA\frac{T_H - T_C}{L} = (401)\left(\pi(0.02)^2\right) 270/1.50$$

which yields H = 90.7 J/s. Using Eq. 20-2, this is associated with an entropy rate-of-decrease of the high temperature reservoir (at 573 K) equal to

$$\Delta S/t = -90.7/573 = -0.158 \text{ (J/K)/s}.$$

And it is associated with an entropy rate-of-increase of the low temperature reservoir (at 303 K) equal to

$$\Delta S/t = +90.7/303 = 0.299 \text{ (J/K)/s}.$$

The net result is (0.299 – 0.158) (J/K)/s = 0.141 (J/K)/s.

62. (a) Eq. 20-14 gives $K = 560/150 = 3.73$.

(b) Energy conservation requires the exhaust heat to be 560 + 150 = 710 J.

63. (a) Eq. 20-15 can be written as $|Q_H| = |Q_L|(1 + 1/K_C) = (35)(1 + \frac{1}{4.6}) = 42.6$ kJ.

(b) Similarly, Eq. 20-14 leads to $|W| = |Q_L|/K = 35/4.6 = 7.61$ kJ.

64. (a) A good way to (mathematically) think of this is: consider the terms when you expand

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$

The coefficients correspond to the multiplicities. Thus, the smallest coefficient is 1.

(b) The largest coefficient is 6.

(c) Since the logarithm of 1 is zero, then Eq. 20-21 gives S = 0 for the least case.

(d) $S = k \ln(6) = 2.47 \times 10^{-23}$ J/K.

65. (a) Eq. 20-2 gives the entropy change for each reservoir (each of which, by definition, is able to maintain constant temperature conditions within itself). The net entropy change is therefore

$$\Delta S = \frac{+|Q|}{273 + 24} + \frac{-|Q|}{273 + 130} = 4.45 \text{ J/K}$$

where we set $|Q| = 5030$ J.

(b) We have assumed that the conductive heat flow in the rod is "steady-state"; that is, the situation described by the problem has existed and will exist for "long times." Thus there are no entropy change terms included in the calculation for elements of the rod itself.

66. Eq. 20-10 gives

$$\left|\frac{Q_{\text{to}}}{Q_{\text{from}}}\right| = \frac{T_{\text{to}}}{T_{\text{from}}} = \frac{300\,\text{K}}{4.0\,\text{K}} = 75.$$

67. We adapt the discussion of §20-7 to 3 and 5 particles (as opposed to the 6 particle situation treated in that section).

(a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, using Eq. 20-20, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box *B*, $W = 5!/(5!0!) = 1$ in the "least" case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Thus,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k\ln W = \left(1.38\times10^{-23}\right)\ln 3 = 1.5\times10^{-23}\ \text{J/K}.$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k \ln 10 = 3.2 \times 10^{-23}\ \text{J/K}.$$

68. A metric ton is 1000 kg, so that the heat generated by burning 380 metric tons during one hour is $(380000 \text{ kg})(28 \text{ MJ/kg}) = 10.6\times10^6 \text{ MJ}$. The work done in one hour is

$$W = (750 \text{ MJ/s})(3600 \text{ s}) = 2.7\times10^6 \text{ MJ}$$

where we use the fact that a Watt is a Joule-per-second. By Eq. 20-11, the efficiency is

$$\varepsilon = \frac{2.7\times10^6 \text{ MJ}}{10.6\times10^6 \text{ MJ}} = 0.253 = 25\%.$$

69. Since the volume of the monatomic ideal gas is kept constant it does not do any work in the heating process. Therefore the heat Q it absorbs is equal to the change in its inertial energy: $dQ = dE_{\text{int}} = \frac{3}{2} n\,R\,dT$. Thus,

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3nR/2)\,dT}{T} = \frac{3}{2} nR \ln\left(\frac{T_f}{T_i}\right) = \frac{3}{2}(1.00\text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400\text{ K}}{300\text{ K}}\right)$$

$$= 3.59\text{ J/K}.$$

70. With the pressure kept constant,

$$dQ = nC_p dT = n(C_V + R)\,dT = \left(\frac{3}{2}nR + nR\right)dT = \frac{5}{2}nRdT,$$

so we need to replace the factor 3/2 in the last problem by 5/2. The rest is the same. Thus the answer now is

$$\Delta S = \frac{5}{2}nR\ln\left(\frac{T_f}{T_i}\right) = \frac{5}{2}(1.00\ \text{mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400\ \text{K}}{300\ \text{K}}\right) = 5.98\ \text{J/K}.$$

71. The change in entropy in transferring a certain amount of heat Q from a heat reservoir at T_1 to another one at T_2 is $\Delta S = \Delta S_1 + \Delta S_2 = Q(1/T_2 - 1/T_1)$.

(a) ΔS = (260 J)(1/100 K – 1/400 K) = 1.95 J/K.

(b) ΔS = (260 J)(1/200 K – 1/400 K) = 0.650 J/K.

(c) ΔS = (260 J)(1/300 K – 1/400 K) = 0.217 J/K.

(d) ΔS = (260 J)(1/360 K – 1/400 K) = 0.072 J/K.

(e) We see that as the temperature difference between the two reservoirs decreases, so does the change in entropy.

72. The Carnot efficiency (Eq. 20-13) depends linearly on T_L so that we can take a derivative

$$\varepsilon = 1 - \frac{T_{\rm L}}{T_{\rm H}} \Rightarrow \frac{d\varepsilon}{dT_{\rm L}} = -\frac{1}{T_{\rm H}}$$

and quickly get to the result. With $d\varepsilon \to \Delta\varepsilon = 0.100$ and $T_{\rm H}$ = 400 K, we find $dT_{\rm L} \to \Delta T_{\rm L}$ = –40 K.

73. (a) We use Eq. 20-16. For configuration *A*

$$W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{50!}{(25!)(25!)} = 1.26\times10^{14}.$$

(b) For configuration *B*

$$W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{50!}{[0.6(50)]![0.4(50)]!} = 4.71\times10^{13}.$$

(c) Since all microstates are equally probable,

$$f = \frac{W_B}{W_A} = \frac{1265}{3393} \approx 0.37.$$

We use these formulas for $N = 100$. The results are

(d) $W_A = \dfrac{N!}{(N/2)!(N/2)!} = \dfrac{100!}{(50!)(50!)} = 1.01\times10^{29}.$

(e) $W_B = \dfrac{N!}{(0.6N)!(0.4N)!} = \dfrac{100!}{[0.6(100)]![0.4(100)]!} = 1.37\times10^{28}.$

(f) and $f \; W_B/W_A \approx 0.14$.

Similarly, using the same formulas for $N = 200$, we obtain

(g) $W_A = 9.05 \times 10^{58}$,

(h) $W_B = 1.64 \times 10^{57}$,

(i) and $f = 0.018$.

(j) We see from the calculation above that f decreases as N increases, as expected.

74. (a) From Eq. 20-1, we infer $Q = \int T\,dS$, which corresponds to the "area under the curve" in a T-S diagram. Thus, since the area of a rectangle is (height)×(width), we have $Q_{1\to2} = (350)(2.00) = 700\text{J}$.

(b) With no "area under the curve" for process $2 \to 3$, we conclude $Q_{2\to3} = 0$.

(c) For the cycle, the (net) heat should be the "area inside the figure," so using the fact that the area of a triangle is ½ (base) × (height), we find

$$Q_{\text{net}} = \frac{1}{2}(2.00)(50) = 50\ \text{J}\ .$$

(d) Since we are dealing with an ideal gas (so that $\Delta E_{\text{int}} = 0$ in an isothermal process), then

$$W_{1\to2} = Q_{1\to2} = 700\ \text{J}\ .$$

(e) Using Eq. 19-14 for the isothermal work, we have

$$W_{1\to2} = nRT\ln\frac{V_2}{V_1}\ .$$

where $T = 350$ K. Thus, if $V_1 = 0.200\ \text{m}^3$, then we obtain

$$V_2 = V_1 \exp(W/nRT) = (0.200)\,e^{0.12} = 0.226\ \text{m}^3\ .$$

(f) Process $2 \to 3$ is adiabatic; Eq. 19-56 applies with $\gamma = 5/3$ (since only translational degrees of freedom are relevant, here).

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

This yields $V_3 = 0.284\ \text{m}^3$.

(g) As remarked in part (d), $\Delta E_{\text{int}} = 0$ for process $1 \to 2$.

(h) We find the change in internal energy from Eq. 19-45 (with $C_V = \frac{3}{2}R$):

$$\Delta E_{\text{int}} = nC_V(T_3 - T_2) = -1.25\times10^3\ \text{J}\ .$$

(i) Clearly, the net change of internal energy for the entire cycle is zero. This feature of a closed cycle is as true for a T-S diagram as for a p-V diagram.

(j) For the adiabatic ($2 \to 3$) process, we have $W = -\Delta E_{\text{int}}$. Therefore, $W = 1.25\times10^3$ J. Its positive value indicates an expansion.

75. Since the inventor's claim implies that less heat (typically from burning fuel) is needed to operate his engine than, say, a Carnot engine (for the same magnitude of net work), then $Q_H{}' < Q_H$ (See Fig. 20-35(a)) which implies that the Carnot (ideal refrigerator) unit is delivering more heat to the high temperature reservoir than engine X draws from it. This (using also energy conservation) immediately implies Fig. 20-35(b) which violates the second law.